## CHAPTER 5

## ABC REFERENCE FRAME CONTROLLER

### 5.1 Introduction

There are situations in which it is desirable to impress an unbalanced three-phase voltage set to an unbalanced three-phase load in order to ensure a balanced three-phase load current or to use unbalanced three-phase voltage set for voltage or current compensation in active filters in distribution lines. Some fault situations in electric machines and generators manifest as unequal phase impedances or unbalanced induced voltages. In general, four-leg inverters may be used in such applications since the phase currents are not constrained when the star point of the three-phase load is connected to a fourth neutral wire. However when the impressed unbalanced three-phase voltage set is constrained such that the load currents add to zero in star-connected loads, a three-leg inverter can be used. Under such conditions, the expressions for the three modulation signals $M_{i p}$ required to turn on or turn off the switches must be determined given the phase voltages $\mathrm{v}_{\mathrm{an}}, \mathrm{v}_{\mathrm{bn}}, \mathrm{v}_{\mathrm{cn}}$ which are not balanced in general.

This chapter proposes an algorithm to control the currents of an unbalanced threephase load using pulse-width modulation (PWM) based converters. The approach develops a controller compensation technique and modulation signals required to turn on/switch off the switches to obtain the balanced phase currents irrespective of a


Figure 5.1: Three Phase Unbalanced System.
balanced or unbalanced load. The modeling of the unbalanced load system and detailed controller design methodology are set forth.

### 5.2 Model of the Three-Phase Load

In Figure 5.1, the circuit represents a typical three-phase unbalanced system. $\mathrm{v}_{\mathrm{aN}}$, $\mathrm{v}_{\mathrm{bN}}, \mathrm{v}_{\mathrm{cN}}$ are the phase output voltages of a three phase voltage source inverter and $\mathrm{e}_{\mathrm{a}}, \mathrm{e}_{\mathrm{b}}$, and $e_{c}$ are the induced voltages in the load and the general impedances of the load are shown. This load may be balanced or unbalanced.

From the Figure 5.1 it is clear that

$$
\begin{align*}
& v_{a N}=v_{a n}+v_{n N}  \tag{5.1}\\
& v_{b N}=v_{b n}+v_{n N} \tag{5.2}
\end{align*}
$$

$$
\begin{equation*}
v_{c N}=v_{c n}+v_{n N} \tag{5.3}
\end{equation*}
$$

where

$$
\begin{align*}
& v_{a n}=i_{a} R_{a}+L_{a} p i_{a}+e_{a}  \tag{5.4}\\
& v_{b n}=i_{b} R_{b}+L_{b} p i_{b}+e_{b}  \tag{5.5}\\
& v_{c n}=i_{c} R_{c}+L_{c} p i_{c}+e_{c} . \tag{5.6}
\end{align*}
$$

Consider,

$$
\begin{equation*}
v_{a N}-v_{a N}=i_{a} R_{a}+L_{a} p i_{a}+e_{a}-\left(i_{b} R_{b}+L_{b} p i_{b}+e_{b}\right) . \tag{5.7}
\end{equation*}
$$

Similarly,

$$
\begin{equation*}
v_{b N}-v_{c N}=i_{b} R_{b}+L_{b} p i_{b}+e_{b}-\left(i_{c} R_{c}+L_{c} p i_{c}+e_{c}\right) . \tag{5.8}
\end{equation*}
$$

From the current balance equation, $i_{c}=-\left(i_{a}+i_{b}\right)$.
Substituting $\mathrm{i}_{\mathrm{c}}$ in 5.8 and simplifying

$$
\begin{equation*}
v_{b N}-v_{c N}=i_{b}\left(R_{b}+R_{c}\right)+\left(L_{b}+L_{c}\right) p i_{b}+e_{b}+i_{a} R_{c}+L_{c} p i_{a}-e_{c} . \tag{5.10}
\end{equation*}
$$

By solving the above two equations for $\mathrm{pI}_{\mathrm{a}}$ and $\mathrm{pI}_{\mathrm{b}}$
$p i_{a}=\frac{\left(-R_{a} i_{a} L_{b}-R_{a} i_{a} L_{c}-R_{c} i_{b} L_{b}-\left(v_{a N}-e_{c}\right) L_{b}-R_{c} i_{a} L_{b}+\left(v_{a N}-e_{a}\right) L_{b}+\left(v_{a N}-e_{a}\right) L_{c}-\left(v_{b N}-e_{b}\right) L_{c}+R_{b} i_{b} L_{c}\right.}{L_{a} L_{b}+L_{a} L_{c}+L_{c} L_{b}}$

$$
=\sigma_{\mathrm{b}}
$$

$p i_{b}=\frac{-\left(R_{b} i_{b} L_{b}+R_{c} i_{b} L_{a}-\left(v_{b N}-e_{b}\right) L_{a}+\left(v_{b N}-e_{b}\right) L_{a}+\left(v_{a N}-e_{a}\right) L_{c}-\left(v_{b N}-e_{b}\right) L_{c}+R_{c} i_{a} L_{a}-R_{a} i_{a} L_{c}+R_{b} i_{b} L_{c}\right.}{L_{a} L_{b}+L_{a} L_{c}+L_{c} L_{b}}$ $=\sigma_{b}$.

In case of balance load conditions i.e., $\mathrm{R}_{\mathrm{a}}=\mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\mathrm{c}}=\mathrm{R}, \mathrm{L}_{\mathrm{a}}=\mathrm{L}_{\mathrm{b}}=\mathrm{L}_{\mathrm{c}}=\mathrm{L}$, and $e_{a}=e_{b}=e_{c}=E$

$$
\begin{equation*}
p i_{a}=\frac{-3 R i_{a}+2 v_{a N}-v_{b N}-v_{c N}}{3 L}=\sigma_{a} \tag{5.12-a}
\end{equation*}
$$

$$
\begin{equation*}
p i_{b}=\frac{-3 R i_{b}+2 v_{b N}-v_{a N}-v_{c N}}{3 L}=\sigma_{\mathrm{b}} . \tag{5.12-b}
\end{equation*}
$$

### 5.3 Control Scheme

In this particular scheme the current is controlled to obtain three phase balanced load current under unbalanced load conditions. The following block diagram represents the schematic of the control scheme.

The scheme works as follows: The voltage source inverter is connected to an arbitrary load. The main criterion of this scheme is to balance the three phase currents irrespective of what the load is. The control is based on using natural variables of the system without any transformation. In the control scheme the phase currents $i_{a}, i_{b}$, and $i_{c}$ used as feedback signals are compared with the reference currents which are defined. The error current is then passed through the controller whose structure is described in the following section. The outputs of the controllers are taken as the derivatives of the phase


Figure 5.2: Block diagram of the control scheme
currents. Only two controllers are used and the third reference current is obtained from the balance equation of the current, since

$$
\begin{align*}
& i_{a}+i_{b}+i_{c}=0 \\
& \Rightarrow i_{c}=-\left(i_{a}+i_{b}\right) \\
& \text { Also, } p i_{c}=-\left(p i_{b}+p i_{c}\right) . \tag{5.13}
\end{align*}
$$

Using the outputs of the controllers and using Equations (5.4-5.6), the reference phase voltages, which are used to generate the modulating signals to be described in Section 5.5.

### 5.3.1 Structure of the Controller

The block diagram of the controller is outlined in Figure 5.3. The currents of the inverter are being controlled and hence the inverter currents are used as the feedback signals. The currents $i_{a}, i_{b}$, are taken from the inverter and are compared with the reference current signals $i_{a}^{*}, i_{b}^{*}$.

The errors of the currents are converted to positive and negative synchronous reference frames; i.e., $\theta_{x}$ and $-\theta_{x}$. Hence the output of the transformation block is


Figure 5.3: Structure of the controller

$$
\begin{align*}
& i_{q d p}=\left(i_{a}^{*}-i_{a}\right) e^{i \theta_{x}}  \tag{5.14}\\
& i_{q d n}=\left(i_{a}^{*}-i_{a}\right) e^{-i \theta_{x}} \tag{5.15}
\end{align*}
$$

where $\theta_{x}=\omega_{e} t+\theta_{x 0} ; \theta_{x 0}$ - Initial reference angle.

These signals are considered to be constant after the transformations. These signals are passed through two controllers whose transfer functions are given by $K_{p}(p)$ and $K_{n}(p)$. Hence the output of the regulators is

$$
\begin{align*}
& i_{q d p 1}=\left(i_{a}^{*}-i_{a}\right) e^{i \theta_{x}} K_{p}(p)  \tag{5.16}\\
& i_{q d n}=\left(i_{a}^{*}-i_{a}\right) e^{-i \theta_{x}} K_{n}(p) . \tag{5.17}
\end{align*}
$$

Now these signals are again transformed back to the abc reference frame with some delay angle $\phi_{1}$ i.e., $\theta_{y}=\omega_{e} t+\theta_{x 0}-\phi_{1}, \theta_{y}=-\omega_{e} t+\theta_{x 0}-\phi_{1}$. Hence the resulting signals from these transformation blocks are

$$
\begin{align*}
& i_{a p}=\left(i_{a}^{*}-i_{a}\right) e^{i\left(\theta_{x}-\theta_{y}\right)} K_{p}(p-j \omega)  \tag{5.18}\\
& i_{a n}=\left(i_{a}^{*}-i_{a}\right) e^{-i\left(\theta_{x}-\theta_{y}\right)} K_{n}(p+j \omega) . \tag{5.19}
\end{align*}
$$

Let $\theta_{x}-\theta_{y}=\phi_{1}$, then

$$
\begin{align*}
& i_{a p}=\left(i_{a}^{*}-i_{a}\right) e^{i \phi_{1}} K_{p}(p-j \omega)  \tag{5.20}\\
& i_{a n}=\left(i_{a}^{*}-i_{a}\right) e^{-i \phi_{1}} K_{n}(p+j \omega) . \tag{5.21}
\end{align*}
$$

The sum of the two signals $i_{\text {ap }}, i_{\text {an }}$ is taken to get the output of the controller, which is equal to $\mathrm{pi}_{\mathrm{a}}$.

$$
\begin{aligned}
& p i_{a}=\left(i_{a}^{*}-i_{a}\right) e^{i \phi_{1}} K_{p}(p-j \omega)+\left(i_{a}^{*}-i_{a}\right) e^{-i \phi_{1}} K_{n}(p+j \omega) \\
& \Rightarrow\left(i_{a}^{*}-i_{a}\right)\left[e^{i \phi_{1}} K_{p}(p-j \omega)+e^{-i \phi_{1}} K_{n}(p+j \omega)\right]
\end{aligned}
$$

By simplifying the above equation, the transfer function of the system is obtained as

$$
\begin{equation*}
\frac{i_{a}}{i_{a}^{*}}=\frac{e^{i \phi_{1}} K_{p}(p-j \omega)+e^{-i \phi_{1}} K_{n}(p+j \omega)}{p+e^{i \phi_{1}} K_{p}(p-j \omega)+e^{-i \phi_{1}} K_{n}(p+j \omega)} . \tag{5.22}
\end{equation*}
$$

In this particular case, assume the controller to be a PI controller whose transfer function is given as

$$
\begin{align*}
& K_{p}(p)=k_{p}+\frac{k_{i p}}{p}  \tag{5.23}\\
& K_{n}(p)=k_{n}+\frac{k_{i n}}{p} .
\end{align*}
$$

Hence, by substituting the above transfer functions in Eq. (5.22) and simplifying $\frac{i_{a}}{i_{a}^{*}}=\frac{e^{i \phi_{1}}(p+j \omega)\left[k_{p}(p-j \omega)+k_{i p}\right]+e^{-i \phi_{1}}(p-j \omega)\left[k_{n}(p+j \omega)+k_{i n}\right]}{p\left(p^{2}+\omega^{2}\right)+e^{i \phi_{1}}(p+j \omega)\left[k_{p}(p-j \omega)+k_{i p}\right]+e^{-i \phi_{1}}(p-j \omega)\left[k_{n}(p+j \omega)+k_{i n}\right]}$.

For simplicity, if $k_{i p}=k_{i n}=k_{i} ; k_{n}=k_{p}=k_{p}$, then

$$
\begin{equation*}
\frac{i_{a}}{i_{a}^{*}}=\frac{p^{2} 2 k_{p} \cos \phi_{1}+p 2 k_{i} \cos \phi_{1}+2 \cos \phi_{1} \omega^{2} k_{p}-2 k_{i} \omega \sin \phi_{1}}{p^{3}+p^{2} 2 k_{p} \cos \phi_{1}+p\left[2 k_{i} \cos \phi_{1}+\omega^{2}\right]+2 \cos \phi_{1} \omega^{2} k_{p}-2 k_{i} \omega \sin \phi_{1}} . \tag{5.24}
\end{equation*}
$$

In designing the parameters of the controller, compare the denominator of the transfer function with Butterworth Polynomial. The Butterworth polynomial for the third order is as follows

$$
\begin{equation*}
p^{3}+2 p^{2} w_{0}+2 p w_{0}^{2}+w_{0}^{3}=0 . \tag{5.25}
\end{equation*}
$$

Hence by comparing the denominator of the transfer function with above polynomial we get

$$
k_{p}=\frac{w_{0}}{\cos \phi_{1}} ; \quad k_{i}=\frac{2 w_{0}^{2}-\omega^{2}}{2 \cos \phi_{1}} ; w_{0}=\left(2 \cos \phi_{1} \omega^{2} k_{p}-2 k_{i} \omega \sin \phi_{1}\right)^{1 / 3} .
$$



Figure 5.4: I. Effect of the delay angle I. Bode plot (magnitude and phase plots) II. Time response of the system


Figure 5.5: I. Effect of the delay angle I. Root locus of the system for angle $60^{\circ}$ II. Root locus of the system for angle $150^{\circ}$.

I

II

Figure 5.6 Effect of the delay angle I. Variation of the controller parameters $\mathrm{K}_{\mathrm{p}}, \mathrm{K}_{\mathrm{i}} \mathrm{II} . \omega_{0}$


Figure 5.7: Effect of the delay angle on the zeros of the transfer function.

### 5.3.2 Effect of the Delay Angle

Figure 5.4 shows the bode plot of the transfer function for different values of the delay angle. The range of the delay angle is [-pi / 2, pi / 2] and beyond the angle ranges the system become unstable. As seen from Figure 5.4 (I), the phase angle plot, it can be observed that the phase margin becomes negative and which is a situation for instability. Also from the plots, the amount of phase angle added by different delay angles can be observed. Figure 5.4 (II) shows the time response of the system and the effect of the delay angle.

Figure 5.5 shows the root locus of the transfer function and it can be seen for delay angle of $150^{\circ}$ the poles of the system lie on the right half of the plane i.e., the system is unstable. For the delay angle $60^{\circ}$ the poles of the system are on the left half and these poles shift their position with delay angle and hence by varying the delay angle the control characteristics of the system can be changed. Figure 5.6 shows the variation of the control parameters with the variation in the delay angle. The parameter $\mathrm{K}_{\mathrm{i}}$ is scaled down by a factor of 5000 and plotted. Hence choosing any particular point as the operating point, the transfer function of the controller is determined and used to control the variables. Figure 5.7 shows the placement of the zeros of the system with the variation of the delay angle. The zeros of the system are obtained by solving the numerator of the transfer function and these zeros determine the bandwidth of the controller. From Figure 5.7 it can be seen that for delay angle of $120^{\circ}$, the zeros of the system are on the right half of the plane, which signifies that the system is unstable. Hence the choice of the delay angle determines the bandwidth and the stability of the system.

### 5.3.3 Transfer Function of the Controller

To find the transfer function of the controller $\mathrm{G}_{\mathrm{AC}}(\mathrm{s})$, from Figure 5.3

$$
G_{A C}(s)=\frac{p i_{a}}{i_{a}^{*}-i_{a}}
$$

From Figure 5.3

$$
\begin{equation*}
\frac{p i_{a}}{i_{a}^{*}-i_{a}}=\left[e^{i \phi_{1}} K_{p}(p-j \omega)+e^{-i \phi_{1}} K_{n}(p+j \omega)\right] . \tag{5.26}
\end{equation*}
$$

Let

$$
\begin{aligned}
& K_{p}(p)=k_{p}+\frac{k_{i p}}{p} \\
& K_{n}(p)=k_{n}+\frac{k_{i n}}{p} .
\end{aligned}
$$

Hence by substituting the transfer functions of the PI controllers and assuming $k_{i p}=k_{i n}=k_{i} ; k_{n}=k_{p}=k_{p}$ and simplifying

$$
\begin{equation*}
G_{A C}(s)=\frac{p I_{a}}{I_{a}^{*}-I_{a}}=\frac{p^{2} 2 k_{p} \cos \phi_{1}+p 2 k_{i} \cos \phi_{1}+2 k_{p} \cos \phi_{1} \omega^{2}}{p^{2}+\omega^{2}} . \tag{5.27}
\end{equation*}
$$

The $k_{p}$ and $k_{i}$ values are obtained as explained above and therefore the controller is used to control the current.

### 5.4 Generation of the Modulation Signals

Now that the output of the controllers is known; i.e., $\mathrm{pI}_{\mathrm{a}}, \mathrm{pI}_{\mathrm{b}}$ and the third signal $p I_{c}$ is obtained from the balance current equation. From (5.11) and Figure 5.1,

$$
\begin{align*}
& v_{a n}=i_{a} R_{a}+L_{a} \sigma_{a}+e_{a}  \tag{5.28}\\
& v_{b n}=i_{b} R_{b}+L_{b} \sigma_{b}+e_{b}  \tag{5.29}\\
& v_{c n}=i_{c} R_{c}+L_{c} \sigma_{c}+e_{c} . \tag{5.30}
\end{align*}
$$

Also,

$$
\begin{align*}
& v_{a N}=v_{a n}+V_{n N}  \tag{5.31}\\
& v_{b N}=v_{b n}+V_{n N}  \tag{5.32}\\
& v_{c N}=v_{c n}+V_{n N} . \tag{5.33}
\end{align*}
$$

where $\mathrm{v}_{\mathrm{aN}}, \mathrm{v}_{\mathrm{bN}}$, and $\mathrm{v}_{\mathrm{cN}}$ are the output voltages of the inverter.

$$
\begin{align*}
& \frac{V_{d c}}{2}\left(2 S_{a p}-1\right)=i_{a} R_{a}+L_{a} p i_{a}+e_{a}+V_{n N}  \tag{5.34}\\
& \frac{V_{d c}}{2}\left(2 S_{b p}-1\right)=i_{b} R_{b}+L_{b} p i_{b}+e_{b}+V_{n N}  \tag{5.35}\\
& \frac{V_{d c}}{2}\left(2 S_{c p}-1\right)=i_{c} R_{c}+L_{c} p i_{c}+e_{c}+V_{n N} \tag{5.36}
\end{align*}
$$

The switching function can be expressed approximately as a function of the modulation signals such that

$$
\begin{aligned}
& S_{a p}=\frac{1+m_{a p}}{2} \\
& \Rightarrow m_{a p}=2 S_{a p}-1
\end{aligned}
$$

Similarly

$$
\begin{aligned}
& m_{b p}=2 S_{b p}-1 \\
& m_{c p}=2 S_{c p}-1
\end{aligned}
$$

Hence the modulation signals are given by

$$
\begin{align*}
& m_{a p}=\frac{v_{a n}+V_{n N}}{V_{d} / 2}  \tag{5.37}\\
& m_{b p}=\frac{v_{b n}+V_{n N}}{V_{d} / 2}  \tag{5.38}\\
& m_{c p}=\frac{v_{c n}+V_{n N}}{V_{d} / 2} \tag{5.39}
\end{align*}
$$

Where $\mathrm{v}_{\mathrm{an}}, \mathrm{v}_{\mathrm{bn}}$, and $\mathrm{v}_{\mathrm{cn}}$ are obtained from Equation (5.28).

### 5.5 Continuous PWM for Unbalanced Load Voltages

The expressions for the three modulation signals $\mathrm{M}_{\mathrm{ip}}(\mathrm{I}=\mathrm{abc})$ from (5.37-5.39) must be determined by the given phase voltages, which are not balanced in general. There are three linear independent equations to be solved to determine expressions for three unknown modulation signals and $\mathrm{V}_{\mathrm{nN}}$. These equations are under-determined. In view of this indeterminacy, there is an infinite number of solutions which are obtained by various optimizing performance functions defined in terms of the modulation functions. For a set of linear indeterminate equations expressed as $\mathrm{AX}=\mathrm{Y}$, a solution which minimizes the sum of squares of the variable X is obtained using the Moore-Penrose inverse [84]. The solution is given as $X=A^{T}\left[A A^{T}\right]^{-1} Y$. The solution is for the minimization of the sum of the squares of the equally weighted three modulation signals and the square of the normalized neutral voltage $\left(\mathrm{V}^{*}{ }_{\mathrm{nN}}=2 \mathrm{~V}_{\mathrm{nN}} / \mathrm{V}_{\mathrm{d}}\right)$. Equivalently, this is the maximization of the inverter output-input voltage gain, i.e., $\mathrm{M}_{\mathrm{ap}}{ }^{2}+\mathrm{M}_{\mathrm{bp}}{ }^{2}+\mathrm{M}_{\mathrm{cp}}{ }^{2}+\mathrm{V}_{\mathrm{nN}}{ }^{2}$ subject to the constraints in (5.1-5.3). The resulting expressions for the modulation signals are given as

$$
\begin{align*}
& m_{a p}=\frac{1}{2 * V_{d}}\left(3 * v_{a n}-v_{b n}-v_{c n}\right)  \tag{5.40}\\
& m_{b p}=\frac{1}{2 * V_{d}}\left(3 * v_{b n}-v_{c n}-v_{a n}\right)  \tag{5.41}\\
& m_{c p}=\frac{1}{2 * V_{d}}\left(3 * v_{c n}-v_{b n}-v_{a n}\right)  \tag{5.42}\\
& V_{n N}=\frac{1}{2 * V_{\mathrm{d}}}\left(-\mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{cn}}\right) . \tag{5.43}
\end{align*}
$$

### 5.6 Discontinuous PWM for Unbalanced Voltages

An alternative carrier-based discontinuous modulation scheme is obtained by using the space vector methodology to determine the expression for $\mathrm{V}_{\mathrm{nN}}$ in (5.31-5.33). For the unbalanced voltage set, the reference three-phase voltages mapped unto the stationary reference frame $\left(\mathrm{V}_{\mathrm{qdpn}}\right)$ has in addition to the qp and dp voltage components, the nonzero, zero sequence voltage, $\mathrm{V}_{\mathrm{op}}$. This reference voltage is approximated by the time-average over a sampling period (converter switching period, $\mathrm{T}_{\mathrm{s}}$ ) of the two adjacent active states $\left(\mathrm{V}_{\mathrm{qdpna}}, \mathrm{V}_{\mathrm{qdpnb}}\right)$ and the two zero states $\left(\mathrm{V}_{\mathrm{qdpn} 0}, \mathrm{~V}_{\mathrm{qdpn} 7}\right)$. The expressions for the reference average neutral voltage, time-averaging the neutral voltages of the two active also approximates $\mathrm{V}_{\mathrm{nN}}$ and two null modes as expressed in (5.46).

The turn-on and turn-off sequences of any of the switching transistors of the three-phase voltage source inverter shown in Figure 5.1 are represented by an existence function, which has a value of unity when it is turned on and becomes zero when it is turned off. In general, an existence function of a two-level converter is represented by $\mathbf{S}_{\mathbf{i j}}$ , $i=a, b, c$, and $j=p, n$, where $i$ represents the load phase to which the device is connected, and $j$ signifies top (p) and bottom (n) device of an inverter leg. Hence $\mathbf{S}_{\mathbf{a p}}$, $\mathbf{S}_{\mathbf{a n}}$ which take values of zero or unity are, respectively, the existence functions of the top device ( $\mathbf{T}_{\mathrm{ap}}$ ) and bottom device ( $\mathbf{T}_{\mathbf{a n}}$ ) of the inverter leg which are connected to phase ' a ' load. In order to prevent short-circuiting the inverter DC source and thereby inviolate the Kirchoff's voltage law, $\mathbf{T}_{\mathbf{i p}}$ and $\mathbf{T}_{\mathbf{i n}}$ cannot be turned on at the same time. Kirchoff's voltage law constraints the existence functions such that $\mathbf{S}_{\mathbf{i p}}+\mathbf{S}_{\mathbf{i n}}=1$, hence :

$$
\begin{equation*}
0.5 \mathrm{~V}_{\mathrm{d}}\left(2 \mathrm{~S}_{\mathrm{ap}}-1\right)=\mathrm{v}_{\mathrm{an}}+\mathrm{V}_{\mathrm{nN}} \cong 0.5 \mathrm{~V}_{\mathrm{d}} \mathrm{M}_{\mathrm{ap}} \tag{5.44}
\end{equation*}
$$

$$
\begin{align*}
& 0.5 \mathrm{~V}_{\mathrm{d}}\left(2 \mathrm{~S}_{\mathrm{bp}}-1\right)=\mathrm{V}_{\mathrm{bn}}+\mathrm{V}_{\mathrm{nN}} \cong 0.5 \mathrm{~V}_{\mathrm{d}} \mathrm{M}_{\mathrm{bp}}  \tag{5.45}\\
& 0.5 \mathrm{~V}_{\mathrm{d}}\left(2 \mathrm{~S}_{\mathrm{cp}}-1\right)=\mathrm{v}_{\mathrm{cn}}+\mathrm{V}_{\mathrm{nN}} \cong 0.5 \mathrm{~V}_{\mathrm{d}} \mathrm{M}_{\mathrm{cp}} \tag{5.46}
\end{align*}
$$

In Equations (5.44-5.46), $\mathrm{V}_{\mathrm{an}}, \mathrm{V}_{\mathrm{bn},}, \mathrm{V}_{\mathrm{cn}}$ are the phase voltages of the load while the voltage of the load neutral to inverter reference is $\mathrm{V}_{\mathrm{nN}}$. The voltage equations expressed in terms of the modulation signals in (5.44-5.46) are facilitated by the Fourier series approximation of the existence functions, which are approximated as

$$
\begin{equation*}
\mathrm{S}_{\mathrm{ap}} \cong 0.5\left(1+\mathrm{M}_{\mathrm{ap}}\right), \quad \mathrm{S}_{\mathrm{bp}} \cong 0.5\left(1+\mathrm{M}_{\mathrm{bp}}\right), \mathrm{S}_{\mathrm{cp}} \cong 0.5\left(1+\mathrm{M}_{\mathrm{cp}}\right) \tag{5.47}
\end{equation*}
$$

where $\mathrm{M}_{\mathrm{ap}}, \mathrm{M}_{\mathrm{bp}}, \mathrm{M}_{\mathrm{cp}}$ which range between -1 and 1 (for the linear modulation range) are the carrier-based modulation waveforms comprising of fundamental frequency components. In general, the actual existence functions are usually generated by comparing the high frequency triangle waveform, which ranges between -1 and 1 with the modulation waveforms $\left(\mathrm{M}_{\mathrm{ap}}, \mathrm{M}_{\mathrm{bp}}, \mathrm{M}_{\mathrm{cp}}\right)$. The equations for the modulating signals of the top devices from (5.44-5.46) are expressed as

$$
\begin{equation*}
\mathrm{M}_{\mathrm{ip}}=2 \mathrm{v}_{\mathrm{in}} / \mathrm{V}_{\mathrm{d}}+2 \mathrm{~V}_{\mathrm{nN}} / \mathrm{V}_{\mathrm{d}} \quad \mathrm{i}=\mathrm{a}, \mathrm{~b}, \mathrm{c} . \tag{5.48}
\end{equation*}
$$

Table 5.1: Switching modes of the three-phase voltage source inverter and corresponding stationary reference frame qdpn voltages.

| Mode | $\mathbf{S a p}_{\text {ap }}$ | $\mathrm{S}_{\mathrm{bp}}$ | $\mathrm{S}_{\text {cp }}$ | $\mathbf{V}_{\mathbf{q s}}$ | $\mathrm{V}_{\mathrm{ds}}$ | $\mathrm{V}_{\text {os }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{U}_{0}$ | 0 | 0 | 0 | 0 | 0 | $-\mathrm{V}_{\mathrm{d}} / 2$ |
| $\mathrm{U}_{1}$ | 0 | 0 | 1 | $-\mathrm{V}_{\mathrm{d}} / \sqrt{ } 3$ | $\mathrm{V}_{\mathrm{d}} / \sqrt{ } 3$ | $-\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{2}$ | 0 | 1 | 0 | $-\mathrm{V}_{\mathrm{d}} / 3$ | $-\mathrm{V}_{\mathrm{d}} / \sqrt{ } 3$ | $-\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{3}$ | 0 | 1 | 1 | $-2 \mathrm{~V}_{\mathrm{d}} / 3$ | 0 | $\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{4}$ | 1 | 0 | 0 | $2 \mathrm{~V}_{\mathrm{d}} / 3$ | 0 | $-\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{5}$ | 1 | 0 | 1 | $\mathrm{V}_{\mathrm{d}} / 3$ | $-\mathrm{V}_{\mathrm{d}} / \sqrt{ } 3$ | $\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{6}$ | 1 | 1 | 0 | $\mathrm{V}_{\mathrm{d}} / 3$ | $\mathrm{V}_{\mathrm{d}} / \sqrt{ } 3$ | $\mathrm{V}_{\mathrm{d}} / 6$ |
| $\mathrm{U}_{7}$ | 1 | 1 | 1 | 0 | 0 | $\mathrm{V}_{\mathrm{d}} / 2$ |



Figure 5.8: Voltage Space Vector diagram including zero sequence voltages.

The average neutral voltage, $\mathrm{V}_{\mathrm{nN}}$ is determined using the method of space vector modulation. The eight feasible switching modes for the three-phase voltage source inverter are enumerated in Table 5.1.

The stationary reference frame qd and neutral voltages of the switching modes given in Figure 5.4 are expressed in the complex variable form as $\left(a=e^{\mathrm{j} \varsigma}, \varsigma=120^{\circ}\right)$ : $\mathrm{V}_{\mathrm{qdp}}=2 / 3\left(\mathrm{v}_{\mathrm{an}}+\mathrm{a}_{\mathrm{bn}}+\mathrm{a}^{2} \mathrm{v}_{\mathrm{cn}}\right), \quad \mathrm{V}_{\mathrm{qp}}=\mathrm{V}_{\mathrm{d}} / 3\left(2 \mathrm{~S}_{\mathrm{ap}}-\mathrm{S}_{\mathrm{bp}}-\mathrm{S}_{\mathrm{cp}}\right), \quad \sqrt{3} \mathrm{~V}_{\mathrm{dp}}=\mathrm{S}_{\mathrm{cp}}-\mathrm{S}_{\mathrm{bp}}$ $\mathrm{V}_{\mathrm{pn}}=\mathrm{V}_{\mathrm{op}}+\mathrm{V}_{\mathrm{d}} / 3\left(\mathrm{~S}_{\mathrm{ap}}+\mathrm{S}_{\mathrm{bp}}+\mathrm{S}_{\mathrm{cp}}\right)-\mathrm{V}_{\mathrm{d}} / 2, \quad \mathrm{~V}_{\mathrm{op}}=-1 / 3\left(\mathrm{~V}_{\mathrm{an}}+\mathrm{V}_{\mathrm{bn}}+\mathrm{V}_{\mathrm{cn}}\right)$.

Note that $\mathrm{V}_{\text {op }}$ is equal to zero only when the reference voltage set is balanced. In general, the three-phase voltages expressed in the stationary reference frame $\left(\mathrm{V}_{\mathrm{qdpn}}{ }^{*} \Rightarrow \mathrm{~V}_{\mathrm{qp}}{ }^{*}, \mathrm{~V}_{\mathrm{dp}}{ }^{*}\right.$, $\mathrm{V}_{\mathrm{pn}}{ }^{*}$ ) situated in the appropriate sector in Figure 5.8 are approximated by the timeaverage over a sampling period (converter switching period, $\mathrm{T}_{\mathrm{s}}$ ), of the two adjacent active voltage inverter vectors and the two zero states $\mathrm{U}_{0}$ and $\mathrm{U}_{7}$. The switching turn-on times of the two active and two null states are utilized to determine the duty cycle
information to program the active switch gate signals. When the inverter is operating in the linear modulation region, the sum of the times the two active switching modes are utilized is less than the switching period; in which case the remaining time is occupied by using the two null vectors, $\mathrm{U}_{0}$ and $\mathrm{U}_{7}$. If the four voltage vectors $\mathrm{V}_{\mathrm{qdpna}}, \mathrm{V}_{\mathrm{qdpnb}}, \mathrm{V}_{\mathrm{qdpn} 0}$, $\mathrm{V}_{\mathrm{qdpn} 7}$ are called into play for times $\mathrm{t}_{\mathrm{a}}, \mathrm{t}_{\mathrm{b}}, \mathrm{t}_{0}, \mathrm{t}_{7}$ (normalized with respect to modulator sampling time or converter switching period, $\mathrm{T}_{\mathrm{s}}$ ), respectively, then the qp and dp components of the reference voltage $\mathrm{V}_{\mathrm{qdp}}{ }^{*}$ are approximated as
$\mathrm{V}_{\mathrm{qdp}}{ }^{*}=\mathrm{V}_{\mathrm{qp}}{ }^{*}+\mathrm{j} \mathrm{V}_{\mathrm{dp}}{ }^{*}=\mathrm{V}_{\mathrm{qdpa}} \mathrm{t}_{\mathrm{a}}+\mathrm{V}_{\mathrm{qdpb}} \mathrm{t}_{\mathrm{b}}+\mathrm{V}_{\mathrm{qdp} 0} \mathrm{t}_{0}+\mathrm{V}_{\mathrm{qdp} 7} \mathrm{t}_{7}, \mathrm{t}_{\mathrm{c}}=\mathrm{t}_{0}+\mathrm{t}_{7}=1-\mathrm{t}_{\mathrm{a}}-\mathrm{t}_{\mathrm{b}}$.

When separated into real and imaginary parts, (1.44) gives the expressions for $t_{a}$ and $t_{b}$ as $\mathrm{t}_{\mathrm{a}}=\nabla\left[\mathrm{V}_{\mathrm{qp}}{ }^{*} \mathrm{~V}_{\mathrm{dpb}}-\mathrm{V}_{\mathrm{dp}}{ }^{*} \mathrm{~V}_{\mathrm{qpb}}\right], \quad \mathrm{t}_{\mathrm{b}}=\nabla\left[\mathrm{V}_{\mathrm{dp}}{ }^{*} \mathrm{~V}_{\mathrm{qpa}}-\mathrm{V}_{\mathrm{qp}}{ }^{*} \mathrm{~V}_{\mathrm{dpa}}\right], \nabla=\left[\mathrm{V}_{\mathrm{dpb}} \mathrm{V}_{\mathrm{qpa}}-\mathrm{V}_{\mathrm{qpb}} \mathrm{V}_{\mathrm{dpa}}\right]^{-1}$.

It is observed that both $\mathrm{V}_{\mathrm{qdp} 0}$ and $\mathrm{V}_{\mathrm{qdp} 7}$ do not influence the values of $\mathrm{t}_{\mathrm{a}}$ and $\mathrm{t}_{\mathrm{b}}$. The times $t_{a}$ and $t_{b}$ expressed in terms of the instantaneous line-line reference voltages are given in Table 5.2 for the six sectors.

The neutral voltage $\mathrm{V}_{\mathrm{nN}}$ averaged over the switching period Ts is given as

$$
\begin{equation*}
\left\langle\mathrm{V}_{\mathrm{nN}}>=\mathrm{V}_{\mathrm{oa}} \mathrm{t}_{\mathrm{a}}+\mathrm{V}_{\mathrm{ob}} \mathrm{t}_{\mathrm{b}}+\mathrm{V}_{\mathrm{o0} 0} \mathrm{t}_{0}+\mathrm{V}_{\mathrm{o} 7} \mathrm{t}_{7}-\mathrm{V}_{\mathrm{op}} .\right. \tag{5.52}
\end{equation*}
$$

It should be noted that $t_{c}$ is partitioned into dwell times for the two null voltage vector $t_{c} \alpha$ for $U_{o}$ and $t_{c}(1-\alpha)$ for $U_{7}$. The averaged neutral voltages for reference voltages in the voltage sectors calculated using (5.52) are shown in Table 5.3.

Table 5.2: Device switching times expressed in terms of reference line-line voltages

| Sector |  | II | III | IV | V | VI |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{V}_{\mathrm{d}} \mathrm{t}_{\mathrm{a}}$ | $\mathrm{v}_{\mathrm{ac}}$ | $\mathrm{v}_{\mathrm{ab}}$ | $\mathrm{v}_{\mathrm{cb}}$ | $\mathrm{v}_{\mathrm{ca}}$ | $\mathrm{v}_{\mathrm{ba}}$ | $\mathrm{V}_{\mathrm{bc}}$ |
| $\mathrm{V}_{\mathrm{d}} \mathrm{t}_{\mathrm{b}}$ | $\mathrm{v}_{\mathrm{cb}}$ | $\mathrm{v}_{\mathrm{ca}}$ | $\mathrm{v}_{\mathrm{ba}}$ | $\mathrm{v}_{\mathrm{bc}}$ | $\mathrm{v}_{\mathrm{ac}}$ | $\mathrm{v}_{\mathrm{ab}}$ |
| Max voltage | $\mathrm{v}_{\mathrm{ap}}$ | $\mathrm{v}_{\mathrm{bp}}$ | $\mathrm{v}_{\mathrm{bp}}$ | $\mathrm{v}_{\mathrm{cp}}$ | $\mathrm{v}_{\mathrm{cp}}$ | $\mathrm{V}_{\mathrm{ap}}$ |
| Min Voltage | $\mathrm{v}_{\mathrm{cp}}$ | $\mathrm{v}_{\mathrm{cp}}$ | $\mathrm{v}_{\mathrm{ap}}$ | $\mathrm{v}_{\mathrm{ap}}$ | $\mathrm{v}_{\mathrm{bp}}$ | $\mathrm{v}_{\mathrm{bp}}$ |

Table 5.3 gives the expression for the averaged neutral voltage $<\mathrm{V}_{\mathrm{nN}}>$ for the six sectors of the space vector. Hence, given the unbalanced voltage set at any instant, $\mathrm{V}^{*}{ }_{\mathrm{qdo}}$ in the stationary reference frame is found and the sector in which $\mathrm{V}_{\mathrm{qd}}{ }^{*}$ is located is determined. The expression for $\mathrm{V}_{\mathrm{nN}}$ is then selected and is subsequently used in (5.375.39) to determine the modulation signals for the three top devices.

Hence, given the unbalanced voltage set at any instant, $V_{q d p}^{*}$ in the stationary reference frame is found and the sector (Table 5.3) in which it is located is determined.

Table 5.3: Expressions for the neutral voltage for the six sectors

| Sector | Neutral Voltage $<\mathrm{V}_{\mathrm{nN}} \gg$ |
| :---: | :---: |
| VI | $\left(2 \mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{cn}}\right) / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{v}_{\mathrm{cn}}-\mathrm{v}_{\mathrm{an}}\right]$ |
| V | $\left(2 \mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{cn}}\right) / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{v}_{\mathrm{cn}}-\mathrm{v}_{\mathrm{bn}}\right]$ |
| IV | $\left(2 \mathrm{v}_{\mathrm{cn}}-\mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{bn})} / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{bn}}\right]\right.$ |
| III | $\left(2 \mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{cn}}\right) / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{van}^{-}-\mathrm{v}_{\mathrm{cn}}\right]$ |
| II | $\left(2 \mathrm{v}_{\mathrm{an}}-\mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{cn}}\right) / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{v}_{\mathrm{bn}}-\mathrm{v}_{\mathrm{n}}\right]$ |
| I | $\left(2 \mathrm{v}_{\mathrm{n}}-\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{\mathrm{n}}\right) / 6+0.5 \mathrm{~V}_{\mathrm{d}}(1-2 \alpha)+0.5(1-2 \alpha)\left[\mathrm{v}_{\mathrm{n}}-\mathrm{v}_{\mathrm{n}}\right]$ |

The corresponding expression for $\mathrm{V}_{\mathrm{nN}}$ is then selected from Table 5.1 and is subsequently used in (5.37-5.39) to determine the modulation signals for the three top devices.

### 5.7 Generalized Discontinuous Carrier-based PWM Scheme for Balanced and Unbalanced Voltage Source Inverter

This section of the chapter presents the unified theory for the discontinuous carrier-based PWM methodology for balanced and unbalanced voltage source inverter while retaining the benefits of the generalized discontinuous modulation. This scheme eliminates the computational burden that is being caused in case of the unbalanced situation, which was observed in the above methodology; i.e., in case of the unbalanced situation, the zero sequence voltage $\mathrm{V}_{\mathrm{nN}}$ has to be calculated in each of the sectors as the magnitude varies depending on where the reference vector lies. The different expression of the zero sequence voltage is tabulated in Table 5.3. In the present case a generalized zero sequence voltage expression is being derived which is applicable for both the balanced and unbalanced conditions.

Under balanced conditions three-phase reference voltage set, Table 5.4, the average neutral voltage expressions for the six sectors unifies to Eq. (5.47)

$$
\begin{equation*}
<V_{n N}>=0.5(1-2 \alpha)-\alpha v_{\min }+v_{\max }(\alpha-1) . \tag{5.47}
\end{equation*}
$$

When the zero sequence component of the voltage is removed from each of the three-phase unbalanced fundamental voltages, the resulting set of three-phase signals is

Table 5.4: Device Switching times expressed in terms of reference line-line Voltages

| Sector | I | II | III | IV | V | VI |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{V}_{\mathrm{d}} \mathrm{t}_{\mathrm{a}}$ | $\mathrm{v}_{\mathrm{ac}}$ | $\mathrm{v}_{\mathrm{ab}}$ | $\mathrm{v}_{\mathrm{cb}}$ | $\mathrm{v}_{\mathrm{ca}}$ | $\mathrm{v}_{\mathrm{ba}}$ | $\mathrm{v}_{\mathrm{bc}}$ |
| $\mathrm{V}_{\mathrm{d}} \mathrm{t}_{\mathrm{b}}$ | $\mathrm{v}_{\mathrm{cb}}$ | $\mathrm{v}_{\mathrm{ca}}$ | $\mathrm{v}_{\mathrm{ba}}$ | $\mathrm{v}_{\mathrm{bc}}$ | $\mathrm{v}_{\mathrm{ac}}$ | $\mathrm{v}_{\mathrm{ab}}$ |
| Max voltage | $\mathrm{v}_{\mathrm{ap}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{bp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{bp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{cp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{cp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{V}_{\mathrm{ap}}-\mathrm{V}_{\mathrm{nN}}$ |
| Min Voltage | $\mathrm{v}_{\mathrm{cp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{cp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{ap}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{ap}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{bp}}-\mathrm{V}_{\mathrm{nN}}$ | $\mathrm{v}_{\mathrm{bp}}-\mathrm{V}_{\mathrm{nN}}$ |

balanced with $120^{\circ}$ symmetry. This property makes it possible to locate the sectors of qd components of unbalanced voltage as shown in Table 5.5. Hence if the unbalanced phase voltages are instantaneously diminished by the value of the zero sequence voltage, the modified phase voltages are balanced and can be used to generalize Table 5.4 to give the average neutral voltage as

$$
\begin{equation*}
<V_{n N}>=0.5(1-2 \alpha)-\alpha v_{\min }^{\prime}+v_{\max }{ }^{\prime}(\alpha-1) . \tag{5.48}
\end{equation*}
$$

where $\mathrm{v}_{\text {min }}^{\prime}=\operatorname{Minimum}\left(\mathrm{v}_{\mathrm{an}}^{\prime}, \mathrm{v}_{\mathrm{bn}}^{\prime}, \mathrm{v}_{\mathrm{cn}}{ }^{\prime}\right), \mathrm{v}_{\text {man }}{ }^{\prime}=\operatorname{Maximum}\left(\mathrm{v}_{\mathrm{an}}^{\prime}, \mathrm{v}_{\mathrm{bn}}{ }^{\prime}, \mathrm{v}_{\mathrm{cn}}{ }^{\prime}\right)$ and $\mathrm{v}_{\mathrm{in}}{ }^{\prime}=\mathrm{v}_{\mathrm{in}}-\mathrm{V}_{\mathrm{nN}} \mathrm{i}=\mathrm{a}, \mathrm{b}, \mathrm{c}$.

Hence using the above generalized zero sequence voltage expression the discontinuous modulation can be implemented. Hence the approach makes the implementation of the unbalanced system easy without any complex computations to calculate the turn on times of the devices. Also there is no need of search to find the combination of the switch mode and the subsequent sequencing of switching vectors.



Figure 5.9: Continuous modulation I. Three phase balanced currents II. Three phase modulation signals.


Figure 5.10: Continuous modulation: I (a) (b) (c) Three phase voltages II. (a) (b)Tracking of the reference currents by the controller
(a)

(b)

(c)



Figure 5.11: Discontinuous modulation for $\delta=-\frac{\pi}{3}$, initial transients of the system. I. Three phase modulation signals II. Three phase-balanced currents.
(a)

(b)

(a)


(b)

(c)
I

Figure 5.12: Discontinuous modulation for $\delta=-\frac{\pi}{3}$, initial transients of the system.
I. (a) (b) Tracking of the reference currents II. (a) ,(b), (c) Three-phase voltages.


(b)

(c)


Figure 5.13: Discontinuous modulation for $\delta=-\frac{\pi}{3}$, transients of the system during the load change. I. Change of back-emf from sine to a square wave. II. Three phase modulation signals.



Figure 5.14: Discontinuous modulation for $\delta=-\frac{\pi}{3}$, transients of the system during the load change. I Three-phase balanced currents. II. Three-phase voltages.

### 5.8 Simulation Results

Figures $5.9-5.10$ shows the simulation results for the control scheme using continuous modulation scheme. Figure 5.9 II shows the three phase modulation signals which when modulated using the carrier-based PWM generates the switching for the devices to produce the balanced phase currents. Figure 5.9 I show the balanced currents produced by the scheme. Figure 5.10 I show the unbalanced three phase voltages impressed across the load. Figure 5.10 II shows the reference current and the actual phase current on the same plot and hence it is clear that the phase currents so produced are balanced currents since the controller is tracking the assumed balanced currents. The load in the particular case is, for $t=0 \sec$ to $t=1 \sec R_{a}=1 \Omega, R_{b}=3 \Omega, R_{c}=4 \Omega, L_{a}=0.025 H$, $L_{b}=0.05 H, L_{c}=0.1 H, e_{a}=100 \sin (\omega t), e_{b}=100 \sin \left(\omega t-120^{\circ}\right)$, and $e_{c}=100 \sin$ $\left(\omega \mathrm{t}+120^{\circ}\right)$. The dc supply voltage used is in this case is $\mathrm{V}_{\mathrm{d}}=400 \mathrm{~V}$.

In the discontinuous mode, assume $\delta=-60^{\circ}$. In this case $V_{d}=200 \mathrm{~V}$. Figures 5.11 - 5.12 illustrates the initial transients of the system. Figure 5.11 I shows the discontinuous modulation signals required to generate the balanced currents and as seen from the figure the devices will be clamped to the negative and the positive dc rails for $120^{0}$ and depending on the unbalance the clamping period will shift in the cycle. Figure 5.11 II show the three phase balanced currents generated. Figure 5.12 I shows the three phase unbalanced voltages impressed across the load and the Figure 5.12 II shows the effectiveness of the controller to track the reference currents. After reaching the steady state the back-emf connected as the load is changed from the sine wave to a square wave to create the extreme unbalance condition. The change in load occurs at $t=0.2 \mathrm{sec}$. The
magnitude of the square is 50 V . Even under this unbalanced condition the controller tracks the reference currents. Figures $5.13-5.14$ shows the transients of the system during the change of the load.

### 5.8 Unbalance Operation of a Two-Level Three-Phase Rectifier

In recent years, there was a significant increase in the usage of the rectifiers in converting the power generated. Especially, diode and thyristor rectifiers are commonly applied in the front end of dc-link power converters as an interface with the ac power line. The rectifiers are nonlinear in nature and consequently generate harmonic currents in the ac power line. The high harmonic content of the line current and the resulting low power factor of the load cause a number of problems in the power distribution.

- Voltage distortion and electromagnetic interface affecting other users of the power system.
- Increase voltampere (VA) ratings of the power system equipment to handle the harmonics.

In real world, no system is balanced and hence the significance of the study of the unbalanced system increasing day-by-day. In the present section, the unbalance operation of the two-level rectifier is considered.

### 5.9 Circuit Configuration

The circuit configuration is based on the general three-phase two-level converter with unbalanced source and input impedances as shown in Figure 5.15. The converter consists of boost inductors $L_{a}, L_{b}$, and $L_{c}$ on the ac side to filter out the input harmonic current and achieve sinusoidal current waveforms. $\mathrm{R}_{\mathrm{a}}, \mathrm{R}_{\mathrm{b}}$, and $\mathrm{R}_{\mathrm{c}}$ are the series equivalent resistors. Six switching devices with a rating of $\mathrm{V}_{\mathrm{dc}}$ are used.

Applying the KVL for the input side, the supply voltage can be written as the sum of the voltage drop across the input side impedance

$$
\begin{align*}
& v_{a}=i_{a} R_{a}+L_{a} p i_{a}+v_{a o}  \tag{5.48}\\
& v_{b}=i_{b} R_{b}+L_{b} p i_{b}+v_{b o}  \tag{5.49}\\
& v_{c}=i_{c} R_{c}+L_{c} p i_{c}+v_{c o} . \tag{5.50}
\end{align*}
$$

The voltage $v_{a o}$ is given by

$$
\begin{gather*}
v_{a o}=S_{a p} V_{k N}+S_{a n} V_{N o}  \tag{5.51}\\
v_{a o}=S_{a p} V_{k N}+S_{a n} V_{N o}  \tag{5.52}\\
v_{a o}=S_{a p} V_{k N}+S_{a n} V_{N o} . \tag{5.53}
\end{gather*}
$$

To avoid shorting of the output capacitor the following constraint has to be followed while switching the devices.

$$
\begin{align*}
& S_{a p}+S_{a n}=1 \Rightarrow S_{a p}=1-S_{a n}  \tag{5.54}\\
& S_{b p}+S_{b n}=1 \Rightarrow S_{b p}=1-S_{b n}  \tag{5.55}\\
& S_{c p}+S_{c n}=1 \Rightarrow S_{c p}=1-S_{c n} . \tag{5.56}
\end{align*}
$$



Figure 5.15: Schematic of the three-phase two-level rectifier.
Hence by substituting the expression in Eqs. (5.51-5.53) into Eqs. (5.48-5.50)

$$
\begin{align*}
& v_{a}=i_{a} R_{a}+L_{a} p i_{a}+\left(S_{a p}-S_{a n}\right) \frac{V_{k N}}{2}+V_{N 0}  \tag{5.57}\\
& v_{b}=i_{b} R_{b}+L_{b} p i_{b}+\left(S_{b p}-S_{b n}\right) \frac{V_{k N}}{2}+V_{N 0}  \tag{5.58}\\
& v_{c}=i_{c} R_{c}+L_{c} p i_{c}+\left(S_{c p}-S_{c n}\right) \frac{V_{k N}}{2}+V_{N 0} \tag{5.59}
\end{align*}
$$

Hence

$$
\begin{align*}
& \left(S_{a p}-S_{a n}\right)=\frac{v_{a}-i_{a} R_{a}-L_{a} p i_{a}-V_{N 0}}{\frac{V_{k N}}{2}}  \tag{5.60}\\
& \left(S_{b p}-S_{b n}\right)=\frac{v_{b}-i_{b} R_{b}-L_{b} p i_{b}-V_{N 0}}{\frac{V_{k N}}{2}} \tag{5.61}
\end{align*}
$$

$$
\begin{equation*}
\left(S_{c p}-S_{c n}\right)=\frac{v_{c}-i_{c} R_{c}-L_{c} p i_{c}-V_{N 0}}{\frac{V_{k N}}{2}} \tag{5.62}
\end{equation*}
$$

The relationship between the switching function and the modulation signals can be expressed as

$$
\begin{align*}
& S_{a p}=\frac{\left(1+M_{a}\right)}{2}  \tag{5.63}\\
& S_{b p}=\frac{\left(1+M_{b}\right)}{2}  \tag{5.64}\\
& S_{c p}=\frac{\left(1+M_{c}\right)}{2} \tag{5.65}
\end{align*}
$$

By substituting the conditions in Eqs. (5.63-5.65) and Eqs. (5.54-5.56) in Eqs. (5.60 5.62) gives the modulation signals as

$$
\begin{align*}
& M_{a p}=\frac{v_{a}-I_{a} R_{a}-L_{a} p I_{a}-V_{N 0}}{V_{k N}}  \tag{5.66}\\
& M_{b p}=\frac{v_{b}-i_{b} R_{b}-L_{b} p i_{b}-V_{N 0}}{V_{k N}}  \tag{5.67}\\
& M_{c p}=\frac{v_{c}-i_{c} R_{c}-L_{c} p i_{c}-V_{N 0}}{V_{k N}} \tag{5.68}
\end{align*}
$$

The current flowing through the capacitor is given by

$$
\begin{equation*}
C p V_{d}=-I_{d c}+\left(M_{a p} i_{a}+M_{b p} i_{b}+M_{c p} i_{c}\right) . \tag{5.69}
\end{equation*}
$$

### 5.10 Control Scheme

The control structure is as shown in Figure 5.16. As seen the structure is similar to that of the three-level rectifier. Similar control methodology is applied to derive the reference phase currents.

Now

$$
\begin{align*}
& i_{a} R_{a}+L_{a} p i_{a}=\sigma_{a}  \tag{5.70}\\
& i_{b} R_{b}+L_{b} p i_{b}=\sigma_{b}  \tag{5.71}\\
& i_{c} R_{c}+L_{c} p i_{c}=\sigma_{c} . \tag{5.72}
\end{align*}
$$

By substituting the above assumption in Eqs. (5.66-5.68) and simplifying, the expressions for the modulation signals are obtained as

$$
\begin{align*}
& M_{a p}=\frac{v_{a}-\sigma_{a}-V_{N 0}}{V_{d c}}  \tag{5.73}\\
& M_{b p}=\frac{v_{b}-\sigma_{b}-V_{N 0}}{V_{d c}}  \tag{5.74}\\
& M_{c p}=\frac{v_{c}-\sigma_{c}-V_{N 0}}{V_{d c}} . \tag{5.75}
\end{align*}
$$

The capacitor equation is given by

$$
C p V_{d c}=-I_{d c}+\left(M_{a p} i_{a}+M_{b p} i_{b}+M_{c p} i_{c}\right)
$$

By substituting the expressions for the modulation signals and simplifying

$$
\begin{equation*}
\frac{1}{2} C p V_{d}{ }^{2}+I_{d c} V_{d}=\left(v_{a s} i_{a}+v_{b s}{ }^{\prime} i_{b}+v_{c s} i_{c}-\sigma_{a} i_{a}-\sigma_{b} i_{b}-\sigma_{c} i_{c}\right) . \tag{5.76}
\end{equation*}
$$

The power transferred in a three-phase circuit is given by

$$
\begin{equation*}
P=v_{a s} i_{a}+v_{b s} i_{b}+v_{c s} i_{c} . \tag{5.77}
\end{equation*}
$$

Under balanced conditions, the sum of the three phase currents is equal to zero
$i_{a}+i_{b}+i_{c}=0$
$\Rightarrow i_{c}=-\left(i_{a}+i_{b}\right)$.

Substituting Eq. (5.78) in Eq. (5.77) gives

$$
\begin{equation*}
P=v_{a c} i_{a}+v_{b c} i_{b} . \tag{5.79}
\end{equation*}
$$

The main objective of the control scheme is to transfer constant power and hence the differentiation of power with respect to time is zero; i.e.,

$$
\begin{align*}
& \frac{\partial P}{\partial t}=0 \\
& \Rightarrow \\
& \quad 0=i_{a} p v_{a c}+v_{a c} p i_{a}+I_{b} p v_{b c}+v_{b c} p i_{b} \tag{5.80}
\end{align*}
$$

where $p=\frac{\partial}{\partial t}$.

From Eqs. (5.70-5.71), by substituting the expressions for $\mathrm{pi}_{\mathrm{a}}$ and $\mathrm{pi}_{\mathrm{b}}$ in Eq. (5.80) gives the condition for the constant power transfer

$$
\begin{equation*}
i_{a}\left[L_{a} L_{b} p v_{a c}-L_{b} R_{a} v_{a c}\right]+i_{b}\left[L_{a} L_{b} p v_{b c}-L_{a} R_{b} v_{a c}\right]=-\left[L_{b} v_{a c} \sigma_{a}+L_{a} v_{b c} \sigma_{b}\right] . \tag{5.81}
\end{equation*}
$$

Expressing Eq. (5.76) and (5.81) in the matrix form

$$
\left[\begin{array}{cc}
L_{a} L_{b} p v_{a c}-L_{b} R_{a} v_{a c} & L_{a} L_{b} p v_{b c}-L_{a} R_{b} v_{b c} \\
v_{a c}+\sigma_{c}-\sigma_{a} & v_{b c}+\sigma_{c}-\sigma_{b}
\end{array}\right]\left[\begin{array}{l}
i_{a} \\
i_{b}
\end{array}\right]=\left[\begin{array}{c}
-\left[L_{b} v_{a c} \sigma_{a}+L_{a} v_{b c} \sigma_{b}\right] \\
\sigma_{v}+I_{d c}\left(V_{c 1}+V_{c 2}\right)
\end{array}\right] .
$$

By solving the above matrix for $I_{a}, I_{b}$
$i_{a}=\frac{\left(v_{b c}+\sigma_{c}-\sigma_{b}\right)\left(-\left[L_{b} v_{a c} \sigma_{a}+L_{a} v_{b c} \sigma_{b}\right]\right)-\left(\sigma_{v}+I_{d c}\left(V_{c 1}+V_{c 2}\right)\right)\left(L_{a} L_{b} p v_{b c}-L_{a} R_{b} v_{b c}\right)}{\Delta}$

$$
\begin{equation*}
i_{b}=\frac{\left(L_{a} L_{b} p v_{a c}-L_{b} R_{a} v_{a c}\right)\left(\sigma_{v}+I_{d c}\left(V_{c 1}+V_{c 2}\right)\right)-\left(v_{a c}+\sigma_{c}-\sigma_{a}\right)\left(-\left[L_{b} v_{a c} \sigma_{a}+L_{a} v_{b c} \sigma_{b}\right]\right)}{\Delta} \tag{5.83}
\end{equation*}
$$

where

$$
\Delta=\left(L_{a} L_{b} p v_{a c}-L_{b} R_{a} v_{a c}\right)\left(v_{b c}+\sigma_{c}-\sigma_{b}\right)-\left(L_{a} L_{b} p v_{b c}-L_{a} R_{b} v_{b c}\right)\left(v_{a c}+\sigma_{c}-\sigma_{a}\right) .
$$

Using the above expressions the reference phase currents can be generated. Phase c current can be obtained using the balance condition for the current.

Figure 5.16 shows the schematic of the control scheme. Similar to three level rectifier control scheme, two level rectifiers also have the voltage control as the outer control loop and the current controllers as the inner control loop. In the control scheme the square of the actual dc voltage $V_{d c}{ }^{2}$ is compared with the square of the reference dc voltage $V_{d c}{ }^{* 2}$. The error signal is passed through a PI controller $K_{v}$ whose structure is explained in the next section. The output of this controller is assumed as $\sigma_{v}$. Using $\sigma_{v}$, the expression for constant power and using Equations (5.82-5.83), the reference currents $i_{a}{ }^{*}, i_{b}{ }^{*}$, and $i_{c}{ }^{*}$ can be calculated. These reference currents are compared with the actual currents. The errors are passed through the natural reference frame current controllers $K_{a}, K_{b}$, and $K_{c}$. The structure of these controllers is explained in the next section. The output of these controllers is assumed as $\sigma_{a}, \sigma_{b}$, and $\sigma_{c}$. Using Equations (5.73) -(5.75) the modulation signals can be obtained.


Figure 5.16: Schematic of the control scheme of two-level rectifier.


Figure 5.17: Structure of the abc reference frame controller.

### 5.10.1 Structure of the Controllers

The procedure for determining the transfer functions of the currents is discussed in section 5.3.1. Using the same methodology the transfer functions are derived as follows:

$$
\begin{aligned}
& \frac{i_{a}}{i_{a}{ }^{*}}=\frac{p^{2} 2 K_{p} \operatorname{Cos} \phi_{1}+2 p K_{i} \operatorname{Cos} \phi_{1}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}}{L_{a} p^{3}+p^{2}\left[R_{a}+2 K_{p} \operatorname{Cos} \phi_{1}\right]+p\left[3 \omega_{s}{ }^{2} L_{a}+2 K_{i} \operatorname{Cos} \phi_{1}\right]+\left[R_{a} \omega_{s}{ }^{2}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}\right]} \\
& \frac{i_{b}}{i_{b}{ }^{*}}=\frac{p^{2} 2 K_{p} \operatorname{Cos} \phi_{1}+2 p K_{i} \operatorname{Cos} \phi_{1}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}}{L_{b} p^{3}+p^{2}\left[R_{b}+2 K_{p} \operatorname{Cos} \phi_{1}\right]+p\left[3 \omega_{s}{ }^{2} L_{b}+2 K_{i} \operatorname{Cos} \phi_{1}\right]+\left[R_{b} \omega_{s}{ }^{2}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}\right]} \\
& \frac{i_{c}}{i_{c}{ }^{*}}=\frac{p^{2} 2 K_{p} \operatorname{Cos} \phi_{1}+2 p K_{i} \operatorname{Cos} \phi_{1}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}}{L_{c} p^{3}+p^{2}\left[R_{c}+2 K_{p} \operatorname{Cos} \phi_{1}\right]+p\left[3 \omega_{s}{ }^{2} L_{c}+2 K_{i} \operatorname{Cos} \phi_{1}\right]+\left[R_{c} \omega_{s}{ }^{2}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}\right]} .
\end{aligned}
$$

Comparing the denominator of the transfer function with the Butterworth polynomial is one of the useful techniques in choosing the controller parameters. As seen from the transfer functions, it is clear that the system is a third order system and hence by comparing the denominator with third order polynomial.

$$
\begin{equation*}
p^{3}+2 p^{2} \omega_{o}+2 p \omega_{o}^{2}+\omega_{o}^{3}=0 . \tag{5.84}
\end{equation*}
$$

Following equations are obtained

$$
\begin{align*}
& \frac{R_{a}+2 K_{p} \operatorname{Cos} \phi_{1}}{L_{a}}=2 \omega_{o}  \tag{5.85}\\
& \frac{\omega_{s}^{2} L_{a}+2 K_{i} \operatorname{Cos} \phi_{1}}{L_{a}}=2 \omega_{o}{ }^{2}  \tag{5.86}\\
& \frac{R_{a} \omega_{s}^{2}+2 K_{p} \operatorname{Cos} \phi_{1} \omega^{2}-2 K_{i} \omega \operatorname{Sin} \phi_{1}}{L_{a}}=\omega_{o}{ }^{3} . \tag{5.87}
\end{align*}
$$

Solving the above equations and by varying the delay angle, the controller parameters of the current controller can be obtained.


Figure 5.18: Structure of the voltage controller.

### 5.10.2 Voltage Controller

From Figure 5.18

$$
\begin{aligned}
& \left(V_{d c}{ }^{* 2}-V_{d c}^{2}\right) K_{v}=\frac{1}{2} C p V_{d c}^{2} \\
& \Rightarrow \frac{1}{2} C s V_{d c}{ }^{2}+K_{v} V_{d c}{ }^{2}=K_{v} V_{d c}^{{ }^{2}} \\
& \Rightarrow\left(\frac{1}{2} C s+K_{v}\right) V_{d c}^{2}=K_{v} V_{d c}^{* 2} \\
& \Rightarrow \frac{V_{d c}{ }^{2}}{V_{d c}{ }^{* 2}}=\frac{K_{v}}{K_{v}+\frac{C s}{2}}
\end{aligned}
$$

Assuming the structure of the controller $K_{v}$ as $K_{p}+\frac{K_{i}}{s}$ and by substituting this in to the above transfer function

$$
\begin{aligned}
\frac{V_{d c}{ }^{2}}{V_{d c}{ }^{* 2}} & =\frac{K_{p}+\frac{K_{i}}{s}}{K_{p}+\frac{K_{i}}{s}+\frac{C s}{2}} \\
& =\frac{s K_{p}+K_{i}}{C s^{2}+2 s K_{p}+2 K_{i}}
\end{aligned}
$$



Figure 5.19: Effect of delay angle $\phi_{1}$ on the control parameters of current controller. (I) $\mathrm{K}_{\mathrm{p}}(\mathrm{II}) \mathrm{K}_{\mathrm{i}}(\mathrm{III}) \omega_{o}$.

Figure 5.19 demonstrates the effect of the delay angle on the control parameters of current controller. The control parameters are determined by varying the delay angle from [-pi/2, pi/2].

### 5.11 Circuit Parameters

## Unbalanced Source Impedance Operation:

Input line resistance $R_{a}=0.2 \Omega ; R_{b}=0.4 \Omega ; R_{c}=0.1 \Omega$

Input line inductance $L_{a}=10 \mathrm{mH} ; L_{b}=20 \mathrm{mH} ; L_{c}=5 \mathrm{mH}$

Input Supply Voltage $v_{a}=80 \cos (\omega t)$

$$
\begin{aligned}
& v_{b}=80 \cos \left(\omega t-120^{\circ}\right) \\
& v_{c}=80 \cos \left(\omega t+120^{\circ}\right)
\end{aligned}
$$

Output dc-capacitance $C=2200 \mu F$
Load resistance $R_{L}=75 \Omega$

## Unbalanced Source Voltage and Source Impedance Operation:

Input line resistance $R_{a}=0.2 \Omega ; R_{b}=0.4 \Omega ; R_{c}=0.1 \Omega$

Input line inductance $L_{a}=10 \mathrm{mH} ; L_{b}=20 \mathrm{mH} ; L_{c}=5 \mathrm{mH}$

Input Supply Voltage $v_{a}=80 \cos (\omega t)$

$$
\begin{aligned}
& v_{b}=60 \cos \left(\omega t-120^{\circ}\right) \\
& v_{c}=70 \cos \left(\omega t+120^{\circ}\right)
\end{aligned}
$$

Output dc-capacitance $C=2200 \mu F$
Load resistance $R_{L}=75 \Omega$

### 5.12 Simulation Results

The proposed control scheme is simulated to validate the control methodology. In this, two cases are simulated for unbalanced source impedance with balanced source voltages and the other with unbalanced voltages and unbalanced impedance. Figures 5.20 -5.21 shows the simulation results of the control scheme of the two-level rectifier with unbalanced source impedance. In the control scheme the reference dc bus voltage is taken as 200 V. Figure 5.20 (I) shows the three-phase modulation signals, which when modulated using the carrier-based PWM generates the switching for the power devices. Figure 5.20 (II) shows the dc bus regulation and it is clear that the controller regulates the dc bus voltage effectively. Figure 5.21 (I) (a) (b) (c) illustrates tracking of the reference currents and demonstrates the effectiveness of the current controller even under unbalanced conditions. The other objective of the control scheme is the constant power transfer and it can be observed in Figure 5.21 (II). Figures 5.22 - 5.23 show the simulation results of control scheme for the unbalanced source impedance and source voltages. The simulation results illustrate the effectiveness of the control scheme both in the balanced and unbalanced operation of two-level rectifier.

## Unbalanced Source Impedance Operation:



Figure 5.20: Simulation of the control scheme of two level rectifier with unbalance source impedance (I) Three-phase modulation signals (II) Capacitor voltage $\mathrm{V}_{\mathrm{dc}}$.


Figure 5.21: Simulation of the control scheme of two level rectifier with unbalance source impedance (I) Tracking of the three phase reference currents (II) Power transferred (P).

## Unbalanced Source Voltage and Unbalanced Impedance Operation:



Figure 5.22: Simulation of the control scheme of two level rectifier with unbalance source voltage and source impedance (I) Three-phase modulation signals (II) Capacitor voltage $\mathrm{V}_{\mathrm{dc}}$.
(a) $\mathbf{i}_{\mathbf{a}}{ }^{*} \mathbf{i}_{\mathbf{a}}$

(b) $\mathbf{i}_{\mathrm{b}}{ }^{*} \mathbf{i}_{\mathrm{b}}$

(c)
$\mathbf{i}_{\mathrm{c}}{ }^{*}, \mathbf{i}_{\mathrm{c}}$



Figure 5.23: Simulation of the control scheme of two level rectifier with unbalance source impedance (I) Tracking of the three phase reference currents (II) Power transferred (P).

