CHAPTER 4

THE AVERAGE MODEL OF THE SYMMETRICAL AND SYMMETRICAL MULTIPLE-STAR IPM MACHINES

4.1 Introduction

For many studies the simplified model of a machine is needed. The simplified models can be used for designing the drives, steady state analysis and decoupling of the models [120]. This chapter starts with the modelling of symmetrical and asymmetrical triple-star nine-phase machines using the Fourier series. After generating the general equations for turn functions of the machines phases, the general form of the Fourier series of the winding functions are derived. The Fourier series of the airgap function of the machines are also derived in this chapter and using them the general form of the Fourier series of different inductances of the machines stators are derived. In the next section by neglecting the inductances with the higher order harmonics the simplified inductances are presented. The simplified inductances are then transformed to the rotor reference frame and the general model of the machines is generated. To verify the machines model, they are simulated using the MATLAB Simulink and the simulation results are presented. The general model of the machines is decoupled to remove the coupling terms between different machines and the decoupled models for symmetrical and asymmetrical machines are presented. In the final sections of this chapter an asymmetrical double-star six-phase IPM is also modelled using Fourier series of the machine parameters [83]. The model is generated and transformed to the rotor reference frame and finally the model is decoupled to remove the couplings between the two sets of the three phase machines. The major contribution of this chapter is generating decoupled models and corresponding transformation matrixes for triple-star nine-phase IPM machines (symmetrical and asymmetrical connections) that can be used for designing controllers for triple-star machines without facing the complexities raised by the coupling terms between different sets of the three phase machines.

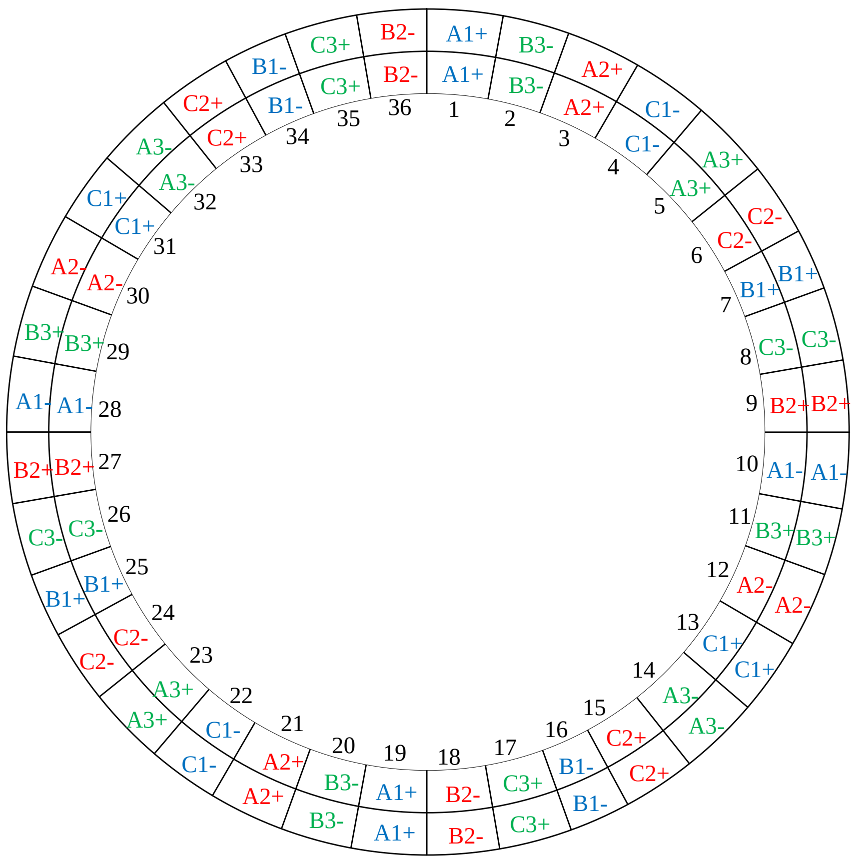


Figure 4.1: The clock diagram of the symmetrical triple-star machine.

4.2 Modelling the Stator Inductances of Triple-Star Machines

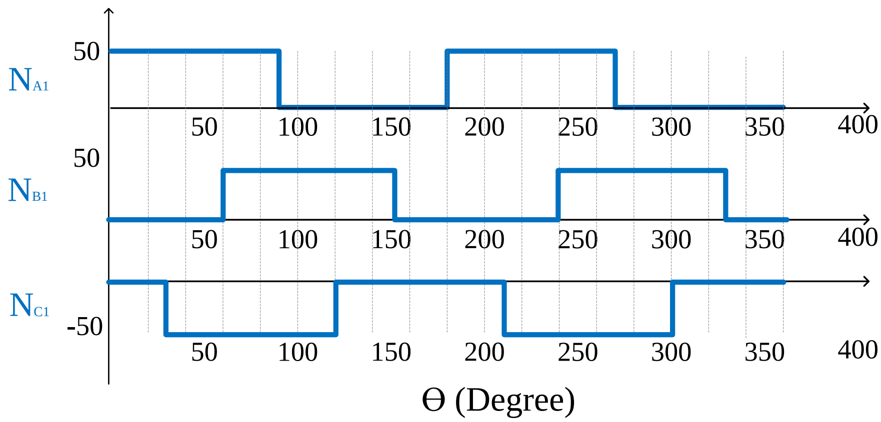


Figure 4.2: The turn functions of the machine 1 phases (symmetrical).

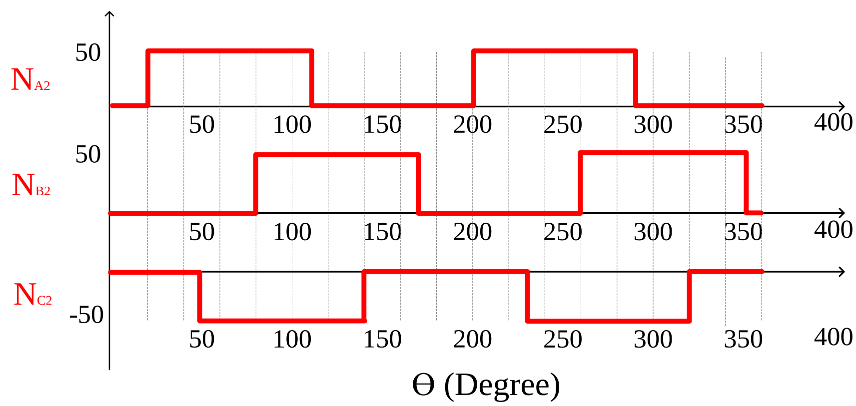


Figure 4.3: The turn functions of the machine 2 phases (symmetrical).

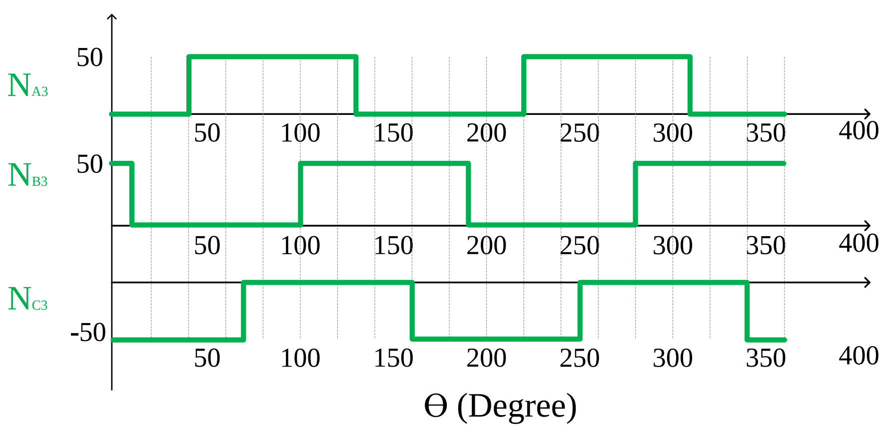


Figure 4.4: The turn functions of the machine 3 phases (symmetrical).

In this section the stator inductances of both symmetrical and asymmetrical triple-star IPM machines are modeled using the Fourier series of the machine parameters such as the winding functions and airgap functions.

Unlike the coupled modelling, there will be some simplifying assumptions made to have a simpler model. For example, in this modelling method, the higher frequency order components of the winding functions and airgap functions are neglected. The modelling can start from the clock diagram and turn functions of the machine. The clock diagram and the turn functions of the symmetrical machine (shown in Figure 4.1) are repeated here in Figures 4.2 to 4.4. Similarly, for the asymmetrical machine the turn functions can be generated using the clock diagram, the clock diagram of the asymmetrical machine is shown in the Figure 4.5.

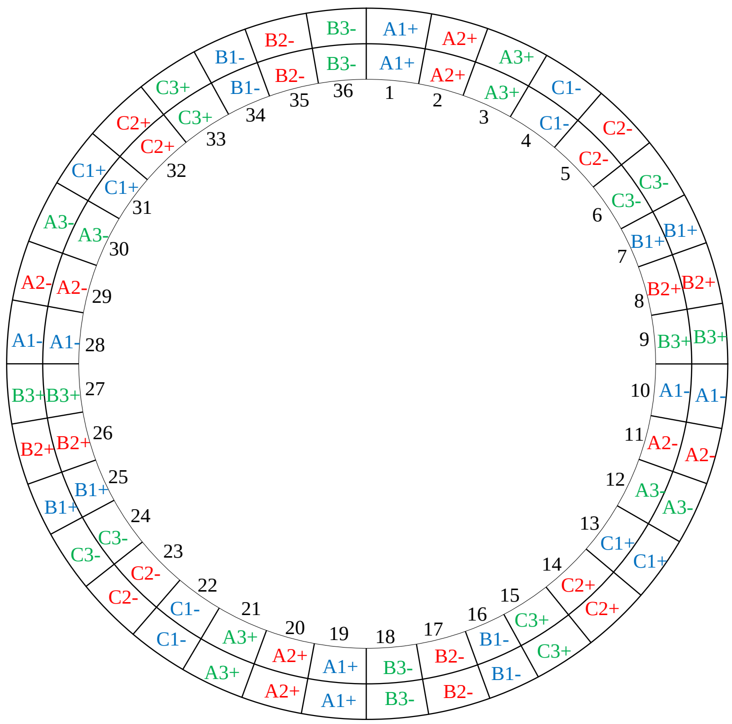


Figure 4.5: The clock diagram of the asymmetrical triple-star machine.

Using the new clock diagram the turn functions can be generated as Figures 4.6 to 4.8.

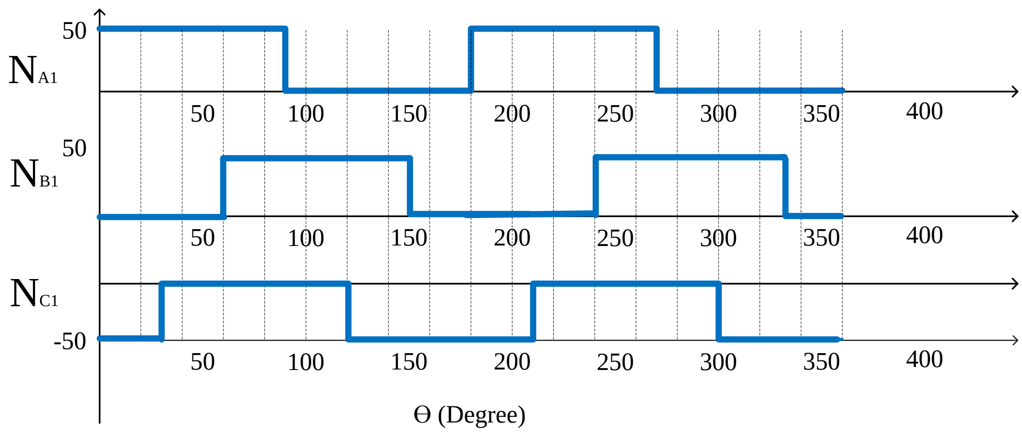


Figure 4.6: The turn functions of the machine 1 phases (asymmetrical).

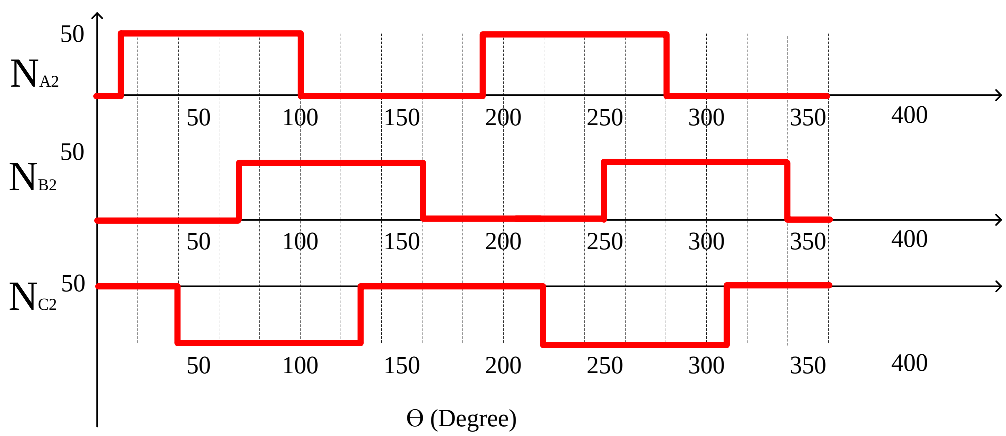


Figure 4.7: The turn functions of the machine 2 phases (asymmetrical).

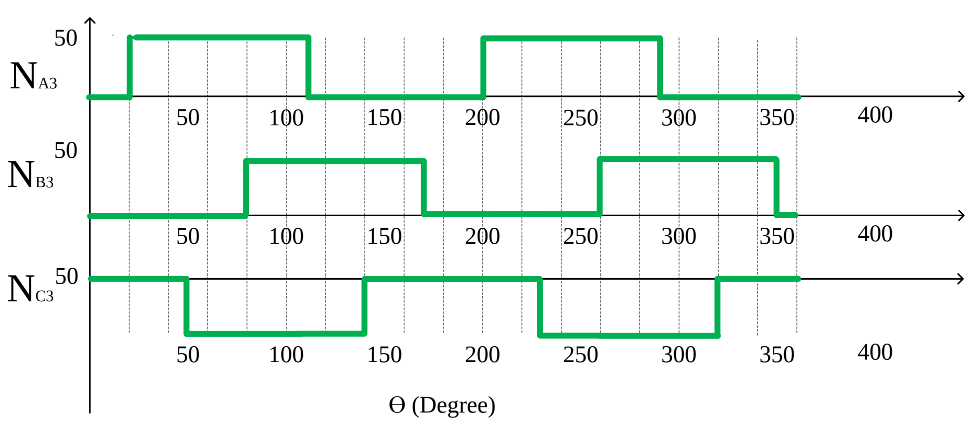


Figure 4.8: The turn functions of the machine 3 phases (asymmetrical).

Based on the above figures the general form of Fourier series of the turn function can be generated as:

$$n_x = N_0 + N_1 \sin(\theta - kd_k\beta) + N_3 \sin 3(\theta - kd_k\beta) +$$
$$N_5 \sin 5(\theta - kd_k\beta) + N_7 \sin 7(\theta - kd_k\beta) \tag{4.1}$$

$$x = a_i, b_i, c_i,\; i = 1,2,3 \quad \beta = \frac{\pi}{9},\; N_0 = \frac{N_x}{2},\; N_n = \frac{4N_x}{n\pi}$$

Where: ' d_k ' equals to 1 and 2 for the symmetrical and asymmetrical machines respectively. Also, for each phase that is placed in slot number 'S' the *'k'* can be defined as:

$$k = S - 1 \tag{4.2}$$

Also, based on Figure 3.6, the Fourier series of the inverse airgap function can be presented as [152]:

$$g^{-1}(\theta, \theta_r) = a_0 + a_1 \cos\left(2(\theta - \theta_r)\right) + a_2 \cos\left(6(\theta - \theta_r)\right) + \tag{4.3}$$

$$a_3 \cos\left(10(\theta - \theta_r)\right) + a_4 \cos\left(14(\theta - \theta_r)\right)$$

Where, a_0, a_1, a_3, a_4 are the Fourier series amplitudes for the inverse air gap function and can be defined as:

$$a_0 = a,\quad a_1 = -b,\quad a_2 = -\frac{b}{3},\quad a_3 = -\frac{b}{5},\quad a_4 = -\frac{b}{7}$$
$$a = \frac{1}{2}\left(\frac{1}{g_b} + \frac{1}{g_b}\right),\; b = \frac{1}{2}\left(\frac{1}{g_b} - \frac{1}{g_a}\right) \tag{4.4}$$

Using the equations (4.1) and (4.3) the winding function of each phase can be calculated as [74]:

$$N_w(\theta) = n_w(\theta) - \frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} \tag{4.5}$$

The different parts of the equation (4.5) can be expressed as:

$$\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta = \int_0^{2\pi} \begin{pmatrix} a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) \\ + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r) \end{pmatrix} d\theta = 2\pi a_0 \tag{4.6}$$

$$\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta = \int_0^{2\pi} \begin{matrix} \left(N_0 + N_1 \sin(\theta - kd_k\beta) + N_3 \sin 3(\theta - kd_k\beta) + N_5 \sin 5(\theta - kd_k\beta) + N_7 \sin 7(\theta - kd_k\beta)\right) \times \\ \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) d\theta \end{matrix} =$$

$$\int_0^{2\pi} \begin{pmatrix} N_0\left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \\ + N_1 \sin(\theta - kd_k\beta)\left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \\ + N_3 \sin 3(\theta - kd_k\beta)\left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \\ + N_5 \sin 5(\theta - kd_k\beta)\left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \\ + N_7 \sin 7(\theta - kd_k\beta)\left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \end{pmatrix} d\theta \tag{4.7}$$

$$= \int_0^{2\pi} \left(N_0(a_0)\right) d\theta = 2\pi N_0 a_0$$

Therefore, equations (4.6) and (4.7) result in:

$$\frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} = \frac{2\pi N_0 a_0}{2\pi a_0} = N_0 \tag{4.8}$$

Using the above equations, the general form of the winding functions of the machines can be expressed as:

$$\begin{aligned} N_w(\theta) &= N_0 + N_1 \sin(\theta - kd_k\beta) + N_3 \sin 3(\theta - kd_k\beta) + N_5 \sin 5(\theta - kd_k\beta) + N_7 \sin 7(\theta - kd_k\beta) - N_0 = \\ & N_1 \sin(\theta - kd_k\beta) + N_3 \sin 3(\theta - kd_k\beta) + N_5 \sin 5(\theta - kd_k\beta) + N_7 \sin 7(\theta - kd_k\beta) \end{aligned} \tag{4.9}$$

Now the general form of the self and mutual inductances (between phases '*j*' and '*i*') of the machines phases can be generated as:

$$L_{ji} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o r l \int_0^{2\pi} \begin{pmatrix} (a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \times \\ (N_1 \sin(\theta - k_j d_k \beta) + N_3 \sin 3(\theta - k_j d_k \beta) + N_5 \sin 5(\theta - k_j d_k \beta) + N_7 \sin 7(\theta - k_j d_k \beta)) \times \\ (N_0 + N_1 \sin(\theta - k_i d_k \beta) + N_3 \sin 3(\theta - k_i d_k \beta) + N_5 \sin 5(\theta - k_i d_k \beta) + N_7 \sin 7(\theta - k_i d_k \beta)) \end{pmatrix} d\theta \quad (4.10)$$

The equation (4.10) is equal to:

$$L_{ji} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o r l \int_0^{2\pi} \begin{pmatrix} (a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \times \\ \left(\begin{pmatrix} N_0 N_1 \sin(\theta - k_j d_k \beta) + N_1 N_1 \sin(\theta - k_j d_k \beta)\sin(\theta - k_i d_k \beta) \\ + N_3 N_1 \sin(\theta - k_j d_k \beta)\sin 3(\theta - k_i d_k \beta) + N_5 N_1 \sin(\theta - k_j d_k \beta)\sin 5(\theta - k_i d_k \beta) \\ + N_7 N_1 \sin(\theta - k_j d_k \beta)\sin 7(\theta - k_i d_k \beta) \end{pmatrix} + \right. \\ \begin{pmatrix} N_0 N_3 \sin 3(\theta - k_j d_k \beta) + N_1 N_3 \sin 3(\theta - k_j d_k \beta)\sin(\theta - k_i d_k \beta) \\ + N_3 N_3 \sin 3(\theta - k_j d_k \beta)\sin 3(\theta - k_i d_k \beta) + N_5 N_3 \sin 3(\theta - k_j d_k \beta)\sin 5(\theta - k_i d_k \beta) \\ + N_7 N_3 \sin 3(\theta - k_j d_k \beta)\sin 7(\theta - k_i d_k \beta) \end{pmatrix} + \\ \begin{pmatrix} N_0 N_5 \sin 5(\theta - k_j d_k \beta) + N_1 N_5 \sin 5(\theta - k_j d_k \beta)\sin(\theta - k_i d_k \beta) \\ + N_3 N_5 \sin 5(\theta - k_j d_k \beta)\sin 3(\theta - k_i d_k \beta) + N_5 N_5 \sin 5(\theta - k_j d_k \beta)\sin 5(\theta - k_i d_k \beta) \\ + N_7 N_5 \sin 5(\theta - k_j d_k \beta)\sin 7(\theta - k_i d_k \beta) \end{pmatrix} + \\ \left. \begin{pmatrix} N_0 N_7 \sin 7(\theta - k_j d_k \beta) + N_1 N_7 \sin 7(\theta - k_j d_k \beta)\sin(\theta - k_i d_k \beta) \\ + N_3 N_7 \sin 7(\theta - k_j d_k \beta)\sin 3(\theta - k_i d_k \beta) + N_5 N_7 \sin 7(\theta - k_j d_k \beta)\sin 5(\theta - k_i d_k \beta) \\ + N_7 N_7 \sin 7(\theta - k_j d_k \beta)\sin 7(\theta - k_i d_k \beta) \end{pmatrix} \right) \end{pmatrix} d\theta \quad (4.11)$$

The non-zero terms are:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right)\times \\ \begin{pmatrix} \left(N_0 N_1 \sin(\theta - k_j d_k \beta) + N_1 N_1 \sin(\theta - k_j d_k \beta)\sin(\theta - k_i d_k \beta)\right)+ \\ \left(N_0 N_3 \sin 3(\theta - k_j d_k \beta) + N_3 N_3 \sin 3(\theta - k_j d_k \beta)\sin 3(\theta - k_i d_k \beta)\right)+ \\ \left(N_0 N_5 \sin 5(\theta - k_j d_k \beta) + N_5 N_5 \sin 5(\theta - k_j d_k \beta)\sin 5(\theta - k_i d_k \beta)\right)+ \\ \left(N_0 N_7 \sin 7(\theta - k_j d_k \beta) + N_7 N_7 \sin 7(\theta - k_j d_k \beta)\sin 7(\theta - k_i d_k \beta)\right) \end{pmatrix} \end{pmatrix} d\theta \qquad (4.12)$$

The equation (4.12) is equal to:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right)\times \\ \begin{pmatrix} \left(N_0 N_1 \sin\left(\theta - k_j d_k \beta\right) + \frac{N_1{}^2}{2}\left(-\cos\left(2\theta - d_k\left(k_j + k_i\right)\beta\right) + \cos\left(d_k\left(k_i - k_j\right)\beta\right)\right)\right)+ \\ \left(N_0 N_3 \sin 3\left(\theta - k_j d_k \beta\right) + \frac{N_3{}^2}{2}\left(-\cos 3\left(2\theta - d_k\left(k_j + k_i\right)\beta\right) + \cos 3\left(d_k\left(k_i - k_j\right)\beta\right)\right)\right)+ \\ \left(N_0 N_5 \sin 5(\theta - k_j d_k \beta) + \frac{N_5{}^2}{2}\left(-\cos 5\left(2\theta - d_k\left(k_j + k_i\right)\beta\right) + \cos 5\left(d_k\left(k_i - k_j\right)\beta\right)\right)\right)+ \\ \left(N_0 N_7 \sin 7(\theta - k_j d\beta) + \frac{N_7{}^2}{2}\left(-\cos 7\left(2\theta - d_k\left(k_j + k_i\right)\beta\right) + \cos 7\left(d_k\left(k_i - k_j\right)\beta\right)\right)\right) \end{pmatrix} \end{pmatrix} d\theta \quad (4.13)$$

And the term with non-zero averages are:

$$L_{ji} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o r l \int_0^{2\pi} \begin{pmatrix} a_0\left(\frac{N_1^{\ 2}}{2}\cos\left(d_k\left(k_i-k_j\right)\beta\right)+\frac{N_3^{\ 2}}{2}\cos 3\left(d_k\left(k_i-k_j\right)\beta\right)+\frac{N_5^{\ 2}}{2}\cos 5\left(d\left(k_i-k_j\right)\beta\right)+\frac{N_7^{\ 2}}{2}\cos 7\left(d_k\left(k_i-k_j\right)\beta\right)\right) \\ -a_1\frac{N_1^{\ 2}}{2}\cos\left(2\theta-d_k\left(k_j+k_i\right)\beta\right)\cos 2(\theta-\theta_r)-a_2\frac{N_3^{\ 2}}{2}\cos 3\left(2\theta-d_k\left(k_j+k_i\right)\beta\right)\cos 6(\theta-\theta_r) \\ -a_3\frac{N_5^{\ 2}}{2}\cos 5\left(2\theta-d_k\left(k_j+k_i\right)\beta\right)\cos 10(\theta-\theta_r)-a_4\frac{N_7^{\ 2}}{2}\cos 7\left(2\theta-d_k\left(k_j+k_i\right)\beta\right)\cos 14(\theta-\theta_r) \end{pmatrix} d\theta \qquad (4.14)$$

The last equation is equal to:

$$L_{ji} = \mu_o r l \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o r l \int_0^{2\pi} \begin{pmatrix} a_0\begin{pmatrix}\frac{N_1^{\ 2}}{2}\cos\left(d_k\left(k_i-k_j\right)\beta\right)+\frac{N_3^{\ 2}}{2}\cos 3\left(d_k\left(k_i-k_j\right)\beta\right) \\ +\frac{N_5^{\ 2}}{2}\cos 5\left(d_k\left(k_i-k_j\right)\beta\right)+\frac{N_7^{\ 2}}{2}\cos 7\left(d_k\left(k_i-k_j\right)\beta\right)\end{pmatrix} \\ -a_1\frac{N_1^{\ 2}}{4}\left(\cos\left(4\theta-2\theta_r-d_k\left(k_j+k_i\right)\beta\right)+\cos\left(2\theta_r-d_k\left(k_j+k_i\right)\beta\right)\right) \\ -a_2\frac{N_3^{\ 2}}{4}\left(\cos\left(12\theta-6\theta_r-3d_k\left(k_j+k_i\right)\beta\right)+\cos\left(6\theta_r-3d_k\left(k_j+k_i\right)\beta\right)\right) \\ -a_3\frac{N_5^{\ 2}}{4}\left(\cos\left(20\theta-10\theta_r-5d_k\left(k_j+k_i\right)\beta\right)+\cos\left(10\theta_r-5d_k\left(k_j+k_i\right)\beta\right)\right) \\ -a_4\frac{N_7^{\ 2}}{4}\left(\cos\left(28\theta-14\theta_r-7d_k\left(k_j+k_i\right)\beta\right)+\cos\left(14\theta_r-7d_k\left(k_j+k_i\right)\beta\right)\right) \end{pmatrix} d\theta \qquad (4.15)$$

And finally the stator inductances can be expressed as:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$2\pi\mu_o rl \begin{pmatrix} a_0 \begin{pmatrix} \frac{N_1^{\ 2}}{2}\cos(d_k(k_i-k_j)\beta)+\frac{N_3^{\ 2}}{2}\cos 3(d_k(k_i-k_j)\beta)+ \\ \frac{N_5^{\ 2}}{2}\cos 5(d_k(k_i-k_j)\beta)+\frac{N_7^{\ 2}}{2}\cos 7(d_k(k_i-k_j)\beta) \end{pmatrix} \\ -a_1\frac{N_1^{\ 2}}{4}\cos(2\theta_r-d_k(k_j+k_i)\beta)-a_2\frac{N_3^{\ 2}}{4}\cos(6\theta_r-3d_k(k_j+k_i)\beta) \\ -a_3\frac{N_5^{\ 2}}{4}\cos(10\theta_r-5d_k(k_j+k_i)\beta)-a_4\frac{N_7^{\ 2}}{4}\cos(14\theta_r-7d_k(k_j+k_i)\beta) \end{pmatrix} \tag{4.16}$$

Where ‘ k_i ’ and ‘ k_j ’ are:

$$k_{i,j} = (Corresponding\ Slot\ Number\ -1) \tag{4.17}$$

The machines inductances have different harmonics including DC, second, sixth, tenth and fourteenth order. Neglecting the harmonics with frequencies higher than two, the general equation for the inductances can be derived as:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$
$$2\pi\mu_o rl\left(a_0\frac{N_1^{\ 2}}{2}\cos(d_k(k_i-k_j)\beta)-a_1\frac{N_1^{\ 2}}{4}\cos(2\theta_r-d_k(k_j+k_i)\beta)\right) \tag{4.18}$$

4.3 Transformation of the Inductances to the Rotor Reference Frame

The inductances of the machines can be arranged inside a 9×9 matrix and be transformed to the rotor reference frame using the transformation presented in section 3.4.

$$
T(\theta_r)L_{ss}T(\theta_r)^{-1}=\frac{2}{3}\begin{pmatrix}
C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0\\
S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & C(\theta_r+\alpha_2) & C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0\\
0 & 0 & 0 & S(\theta_r+\alpha_2) & S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0\\
0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3+\gamma)\\
0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r+\alpha_3) & S(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3+\gamma)\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}\times
$$

$$
\begin{pmatrix}
L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3}\\
L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3}\\
L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3}\\
L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3}\\
L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3}\\
L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3}\\
L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3}\\
L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3}\\
L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3}
\end{pmatrix}\times
\tag{4.19}
$$

$$
\begin{pmatrix}
C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & C(\theta_r+\alpha_2) & S(\theta_r+\alpha_2) & 1 & 0 & 0 & 0\\
0 & 0 & 0 & C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 1 & 0 & 0 & 0\\
0 & 0 & 0 & C(\theta_r+\alpha_2+\gamma) & S(\theta_r+\alpha_2+\gamma) & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & 1\\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & 1\\
0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & 1
\end{pmatrix}=
\begin{pmatrix}
L_{q1q1} & L_{q1d1} & L_{q101} & L_{q1q2} & L_{q1d2} & L_{q102} & L_{q1q3} & L_{q1d3} & L_{q103}\\
L_{d1q1} & L_{d1d1} & L_{d101} & L_{d1q2} & L_{d1d2} & L_{d102} & L_{d1q3} & L_{d1d3} & L_{d103}\\
L_{01q1} & L_{01d1} & L_{0101} & L_{01q2} & L_{01d2} & L_{0102} & L_{01q3} & L_{01d3} & L_{0103}\\
L_{q1q2} & L_{q1d2} & L_{q102} & L_{q2q2} & L_{q2d2} & L_{q202} & L_{q2q3} & L_{q2d3} & L_{q203}\\
L_{d1q2} & L_{d1d2} & L_{d102} & L_{d2q2} & L_{d2d2} & L_{d202} & L_{d2q3} & L_{d2d3} & L_{d203}\\
L_{01q2} & L_{01d2} & L_{0102} & L_{02q2} & L_{02d2} & L_{0202} & L_{02q3} & L_{02d3} & L_{0203}\\
L_{q1q3} & L_{q1d3} & L_{q103} & L_{q2q3} & L_{q2d3} & L_{q203} & L_{q3q3} & L_{q3d3} & L_{q303}\\
L_{d1q3} & L_{d1d3} & L_{d103} & L_{d2q3} & L_{d2d3} & L_{d203} & L_{d3q3} & L_{d3d3} & L_{d303}\\
L_{01q3} & L_{01d3} & L_{0103} & L_{02q3} & L_{02d3} & L_{0203} & L_{03q3} & L_{03d3} & L_{0303}
\end{pmatrix}
$$

In the equation (4.19) 'θ_r' is the rotor angle 'C' represents 'cos', 'S' represents 'sin', $\gamma = \frac{2\pi}{3}$ and α_1, α_2 and α_3 are the arbitrary initial angles for the transformations corresponding to machines 1,2 and 3 respectively. For easier manipulation equation (4.19) can be broken into different parts according to the bellow procedure:

The machines inductances in the rotor reference frame can be considered as:

$$L_{qd} = \begin{bmatrix} L_{qd1} & M_{12} & M_{13} \\ M_{21} & L_{qd2} & M_{23} \\ M_{31} & M_{32} & L_{qd3} \end{bmatrix} \tag{4.20}$$

Now each term of the matrix in the equation (4.20) can be defined as below. The diagonal terms which represent the self-inductances of each set (after adding the leakage inductance) are defined as:

$$L_{qdi} = \frac{2}{3}\begin{bmatrix} C(\theta_r+\alpha_i) & C(\theta_r+\alpha_i-\gamma) & C(\theta_r+\alpha_i+\gamma) \\ S(\theta_r+\alpha_i) & S(\theta_r+\alpha_i-\gamma) & S(\theta_r+\alpha_i-\gamma) \\ 0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} \end{bmatrix}\begin{bmatrix} C(\theta_r+\alpha_i) & S(\theta_r+\alpha_i) & 1 \\ C(\theta_r+\alpha_i-\gamma) & S(\theta_r+\alpha_i-\gamma) & 1 \\ C(\theta_r+\alpha_i+\gamma) & S(\theta_r+\alpha_i-\gamma) & 1 \end{bmatrix} \tag{4.21}$$

$$= \frac{3}{2}\begin{bmatrix} L_{ls}+L_oC(\alpha_i-\alpha_i) & -L_oS(\alpha_i-\alpha_i) & 0 \\ L_oS(\alpha_i-\alpha_i) & L_{ls}+L_oC(\alpha_i-\alpha_i) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} + \frac{3}{2}\begin{bmatrix} L_2C(\alpha_i-\alpha_i) & L_2S(\alpha_i-\alpha_i) & 0 \\ L_2S(\alpha_i-\alpha_i) & -L_2C(\alpha_i-\alpha_i) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

And also the mutual between each two sets of the machines can be defined as:

$$M_{qdkj}=\frac{2}{3}\begin{bmatrix}C(\theta_r+\alpha_k) & C(\theta_r+\alpha_k-\gamma) & C(\theta_r+\alpha_k+\gamma)\\ S(\theta_r+\alpha_k) & S(\theta_r+\alpha_k-\gamma) & S(\theta_r+\alpha_k-\gamma)\\ 0.5 & 0.5 & 0.5\end{bmatrix}\begin{bmatrix}L_{a1a3} & L_{a1b3} & L_{a1c3}\\ L_{b1a3} & L_{b1b3} & L_{b1c3}\\ L_{c1a3} & L_{c1b3} & L_{c1c3}\end{bmatrix}\begin{bmatrix}C(\theta_r+\alpha_j) & S(\theta_r+\alpha_j) & 1\\ C(\theta_r+\alpha_j-\gamma) & S(\theta_r+\alpha_j-\gamma) & 1\\ C(\theta_r+\alpha_j+\gamma) & S(\theta_r+\alpha_j-\gamma) & 1\end{bmatrix}=$$

$$\frac{3}{2}\begin{bmatrix}L_oC(\alpha_k-\alpha_j+(k-j)d_k\beta) & L_oS(\alpha_k-\alpha_j+(k-j)d_k\beta) & 0\\ -L_oS(\alpha_k-\alpha_j+(k-j)d_k\beta) & L_oC(\alpha_k-\alpha_j+(k-j)d_k\beta) & 0\\ 0 & 0 & 0\end{bmatrix} \tag{4.22}$$

$$+\frac{3}{2}\begin{bmatrix}L_2C(\alpha_k+\alpha_j+(k+j-2)d_{k1}\beta) & L_2S(\alpha_k+\alpha_j+(k+j-2)d_{k1}\beta) & 0\\ L_2S(\alpha_k+\alpha_j+(k+j-2)d_{k1}\beta) & -L_2C(\alpha_k+\alpha_j+(k+j-2)d_{k1}\beta) & 0\\ 0 & 0 & 0\end{bmatrix}$$

Where:

$$L_0=\pi\mu_o rlN_1^{\ 2}a_0,\ \ L_2=\pi\mu_o rlN_1^{\ 2}\frac{a_1}{2} \tag{4.23}$$

The non-diagonal terms of the equation (4.21) are equal to zero. To remove coupling between the different axis of the machines, the non-diagonal terms of the equation (4.22) should be equal to zero. By setting them equal to zero the proper initial angles can be calculated as equations (4.24) and (4.25).

$$S(\alpha_k-\alpha_j+(k-j)d_k\beta)=0\Rightarrow\alpha_k-\alpha_j=-(k-j)d_k\beta \tag{4.24}$$

$$S(\alpha_k+\alpha_j+(k+j-2)d_{k1}\beta)=0\Rightarrow\alpha_k+\alpha_j=-(k+j-2)d_{k1}\beta \tag{4.25}$$

For different combinations of '*k*' and '*j*' the initial angles are calculated and given in Table 4.1. This table represents the proper initial values and the coefficients to remove the couplings between *'q'* and *'d'*.

Table 4.1 The initial angle for the transformation d_1 and d_{k1} for different machines.

	d_{k1}	d_1	α_1	α_2	α_3
Symmetrical	4	2	0	$\frac{2\pi}{9}$	$\frac{4\pi}{9}$
Asymmetrical	1	1	0	$\frac{\pi}{9}$	$\frac{2\pi}{9}$

By substituting the values of equation (4.23) in the equations (4.21) and (4.22) and selecting the initial values of Table 4.1 the inductance matrixes change to:

$$L_{qd1} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 + L_{ls} & 0 & 0 \\ 0 & L_0 - L_2 + L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} = \begin{bmatrix} L_{q1q1} & 0 & 0 \\ 0 & L_{d1d1} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{4.26}$$

$$L_{qd2} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 + L_{ls} & 0 & 0 \\ 0 & L_0 - L_2 + L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} = \begin{bmatrix} L_{q2q2} & 0 & 0 \\ 0 & L_{d2d2} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{4.27}$$

$$L_{qd3} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 + L_{ls} & 0 & 0 \\ 0 & L_0 - L_2 + L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} = \begin{bmatrix} L_{q3q3} & 0 & 0 \\ 0 & L_{d3d3} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{4.28}$$

$$M_{qd13} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q1q3} & 0 & 0 \\ 0 & L_{d1d3} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.29}$$

$$M_{qd31} == \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q3q1} & 0 & 0 \\ 0 & L_{d3d1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.30}$$

$$M_{qd12} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q1q2} & 0 & 0 \\ 0 & L_{d1d2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.31}$$

$$M_{qd21} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q2q1} & 0 & 0 \\ 0 & L_{d2d1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.32}$$

$$M_{qd23} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q2q3} & 0 & 0 \\ 0 & L_{d2d3} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.33}$$

$$M_{qd32} = \frac{3}{2}\begin{bmatrix} L_0 + L_2 & 0 & 0 \\ 0 & L_0 - L_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{q3q2} & 0 & 0 \\ 0 & L_{d3d2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.34}$$

Now using equations (4.26) to (4.34) the machines inductances in the rotor reference frame can be presented as equation (4.35). Unlike the models that were generated in the Sections 3.3 and 3.4, in this matrix the mutual inductances between the d and q axis are zero. This fact is due to ignoring the higher order harmonics of the winding functions and airgap function. By substituting the matrix of equation (4.35) in to the model of Section 3.2 the machines model can be expressed as equation (4.36).

$$L_{qd} = \begin{bmatrix} L_{q1q1}+L_{ls} & 0 & 0 & L_{q1q2} & 0 & 0 & L_{q1q3} & 0 & 0 \\ 0 & L_{d1d1}+L_{ls} & 0 & 0 & L_{d1d2} & 0 & 0 & L_{d1d3} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ L_{q2q1} & 0 & 0 & L_{q2q2}+L_{ls} & 0 & 0 & L_{q2q3} & 0 & 0 \\ 0 & L_{d2d1} & 0 & 0 & L_{d2d2}+L_{ls} & 0 & 0 & L_{d2d3} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ L_{q3q1} & 0 & 0 & L_{q3q2} & 0 & 0 & L_{q3q3}+L_{ls} & 0 & 0 \\ 0 & L_{d3d1} & 0 & 0 & L_{d3d2} & 0 & 0 & L_{d3d3}+L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \tag{4.35}$$

$$\begin{aligned}
\lambda_{q1} &= \left(L_{q1q1}+L_{ls}\right)i_{q1} + L_{q1q2}i_{q2} + L_{q1q3}i_{q3} \\
\lambda_{d1} &= \left(L_{d1d1}+L_{ls}\right)i_{d1} + L_{d1d2}i_{d2} + L_{d1d3}i_{d3} + \lambda_{pmd1} \\
\lambda_{o1} &= L_{ls}i_{o1} \\
\lambda_{q2} &= \left(L_{q2q2}+L_{ls}\right)i_{q2} + L_{q2q1}i_{q1} + L_{q2q3}i_{q3} \\
\lambda_{d2} &= \left(L_{d2d2}+L_{ls}\right)i_{d2} + L_{d2d1}i_{d1q1} + L_{d2d3}i_{d3} + \lambda_{pmd2} \\
\lambda_{o2} &= L_{ls}i_{o2} \\
\lambda_{q3} &= \left(L_{q3q3}+L_{ls}\right)i_{q3} + L_{q3q1}i_{q1} + L_{q3q2}i_{q2} \\
\lambda_{d3} &= \left(L_{d3d3}+L_{ls}\right)i_{d3} + L_{d3d1}i_{d1} + L_{d3d2}i_{d2} + \lambda_{pmd3} \\
\lambda_{o3} &= L_{ls}i_{o3}
\end{aligned} \tag{4.36}$$

By substituting the flux linkages of equation (4.36) in to the voltage equations (3.59) to (3.61) the machines voltages in the rotor reference frame can be expressed as equation (4.37). The equivalent circuits of the q and d axis and also zero sequence are shown in Figures 4.9 to 4.11.

$$
\begin{aligned}
\begin{bmatrix} V_{q1} \\ V_{d1} \\ V_{o1} \\ V_{q2} \\ V_{d2} \\ V_{o2} \\ V_{q3} \\ V_{d3} \\ V_{o3} \end{bmatrix}
= r_{se} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \\ i_{q3} \\ i_{d3} \\ i_{o3} \end{bmatrix}
&+ \omega_r \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\left(\begin{bmatrix}
L_{q1q1} & 0 & 0 & L_{q1q2} & 0 & 0 & L_{q1q3} & 0 & 0 \\
0 & L_{d1d1} & 0 & 0 & L_{d1d2} & 0 & 0 & L_{d1d3} & 0 \\
0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\
L_{q2q1} & 0 & 0 & L_{q2q2} & 0 & 0 & L_{q2q3} & 0 & 0 \\
0 & L_{d2d1} & 0 & 0 & L_{d2d2} & 0 & 0 & L_{d2d3} & 0 \\
0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\
L_{q3q1} & 0 & 0 & L_{q3q2} & 0 & 0 & L_{q3q3} & 0 & 0 \\
0 & L_{d3d1} & 0 & 0 & L_{d3d2} & 0 & 0 & L_{d3d3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls}
\end{bmatrix}
\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \\ i_{q3} \\ i_{d3} \\ i_{o3} \end{bmatrix}
+ \begin{bmatrix} 0 \\ \lambda_{pmd1} \\ 0 \\ 0 \\ \lambda_{pmd1} \\ 0 \\ 0 \\ \lambda_{pmd1} \\ 0 \end{bmatrix} \right) \\
&+ p \left(\begin{bmatrix}
L_{q1q1}+L_{ls} & 0 & 0 & L_{q1q2} & 0 & 0 & L_{q1q3} & 0 & 0 \\
0 & L_{d1d1}+L_{ls} & 0 & 0 & L_{d1d2} & 0 & 0 & L_{d1d3} & 0 \\
0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\
L_{q2q1} & 0 & 0 & L_{q2q2}+L_{ls} & 0 & 0 & L_{q2q3} & 0 & 0 \\
0 & L_{d2d1} & 0 & 0 & L_{d2d2}+L_{ls} & 0 & 0 & L_{d2d3} & 0 \\
0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\
L_{q3q1} & 0 & 0 & L_{q3q2} & 0 & 0 & L_{q3q3}+L_{ls} & 0 & 0 \\
0 & L_{d3d1} & 0 & 0 & L_{d3d2} & 0 & 0 & L_{d3d3}+L_{ls} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls}
\end{bmatrix}
\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \\ i_{q3} \\ i_{d3} \\ i_{o3} \end{bmatrix}
+ \begin{bmatrix} 0 \\ \lambda_{pmd1} \\ 0 \\ 0 \\ \lambda_{pmd1} \\ 0 \\ 0 \\ \lambda_{pmd1} \\ 0 \end{bmatrix} \right)
\end{aligned}
\tag{4.37}
$$

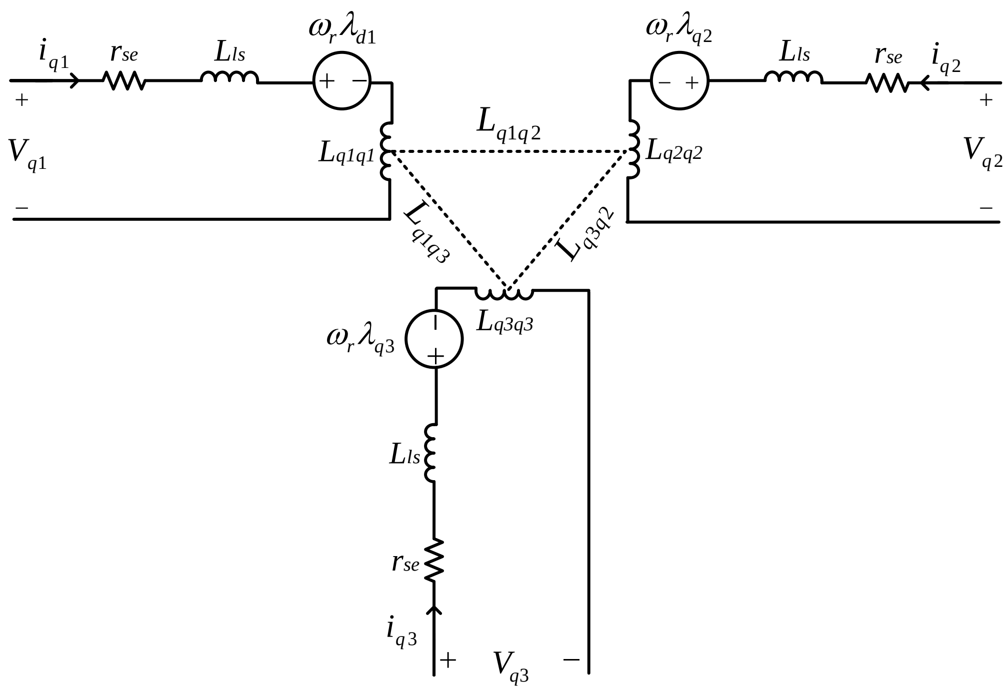


Figure 4.9: The equivalent circuit of the q axis.

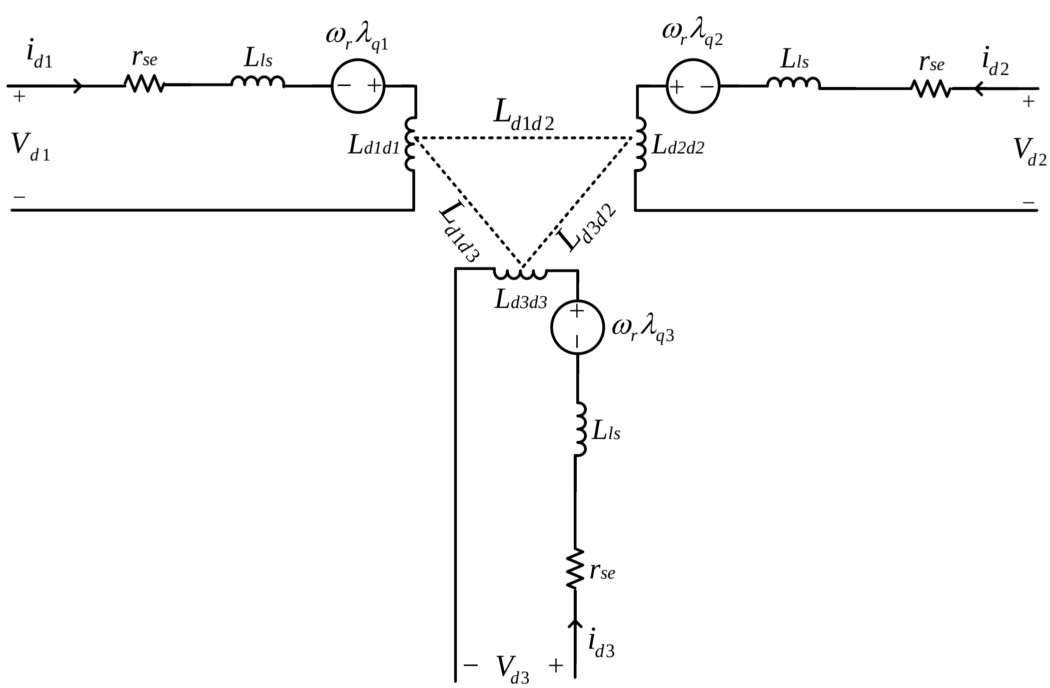


Figure 4.10: The equivalent circuit of the d axis.

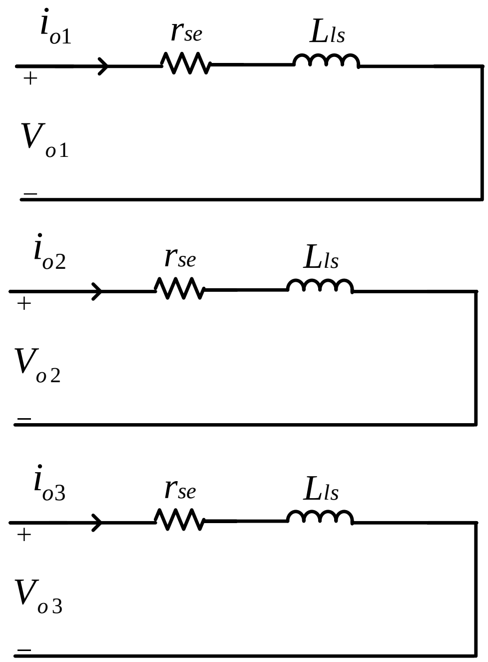


Figure 4.11: The equivalent circuit of the zero sequence.

Also, substituting the same inductances in to the torque equations presented in section 3.2 results in [122]:

$$T_{e1} = \frac{3}{2}\frac{P}{2}\left[\left(L_{d1d1}i_{d1} + L_{d1d2}i_{d2} + L_{d1d3}i_{d3}\right)i_{q1} - \left(L_{q1q1}i_{q1} + L_{q1q2}i_{q2} + L_{q1q3}i_{q3}\right)i_{d1}\right] + \frac{3}{2}\frac{P}{2}\left(\lambda_{pmd1}i_{q1}\right) \tag{4.38}$$

$$T_{e2} = \frac{3}{2}\frac{P}{2}\left[\left(L_{d1d2}i_{d1} + L_{d22}i_{d2} + L_{d2d3}i_{d3}\right)i_{q2} - \left(L_{q1q2}i_{q1} + L_{q22}i_{q2} + L_{q2q3}i_{q3}\right)i_{d2}\right] + \frac{3}{2}\frac{P}{2}\left(\lambda_{pmd2}i_{q2}\right) \tag{4.39}$$

$$T_{e3} = \frac{3}{2}\frac{P}{2}\left[\left(L_{d1d3}i_{d1} + L_{d2d3}i_{d2} + L_{d3d3}i_{d3}\right)i_{q3} - \left(L_{q1q3}i_{q1} + L_{q2q3}i_{q2} + L_{q3q3}i_{q3}\right)i_{d3}\right] + \frac{3}{2}\frac{P}{2}\left(\lambda_{pmd3}i_{q3}\right) \tag{4.40}$$

And finally the mechanical dynamic equation can be expressed as:

$$T_e = T_{e1} + T_{e2} + T_{e3} = J\left(\frac{2}{P}\right)p\omega_r + T_L + B\omega_r \tag{4.41}$$

Table 4.2 shows the inductances of the machines in the rotor reference frame for the cases of symmetrical and asymmetrical connection. It can be seen that the inductances have the same values for q and d axis of the rotor reference frame.

Table 4.2 The inductances of the symmetrical and asymmetrical machines in rotor reference frame.

$L_0 = \pi\mu_o rlN_1{}^2 a_0$, $L_2 = \pi\mu_o rlN_1{}^2 \frac{a_1}{2}$,		
The Inductance in Rotor Reference Frame	Symmetrical	Asymmetrical
L_{q11}	$\frac{3}{2}(L_0 + L_2)$	$\frac{3}{2}(L_0 + L_2)$
L_{q1d1}	0	0
L_{d1q1}	0	0
L_{d11}	$\frac{3}{2}(L_0 - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{q22}	$\frac{3}{2}(L_0 + L_2)$	$\frac{3}{2}(L_0 + L_2)$
L_{q2d2}	0	0
L_{d2q2}	0	0
L_{d22}	$\frac{3}{2}(L_0 - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{q33}	$\frac{3}{2}(L_0 + L_2)$	$\frac{3}{2}(L_0 + L_2)$
L_{q3d3}	0	0
L_{d3q3}	0	0
L_{d33}	$\frac{3}{2}(L_0 - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d1d3}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d3d1}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d1q3}	0	0

L_{d3q1}	0	0
L_{q1d3}	0	0
L_{q3d1}	0	0
L_{q1q3}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$
L_{q3q1}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$
L_{d2d3}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d3d2}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d2q3}	0	0
L_{d3q2}	0	0
L_{q2d3}	0	0
L_{q3d2}	0	0
L_{q2q3}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$
L_{q3q2}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$
L_{d1d2}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d2d1}	$\frac{3}{2}(L_o - L_2)$	$\frac{3}{2}(L_0 - L_2)$
L_{d1q2}	0	0
L_{d2q1}	0	0
L_{q1d2}	0	0
L_{q2d1}	0	0
L_{q1q2}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$
L_{q2q1}	$\frac{3}{2}(L_o + L_2)$	$\frac{3}{2}(L_o + L_2)$

4.3.1 The MMF Analysis

In this section using the winding functions generated for the symmetrical and asymmetrical machines the general equations for the stator MMF is generated for symmetrical and asymmetrical machines. The harmonic currents generated by a polluted voltage source such as an inverter or a grid with voltage harmonics in winding 'w' of a machine can be defined as in equation 4.42 [139].

$$I_w(t) = I_1 \sin(\omega_s t - kd_k\beta) + I_3 \sin 3(\omega_s t - kd_k\beta) + I_5 \sin 5(\omega_s t - kd_k\beta) + I_7 \sin 7(\omega_s t - kd_k\beta) \quad (4.42)$$

The winding function of the winding 'w' is presented in equation (4.9). Using the equations (4.9) and (4.42) the MMF of the winding 'w' can be expressed as:

$$\begin{aligned} N_w(\theta)I_w(t) = &\left(N_1 \sin(\theta - kd_k\beta) + N_3 \sin 3(\theta - kd_k\beta) + N_5 \sin 5(\theta - kd_k\beta) + N_7 \sin 7(\theta - kd_k\beta)\right)\times \\ &\left(I_1 \sin(\omega_s t - kd_k\beta) + I_3 \sin 3(\omega_s t - kd_k\beta) + I_5 \sin 5(\omega_s t - kd_k\beta) + I_7 \sin 7(\omega_s t - kd_k\beta)\right) \end{aligned} \quad (4.43)$$

Expanding the equation (4.43) results in:

$$
N_w(\theta)I_w(t) =
$$

$$
\frac{1}{2}\begin{pmatrix} I_1N_1\left(\cos\left(\omega_s t-\theta\right)-\cos\left(\omega_s t+\theta-2kd_k\beta\right)\right)+ \\ I_1N_3\left(\cos\left(\omega_s t-3\theta+2kd_k\beta\right)-\cos\left(\omega_s t+3\theta-4kd_k\beta\right)\right)+ \\ I_1N_5\left(\cos\left(\omega_s t-5\theta+4kd_k\beta\right)-\cos\left(\omega_s t+5\theta-6kd_k\beta\right)\right)+ \\ I_1N_7\left(\cos\left(\omega_s t-7\theta+6kd_k\beta\right)-\cos\left(\omega_s t+7\theta-8kd_k\beta\right)\right) \end{pmatrix}+\frac{1}{2}\begin{pmatrix} I_3N_1\left(\cos\left(3\omega_s t-\theta-2kd_k\beta\right)-\cos\left(3\omega_s t+\theta-4kd_k\beta\right)\right)+ \\ I_3N_3\left(\cos\left(3\omega_s t-3\theta\right)-\cos\left(3\omega_s t+3\theta-6kd_k\beta\right)\right)+ \\ I_3N_5\left(\cos\left(3\omega_s t-5\theta+2kd_k\beta\right)-\cos\left(3\omega_s t+5\theta-8kd_k\beta\right)\right)+ \\ I_3N_7\left(\cos\left(3\omega_s t-7\theta+4kd_k\beta\right)-\cos\left(3\omega_s t+7\theta-10kd_k\beta\right)\right) \end{pmatrix}+
$$

$$
\frac{1}{2}\begin{pmatrix} I_5N_1\left(\cos\left(5\omega_s t-4kd_k\beta-\theta\right)-\cos\left(5\omega_s t-6kd_k\beta+\theta\right)\right)+ \\ I_5N_3\left(\cos\left(5\omega_s t-2kd_k\beta-3\theta\right)-\cos\left(5\omega_s t-8kd_k\beta+3\theta\right)\right)+ \\ I_5N_5\left(\cos\left(5\omega_s t-5\theta\right)-\cos\left(5\omega_s t-10kd_k\beta+5\theta\right)\right)+ \\ I_5N_7\left(\cos\left(5\omega_s t+2kd_k\beta-7\theta\right)-\cos\left(5\omega_s t-12kd_k\beta+7\theta\right)\right) \end{pmatrix}+\frac{1}{2}\begin{pmatrix} I_7N_1\left(\cos\left(7\omega_s t-6kd_k\beta-\theta\right)-\cos\left(7\omega_s t-8kd_k\beta+\theta\right)\right)+ \\ I_7N_3\left(\cos\left(7\omega_s t-4kd_k\beta-3\theta\right)-\cos\left(7\omega_s t-10kd_k\beta+3\theta\right)\right)+ \\ I_7N_5\left(\cos\left(7\omega_s t-2kd_k\beta-5\theta\right)-\cos\left(7\omega_s t-12kd_k\beta+5\theta\right)\right)+ \\ I_7N_7\left(\cos\left(7\omega_s t-7\theta\right)-\cos\left(7\omega_s t-14kd_k\beta+7\theta\right)\right) \end{pmatrix} \tag{4.44}
$$

For each of the machines there are nine equations like equation (4.44) to describe the MMF of each phase of the machine. For each machine (three phase set) the total MMF of the stator is the sum of the corresponding phases MMF.

$$MMF = \sum_{w=i,i+3,i+6} N_w(\theta) I_w(t) \tag{4.45}$$

By substituting the different parameters (K and d_k) from equation (4.17) and Table 4.1 into equation (4.45) and using MATLAB/Symbolic for simplifying that the MMF for each machine (three phase) set of symmetrical and asymmetrical machines could be presented. For symmetrical case the MMF of the machines 1,2 and 3 are presented as equation (4.46), (4.47) and (4.48) respectively.

$$MMF_1 = \sum_{w=1,4,7} N_w(\theta) I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1\left(\cos\left(\omega_s t-\theta\right)\right)+ \\ I_1N_3\left(\cos\left(\omega_s t-3\theta+24\beta\right)\right)+ \\ I_1N_5\left(\cos\left(\omega_s t-5\theta+48\beta\right)\right)+ \\ I_1N_7\left(\cos\left(\omega_s t-7\theta+72\beta\right)\right) \end{pmatrix}+\frac{3}{2}\begin{pmatrix} I_3N_1\left(\cos\left(3\omega_s t-\theta-24\beta\right)\right)+ \\ I_3N_3\left(\cos\left(3\omega_s t-3\theta\right)\right)+ \\ I_3N_5\left(\cos\left(3\omega_s t-5\theta+24\beta\right)\right)+ \\ I_3N_7\left(\cos\left(3\omega_s t-7\theta+48\beta\right)\right) \end{pmatrix}+ \tag{4.46}$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1\left(\cos\left(5\omega_s t-48\beta-\theta\right)\right)+ \\ I_5N_3\left(\cos\left(5\omega_s t-24\beta-3\theta\right)\right)+ \\ I_5N_5\left(\cos\left(5\omega_s t-5\theta\right)\right)+ \\ I_5N_7\left(\cos\left(5\omega_s t+24\beta-7\theta\right)\right) \end{pmatrix}+\frac{3}{2}\begin{pmatrix} I_7N_1\left(\cos\left(7\omega_s t-72\beta-\theta\right)\right)+ \\ I_7N_3\left(\cos\left(7\omega_s t-48\beta-3\theta\right)\right)+ \\ I_7N_5\left(\cos\left(7\omega_s t-24\beta-5\theta\right)\right)+ \\ I_7N_7\left(\cos\left(7\omega_s t-7\theta\right)\right) \end{pmatrix}$$

$$MMF_2 = \sum_{w=2,5,8} N_w(\theta) I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t - \theta)) + \\ I_1N_3(\cos(\omega_s t - 3\theta + 42\beta)) + \\ I_1N_5(\cos(\omega_s t - 5\theta + 66\beta)) + \\ I_1N_7(\cos(\omega_s t - 7\theta + 90\beta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t - \theta - 42\beta)) + \\ I_3N_3(\cos(3\omega_s t - 3\theta)) + \\ I_3N_5(\cos(3\omega_s t - 5\theta + 42\beta)) + \\ I_3N_7(\cos(3\omega_s t - 7\theta + 84\beta)) \end{pmatrix} + \tag{4.47}$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t - 66\beta - \theta)) + \\ I_5N_3(\cos(5\omega_s t - 42\beta - 3\theta)) + \\ I_5N_5(\cos(5\omega_s t - 5\theta)) + \\ I_5N_7(\cos(5\omega_s t + 42\beta - 7\theta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t - 90\beta - \theta)) + \\ I_7N_3(\cos(7\omega_s t - 66\beta - 3\theta)) + \\ I_7N_5(\cos(7\omega_s t - 42\beta - 5\theta)) + \\ I_7N_7(\cos(7\omega_s t - 7\theta)) \end{pmatrix}$$

$$MMF_3 = \sum_{w=3,6,9} N_w(\theta) I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t - \theta)) + \\ I_1N_3(\cos(\omega_s t - 3\theta + 24\beta)) + \\ I_1N_5(\cos(\omega_s t - 5\theta + 48\beta)) + \\ I_1N_7(\cos(\omega_s t - 7\theta + 72\beta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t - \theta - 24\beta)) + \\ I_3N_3(\cos(3\omega_s t - 3\theta)) + \\ I_3N_5(\cos(3\omega_s t - 5\theta + 24\beta)) + \\ I_3N_7(\cos(3\omega_s t - 7\theta + 48\beta)) \end{pmatrix} + \tag{4.48}$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t - 48\beta - \theta)) + \\ I_5N_3(\cos(5\omega_s t - 24\beta - 3\theta)) + \\ I_5N_5(\cos(5\omega_s t - 5\theta)) + \\ I_5N_7(\cos(5\omega_s t + 24\beta - 7\theta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t - 72\beta - \theta)) + \\ I_7N_3(\cos(7\omega_s t - 48\beta - 3\theta)) + \\ I_7N_5(\cos(7\omega_s t - 24\beta - 5\theta)) + \\ I_7N_7(\cos(7\omega_s t - 7\theta)) \end{pmatrix}$$

The MMF of the machine is equal to the sum of the equations (4.46) to (4.48) which is presented in equation (4.49)

$$MMF = \sum_{w=1}^{9} N_w(\theta)I_w(t) =$$

$$\frac{9}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t-\theta))+I_1N_3(\cos(\omega_s t-3\theta+24\beta))+ \\ I_1N_5(\cos(\omega_s t-5\theta+48\beta))+I_1N_7(\cos(\omega_s t-7\theta+72\beta)) \end{pmatrix}+$$

$$\frac{9}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t-\theta-24\beta))+I_3N_3(\cos(3\omega_s t-3\theta))+ \\ I_3N_5(\cos(3\omega_s t-5\theta+24\beta))+I_3N_7(\cos(3\omega_s t-7\theta+48\beta)) \end{pmatrix}+ \qquad (4.49)$$

$$\frac{9}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t-48\beta-\theta))+I_5N_3(\cos(5\omega_s t-24\beta-3\theta))+ \\ I_5N_5(\cos(5\omega_s t-5\theta))+I_5N_7(\cos(5\omega_s t+24\beta-7\theta)) \end{pmatrix}+$$

$$\frac{9}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t-72\beta-\theta))+I_7N_3(\cos(7\omega_s t-48\beta-3\theta))+ \\ I_7N_5(\cos(7\omega_s t-24\beta-5\theta))+I_7N_7(\cos(7\omega_s t-7\theta)) \end{pmatrix}$$

For asymmetrical case, the MMF of the machines 1,2 and 3 are presented as equation (4.49), (4.50) and (4.51) respectively.

$$MMF_1 = \sum_{w=1,4,7} N_w(\theta)I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t-\theta))+ \\ I_1N_3(\cos(\omega_s t-3\theta+24\beta))+ \\ I_1N_5(\cos(\omega_s t-5\theta+48\beta))+ \\ I_1N_7(\cos(\omega_s t-7\theta+72\beta)) \end{pmatrix}+\frac{3}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t-\theta-24\beta))+ \\ I_3N_3(\cos(3\omega_s t-3\theta))+ \\ I_3N_5(\cos(3\omega_s t-5\theta+24\beta))+ \\ I_3N_7(\cos(3\omega_s t-7\theta+48\beta)) \end{pmatrix}+ \qquad (4.50)$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t-48\beta-\theta))+ \\ I_5N_3(\cos(5\omega_s t-24\beta-3\theta))+ \\ I_5N_5(\cos(5\omega_s t-5\theta))+ \\ I_5N_7(\cos(5\omega_s t+24\beta-7\theta)) \end{pmatrix}+\frac{3}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t-72\beta-\theta))+ \\ I_7N_3(\cos(7\omega_s t-48\beta-3\theta))+ \\ I_7N_5(\cos(7\omega_s t-24\beta-5\theta))+ \\ I_7N_7(\cos(7\omega_s t-7\theta)) \end{pmatrix}$$

$$MMF_2 = \sum_{w=2,5,8} N_w(\theta) I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t-\theta))+ \\ I_1N_3(\cos(\omega_s t-3\theta+30\beta))+ \\ I_1N_5(\cos(\omega_s t-5\theta+54\beta))+ \\ I_1N_7(\cos(\omega_s t-7\theta+78\beta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t-\theta-30\beta))+ \\ I_3N_3(\cos(3\omega_s t-3\theta))+ \\ I_3N_5(\cos(3\omega_s t-5\theta+30\beta))+ \\ I_3N_7(\cos(3\omega_s t-7\theta+54\beta)) \end{pmatrix} + \tag{4.51}$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t-54\beta-\theta))+ \\ I_5N_3(\cos(5\omega_s t-30\beta-3\theta))+ \\ I_5N_5(\cos(5\omega_s t-5\theta))+ \\ I_5N_7(\cos(5\omega_s t+30\beta-7\theta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t-78\beta-\theta))+ \\ I_7N_3(\cos(7\omega_s t-54\beta-3\theta))+ \\ I_7N_5(\cos(7\omega_s t-30\beta-5\theta))+ \\ I_7N_7(\cos(7\omega_s t-7\theta)) \end{pmatrix}$$

$$MMF_3 = \sum_{w=3,6,9} N_w(\theta) I_w(t) =$$

$$\frac{3}{2}\begin{pmatrix} I_1N_1(\cos(\omega_s t-\theta))+ \\ I_1N_3(\cos(\omega_s t-3\theta+36\beta))+ \\ I_1N_5(\cos(\omega_s t-5\theta+60\beta))+ \\ I_1N_7(\cos(\omega_s t-7\theta+86\beta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_3N_1(\cos(3\omega_s t-\theta-36\beta))+ \\ I_3N_3(\cos(3\omega_s t-3\theta))+ \\ I_3N_5(\cos(3\omega_s t-5\theta+36\beta))+ \\ I_3N_7(\cos(3\omega_s t-7\theta+60\beta)) \end{pmatrix} + \tag{4.52}$$

$$\frac{3}{2}\begin{pmatrix} I_5N_1(\cos(5\omega_s t-60\beta-\theta))+ \\ I_5N_3(\cos(5\omega_s t-36\beta-3\theta))+ \\ I_5N_5(\cos(5\omega_s t-5\theta))+ \\ I_5N_7(\cos(5\omega_s t+36\beta-7\theta)) \end{pmatrix} + \frac{3}{2}\begin{pmatrix} I_7N_1(\cos(7\omega_s t-84\beta-\theta))+ \\ I_7N_3(\cos(7\omega_s t-60\beta-3\theta))+ \\ I_7N_5(\cos(7\omega_s t-36\beta-5\theta))+ \\ I_7N_7(\cos(7\omega_s t-7\theta)) \end{pmatrix}$$

The MMF of the machine is equal to the sum of the equations (4.50) to (4.52) which is presented in equation (4.53).

$$MMF = \sum_{w=1}^{9} N_w(\theta) I_w(t) =$$

$$\frac{9}{2}\left(I_1N_1(\cos(\omega_s t-\theta))+I_3N_3(\cos(3\omega_s t-3\theta))+I_5N_5(\cos(5\omega_s t-5\theta))+I_7N_7(\cos(7\omega_s t-7\theta))\right) \tag{4.53}$$

It could be seen that the asymmetrical machine lacks the MMF harmonics that result from the interactions between the different harmonics of the stator current and the winding function. In the asymmetrical connection these harmonics simply cancel each other's.

4.4 Simulation of the Symmetrical Nine-Phase Machine

In this section the average model that was generated is simulated using MATLAB/Simulink for symmetrical case. First step is to put the machine parameters from Tables 3.1 and 4.1 in to the generated inductances and plugging the resulting inductances into the voltage equations of the Section 4.3. The machine inductances in the rotor reference frame are shown in Figures 4.12 to 4.17.

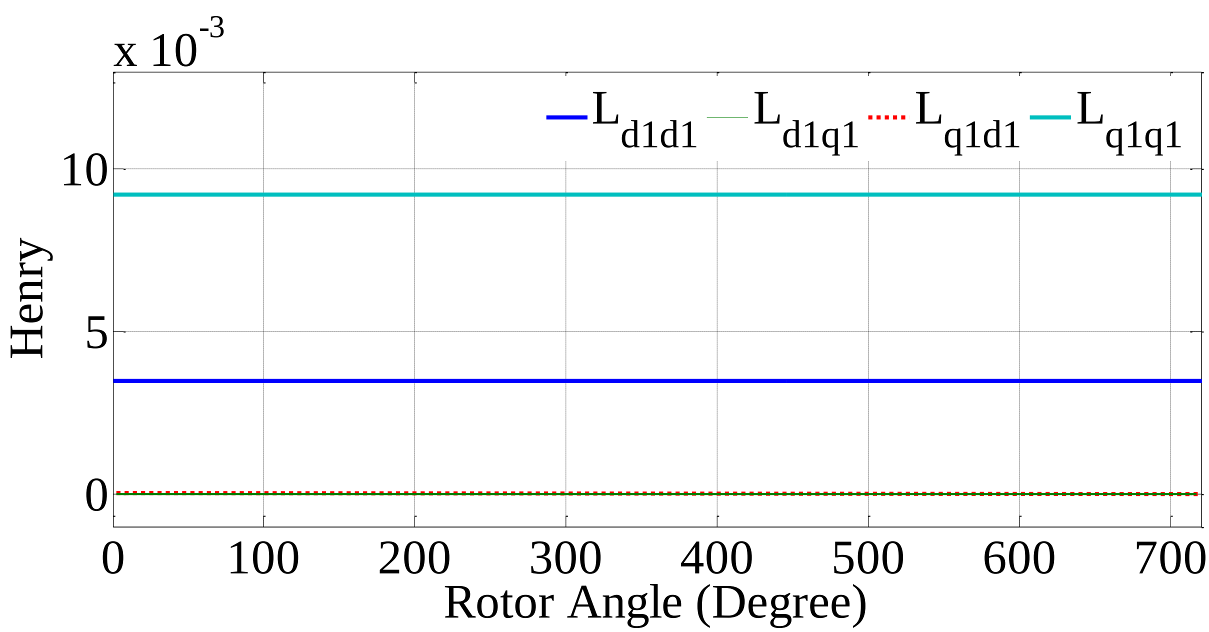


Figure 4.12: The inductances of the machine 1 in the rotor reference frame.

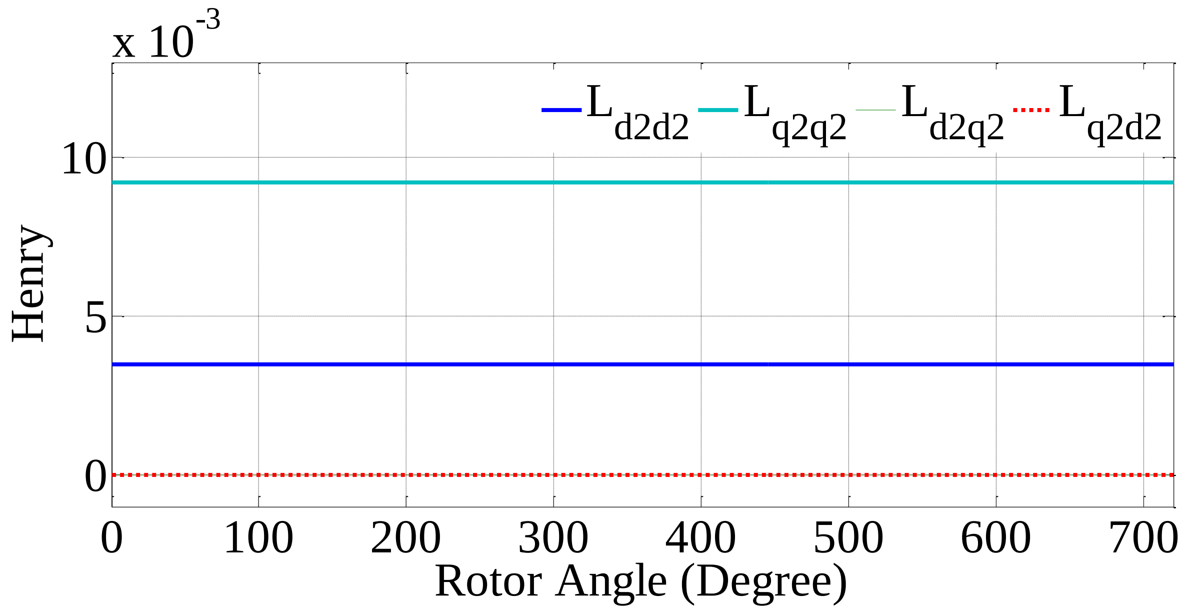


Figure 4.13: The inductances of the machine 2 in the rotor reference frame.

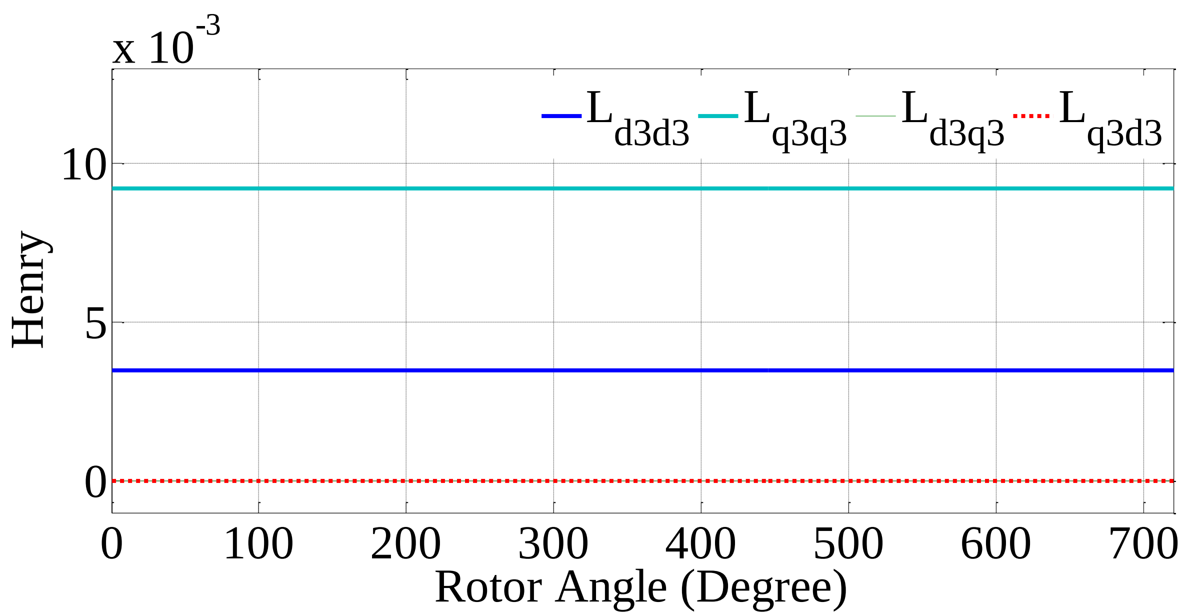


Figure 4.14: The inductances of the machine 3 in the rotor reference frame.

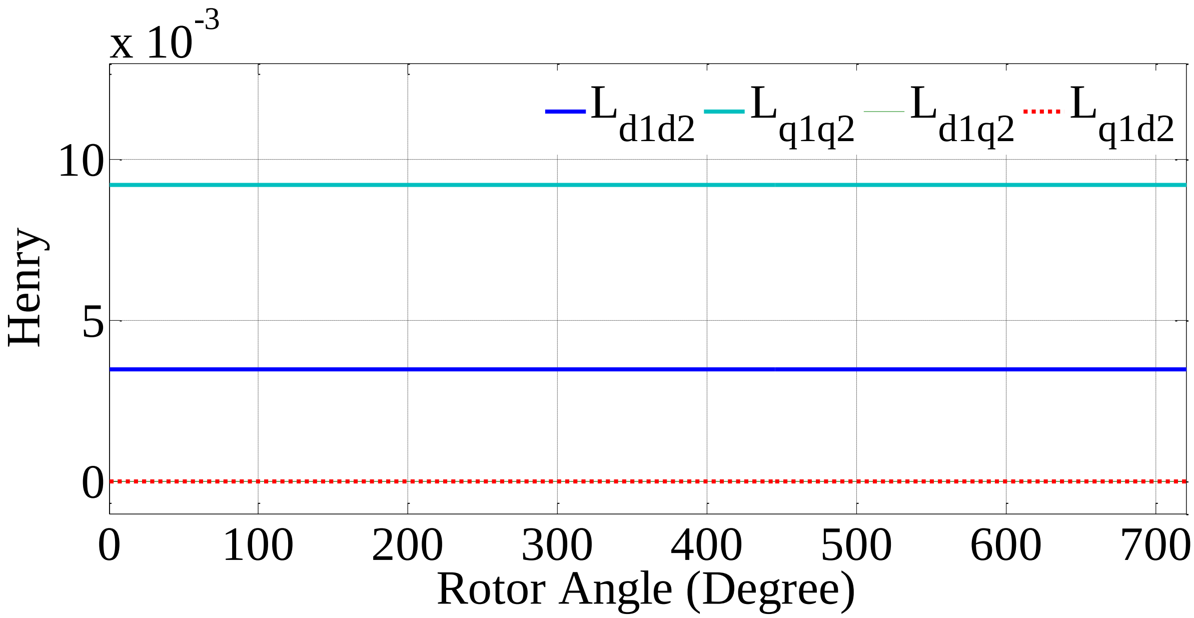


Figure 4.15: The mutual inductances between machines 1 and 2 in the rotor reference frame.

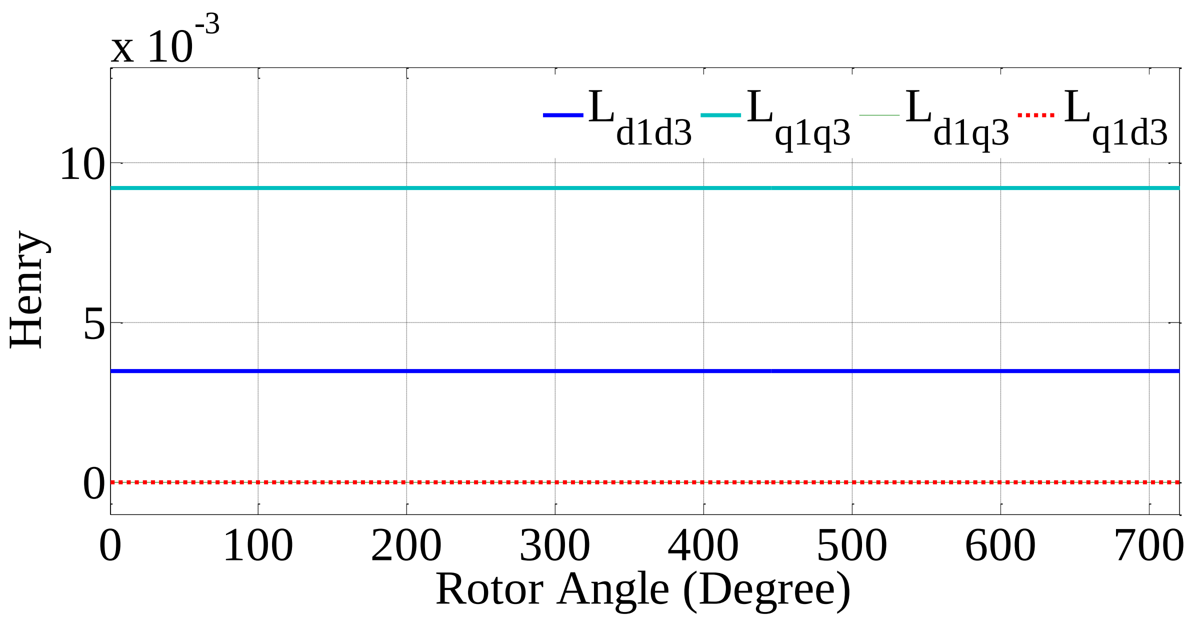


Figure 4.16: The mutual inductances between machines 1 and 3 in the rotor reference frame.

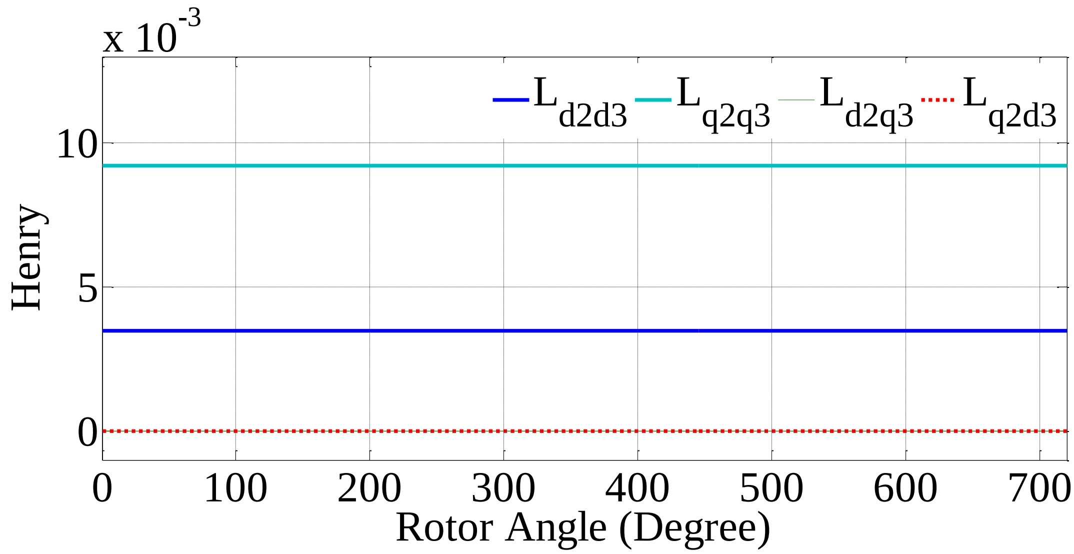


Figure 4.17: The mutual inductances between machines 2 and 3 in the rotor reference frame.

The magnetic flux linkage of the permanent magnet blocks in the rotor reference frame are also shown in Figures 4.18 to 4.20.

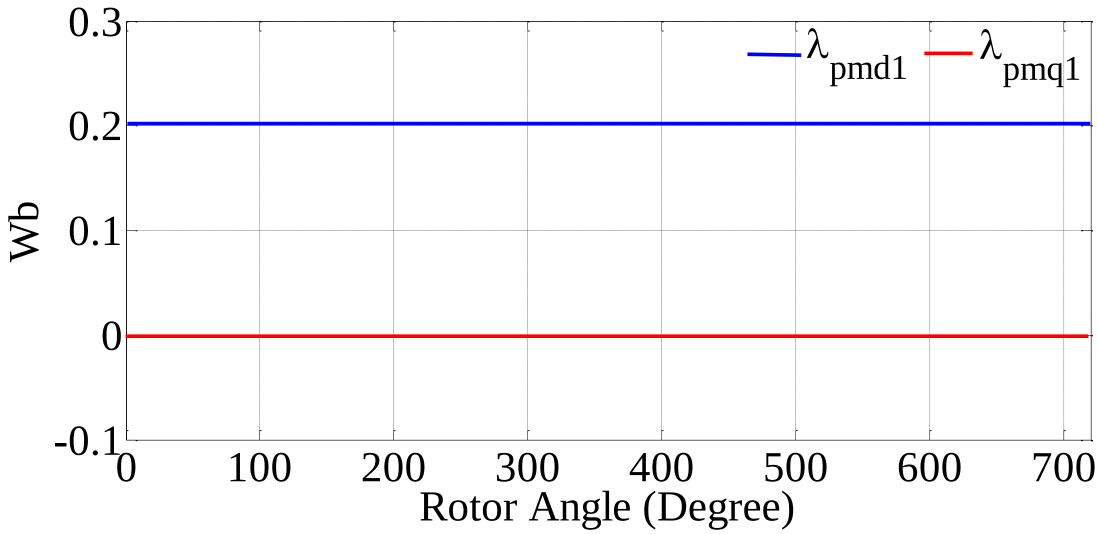


Figure 4.18: The d and q axis flux linkage due to the rotor permanent magnets of machine 1 in rotor reference frame.

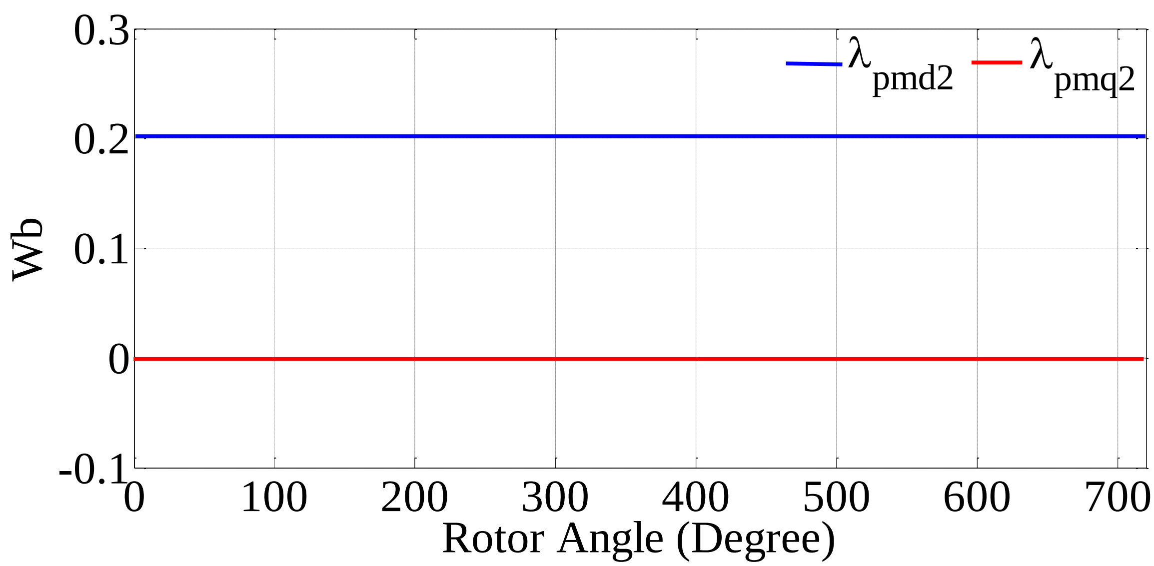


Figure 4.19: The d and q axis flux linkage due to the rotor permanent magnets of machine 2 in rotor reference frame.

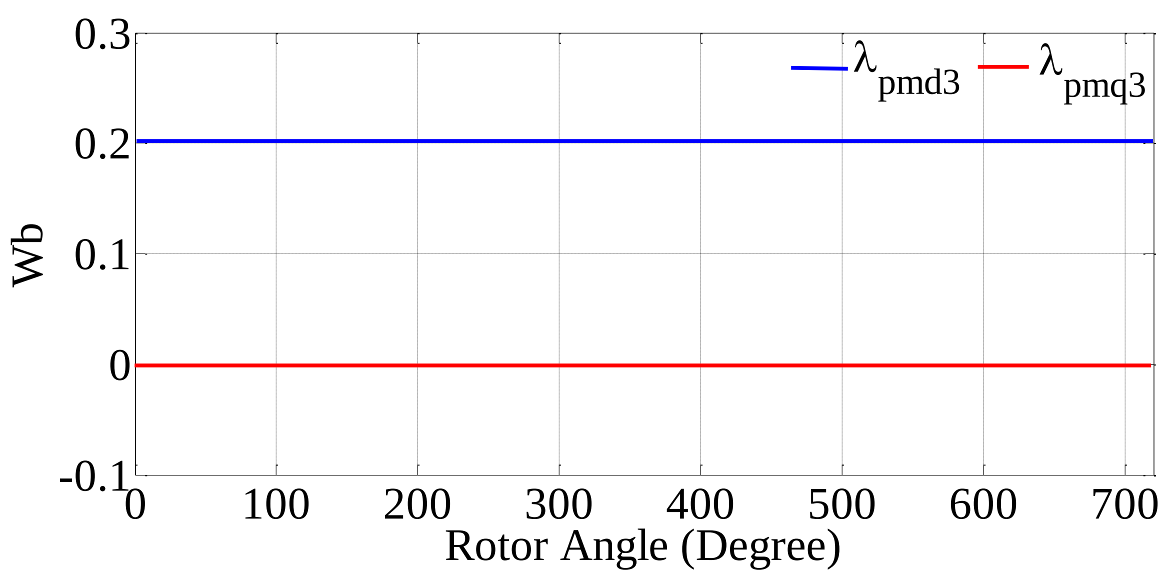


Figure 4.20: The d and q axis flux linkage due to the rotor permanent magnets of machine 3 in rotor reference frame.

Three sets of 60 (Hz) 110 (Volts) three-phase voltages (as shown in Figure 4.21) are applied to the model while the initial rotor speed is 377 rad/sec. After the initial transients are passed the load torque is applied to the machine. Figure 4.22 shows the rotor speed of the machine.

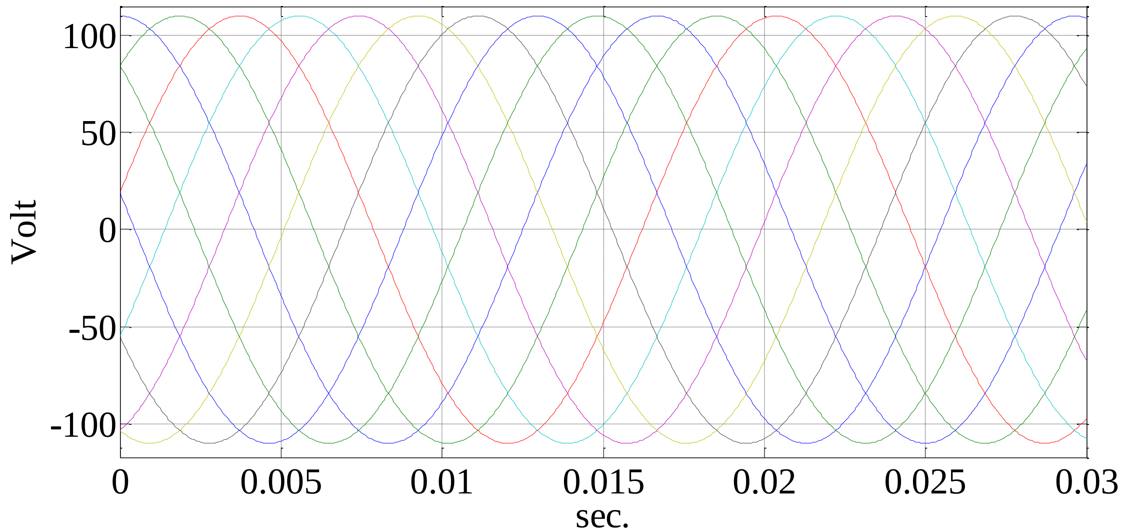


Figure 4.21: The phase voltages.

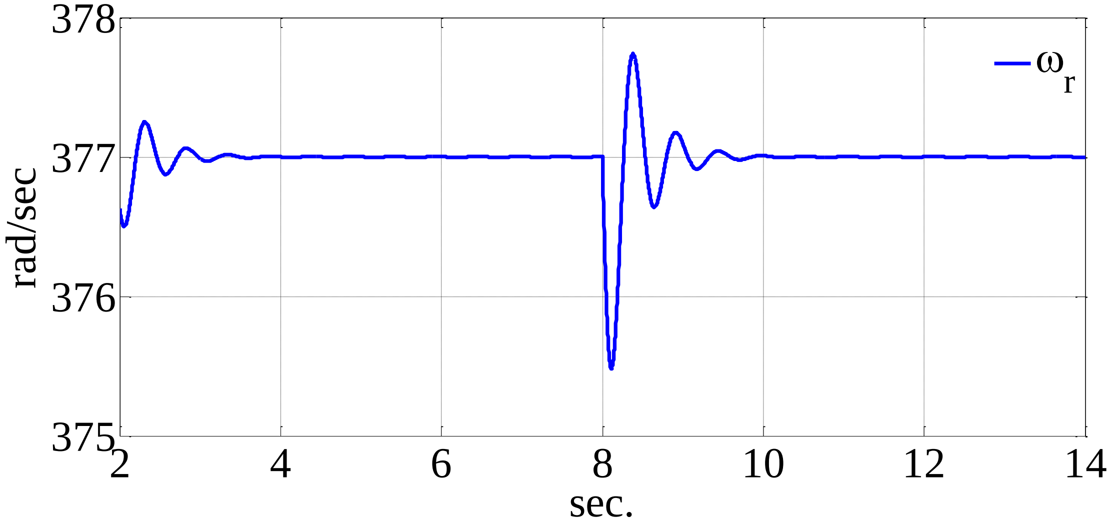
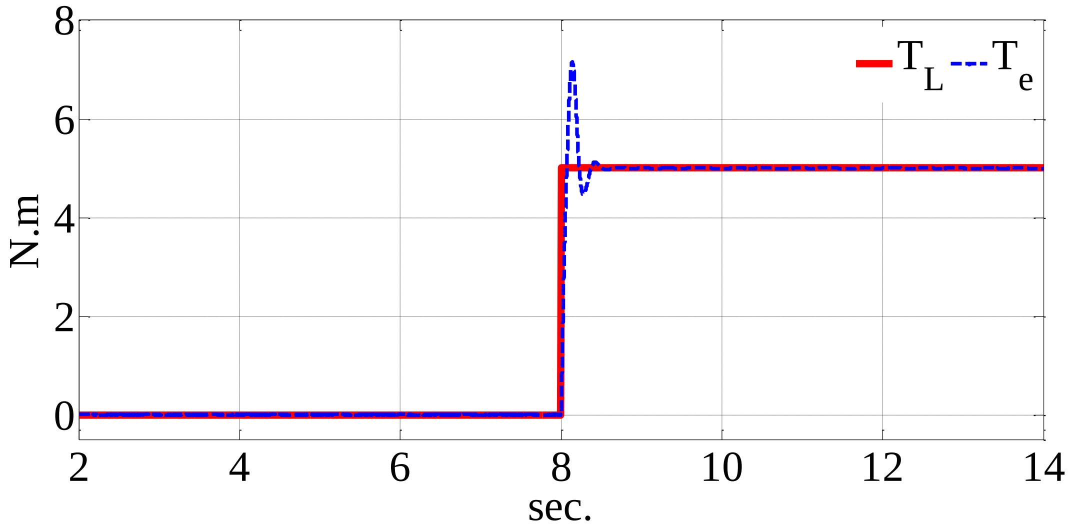
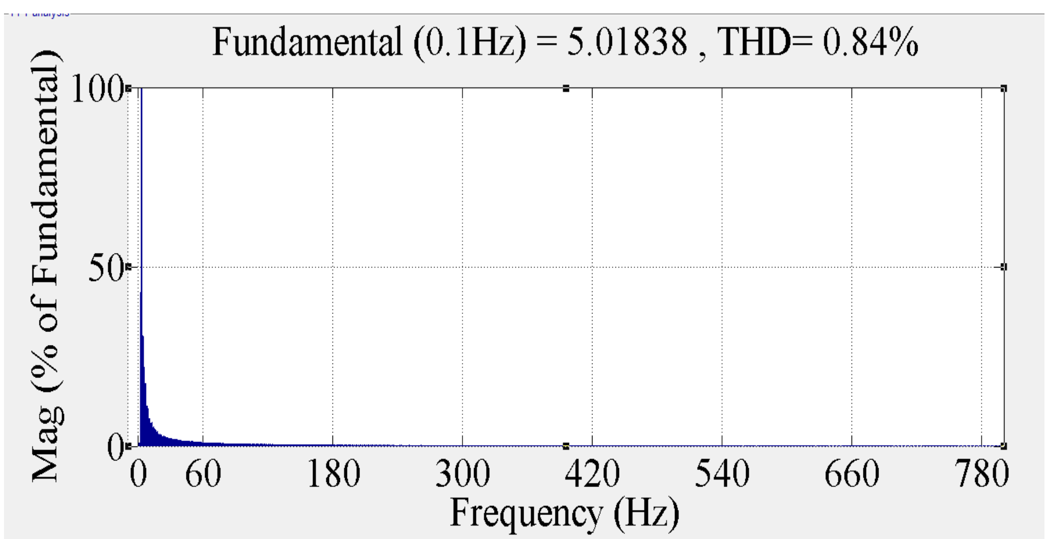


Figure 4.22: The rotor speed.

Figure 4.23 (a) shows the electromagnetic and load torque together. As it can be seen after initial transients have died and the torque goes to zero. After applying the load, the machine starts generating electromagnetic torque to keep the synchronous speed. The spectrum of the electromagnetic torque is shown in the Figure 4.23 (b). The main harmonic frequency is zero and the rest of the higher harmonics have a relatively lower magnitude compared to the main one. The electromagnetic torque is generated by three machines and each of them shares a part of that.

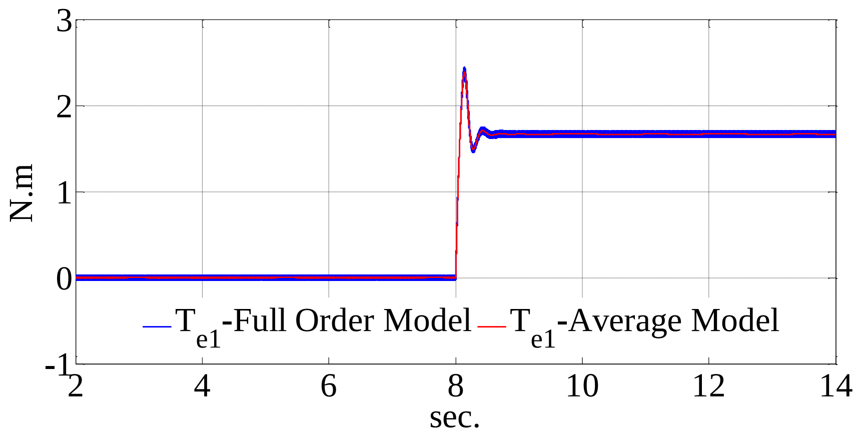


(a)

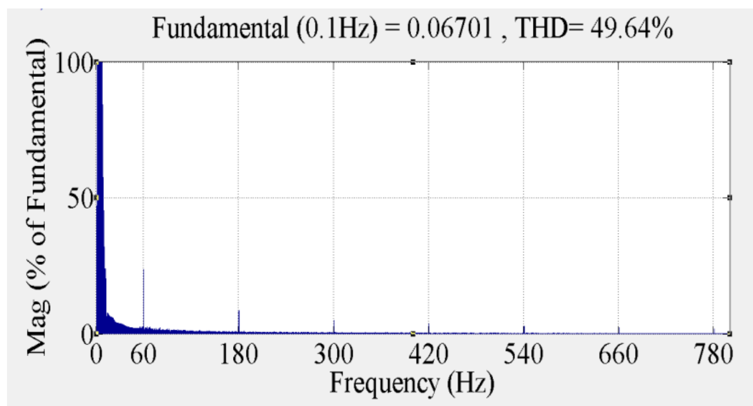


(b)

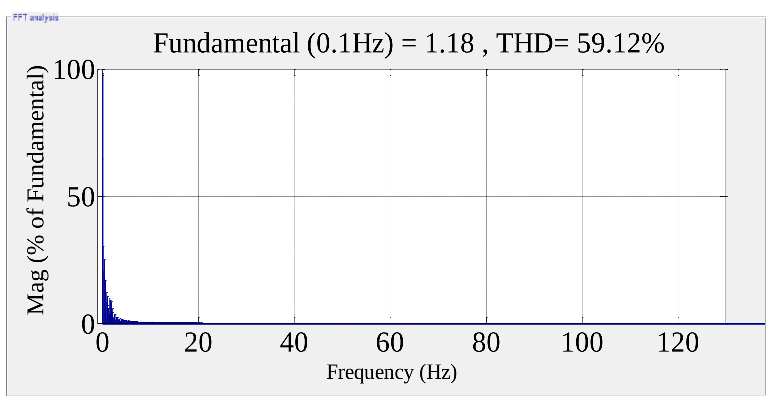
Figure 4.23 (a) The total electromagnetic and load torque, (b) The spectrum of the electromagnetic torque of the machine.



(a)

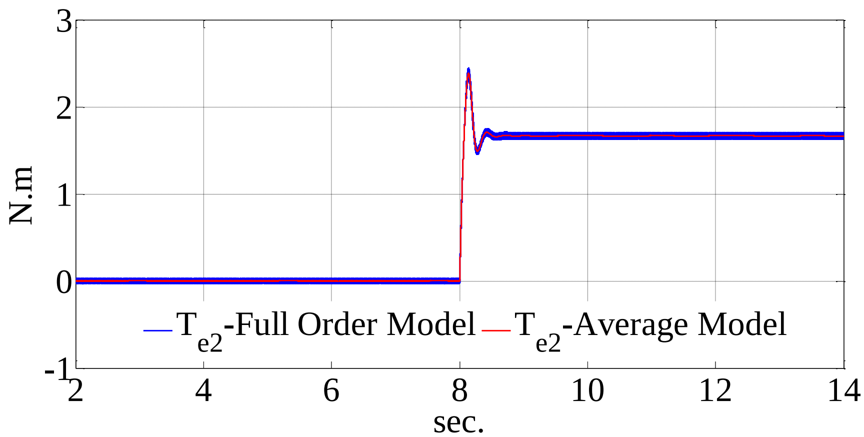


(b)

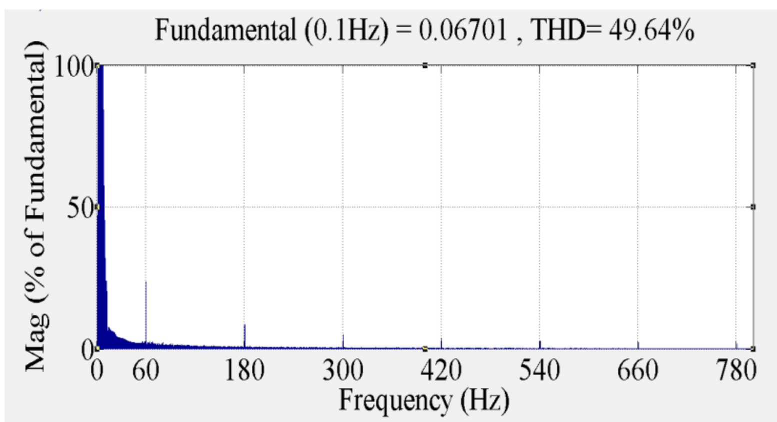


(c)

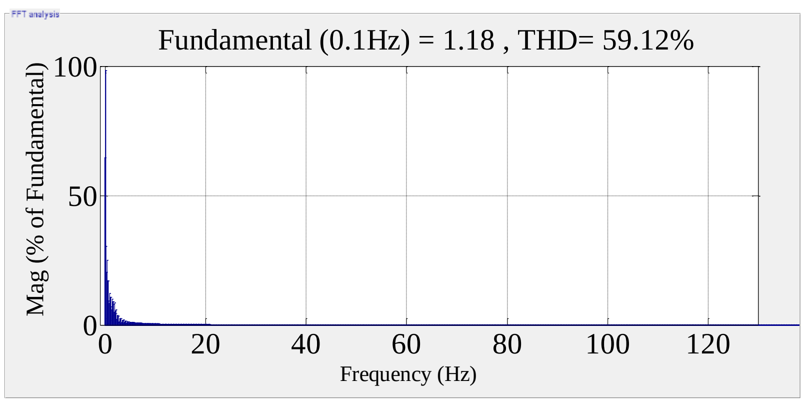
Figure 4.24: (a) The electromagnetic torque generated by machine 1 for average and full order model, (b) The spectrum of the electromagnetic torque of the full order model, (c) The spectrum of the electromagnetic torque for the average model.



(a)

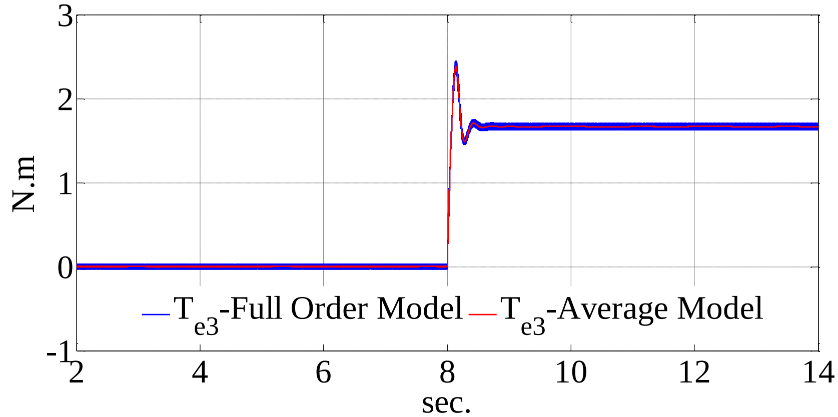


(b)

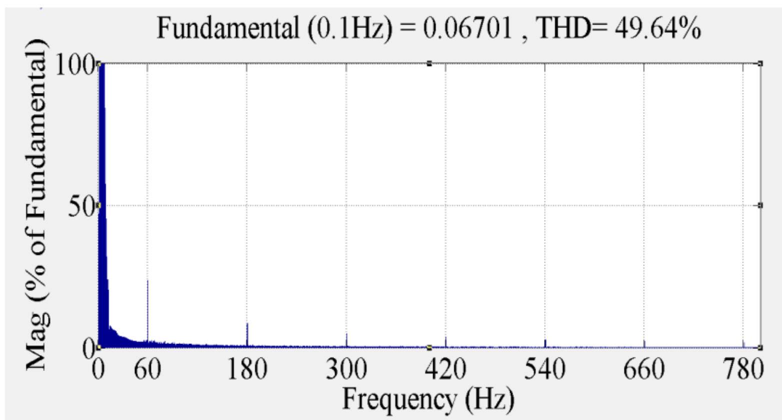


(c)

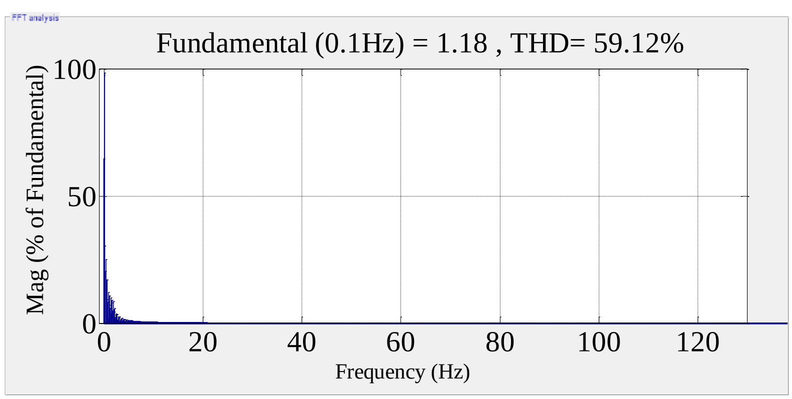
Figure 4.25: (a) The electromagnetic torque generated by machine 2 for average and full order model, (b) The spectrum of the electromagnetic torque of the full order model, (c) The spectrum of the electromagnetic torque for the average model.



(a)



(b)



(c)

Figure 4.26: (a) The electromagnetic torque generated by machine 3 for average and full order model, (b) The spectrum of the electromagnetic torque of the full order model, (c) The spectrum of the electromagnetic torque for the average model.

Figures 4.24 to 4.26 show the electromagnetic torque of each machine along with the electromagnetic torque of the machine from full order modelling in chapter 3. Also the spectrums of the electromagnetic torques of the average and full order model are shown in the same figures. It can be seen that the full order model has some harmonics around the voltage source frequency while the average model does not generate that harmonics.

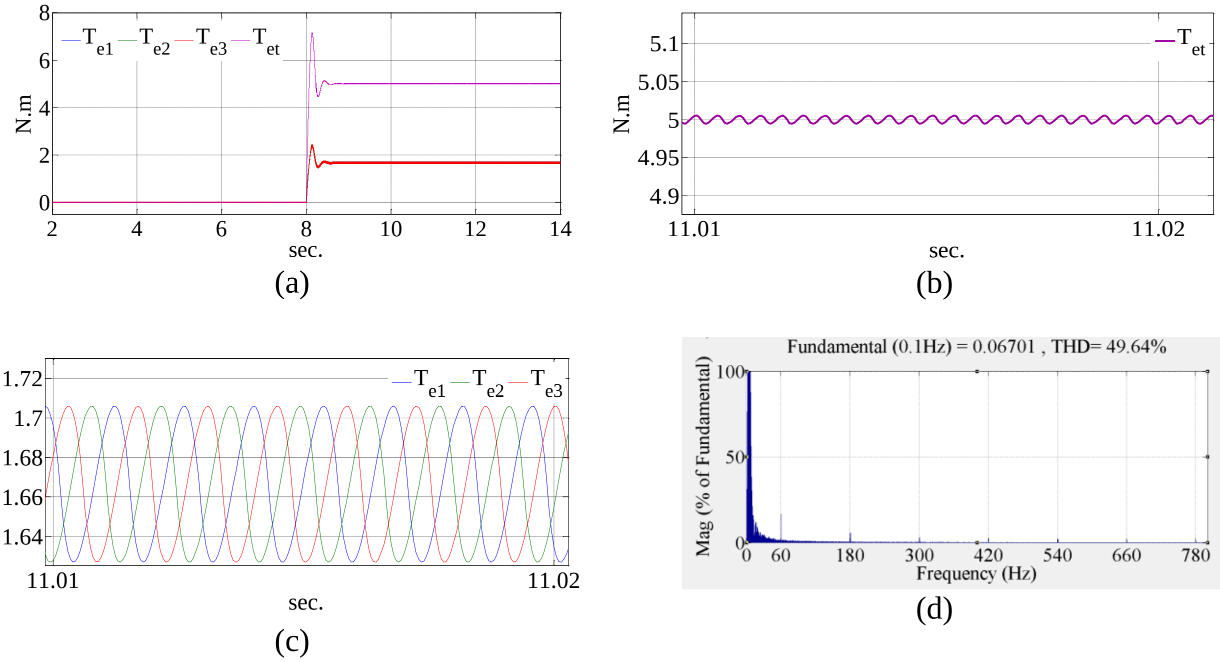


Figure 4.27: (a) The electromagnetic torque generated by all machines, (b) The zoomed view of the total torque, (d) The zoomed view of the torques of the individual machines (c) The spectrum of the total electromagnetic torque of the full order model.

For the full order model, generated in chapter 3, the total electromagnetic torque and the spectrum of that are shown in the Figure 4.27. It can be seen that the total torque of the machine has less ripple compared to the electromagnetic torque of each machine. It also can be seen from the spectrum of the torque shown in the figure 4.27 (d), with comparing the harmonics of the torque around the source frequency by that of each individual machine in Figures 4.24 to 4.26. The spectrum of the airgap flux linkage of different machines and the total are shown in the Figure 4.28. The main component is equal to the source frequency and the harmonics can be seen in the zoomed view of the figure.

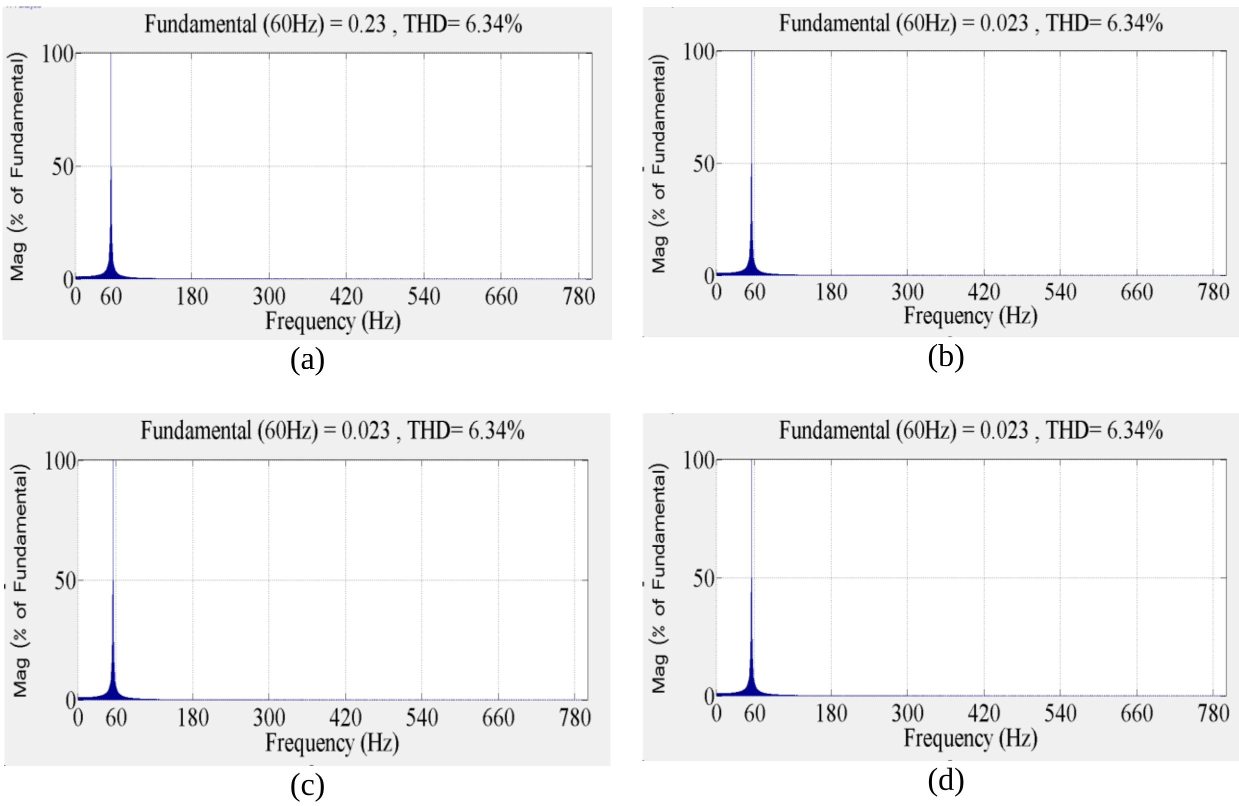


Figure 4.28: The spectrum of the airgap flux linkage from the average model for, (a) Machine '1', (b) Machine '2', (c) Machine '3', (d) Total.

The stator currents in natural quantities are shown in the Figures 4.29 to 4.31 along with the currents of the full order modelling. By comparing the currents, the harmonics of the full order model currents can be seen in these figures.

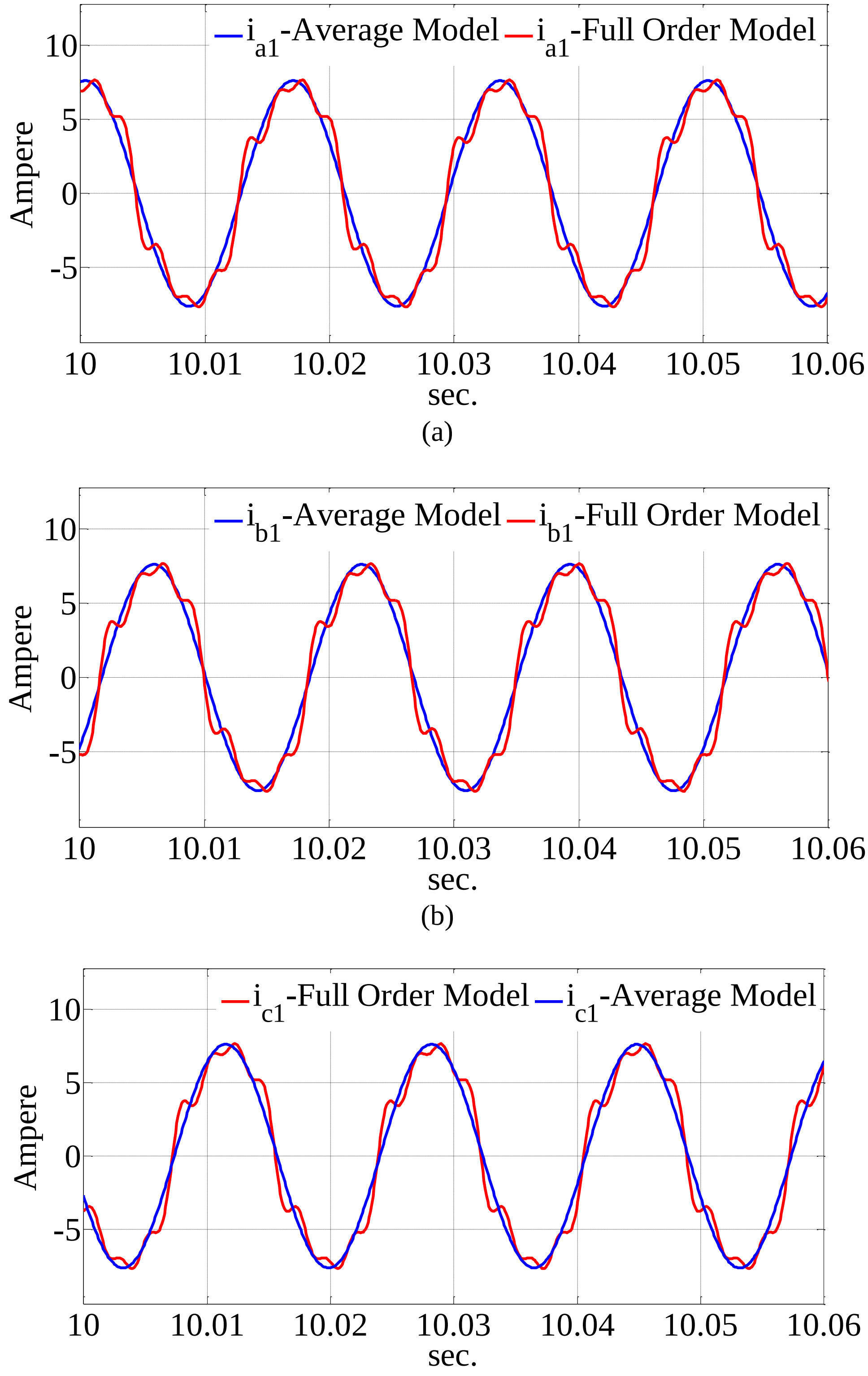


Figure 4.29: The stator currents of machine ‘1’ at steady state for average and full order model,

(a) Phase ‘a’, (b) Phase ‘b’, (c) Phase ‘c’.

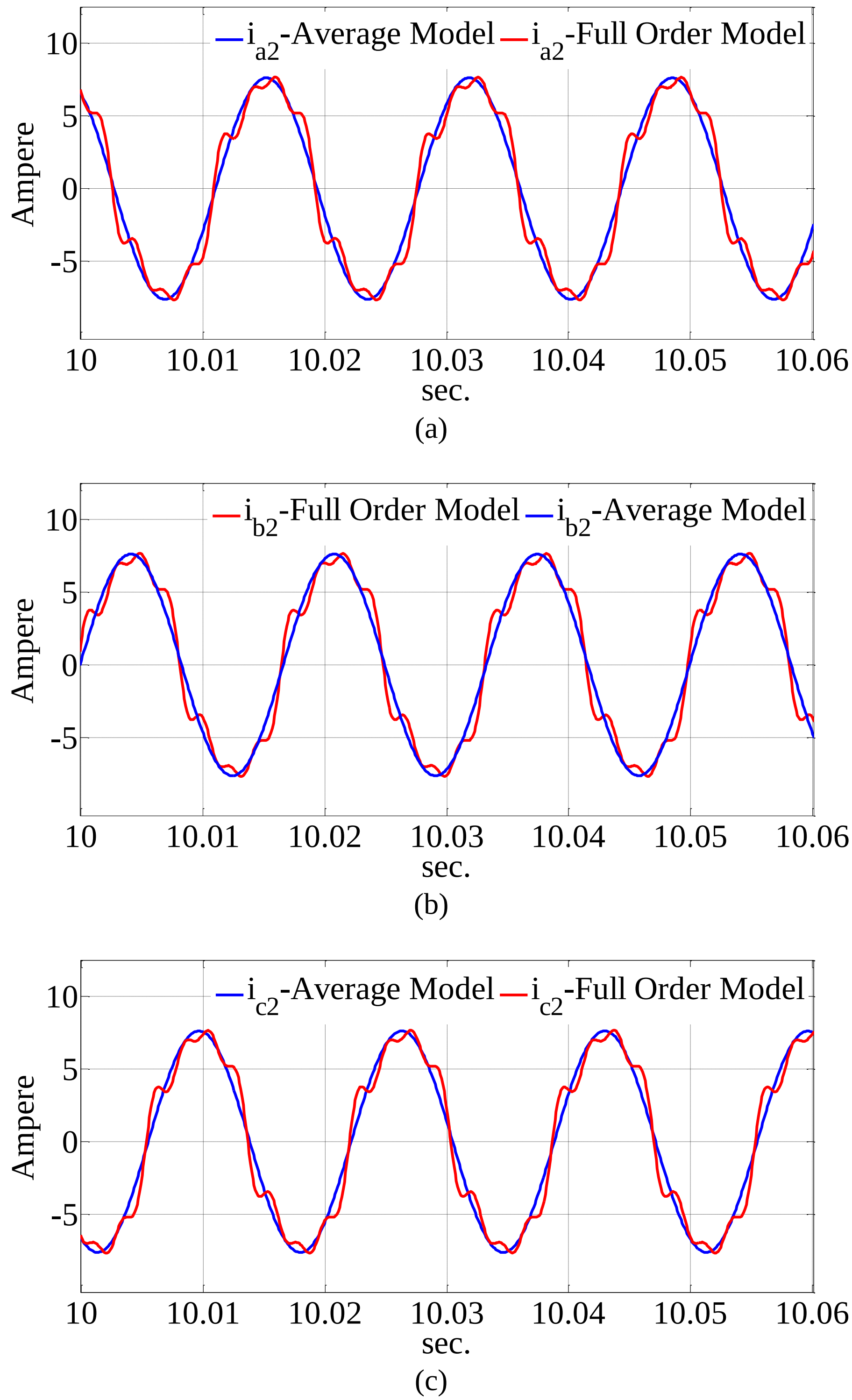


Figure 4.30: The stator currents of machine ‘2’ at steady state for average and full order model, (a) Phase ‘a’, (b) Phase ‘b’, (c) Phase ‘c’.

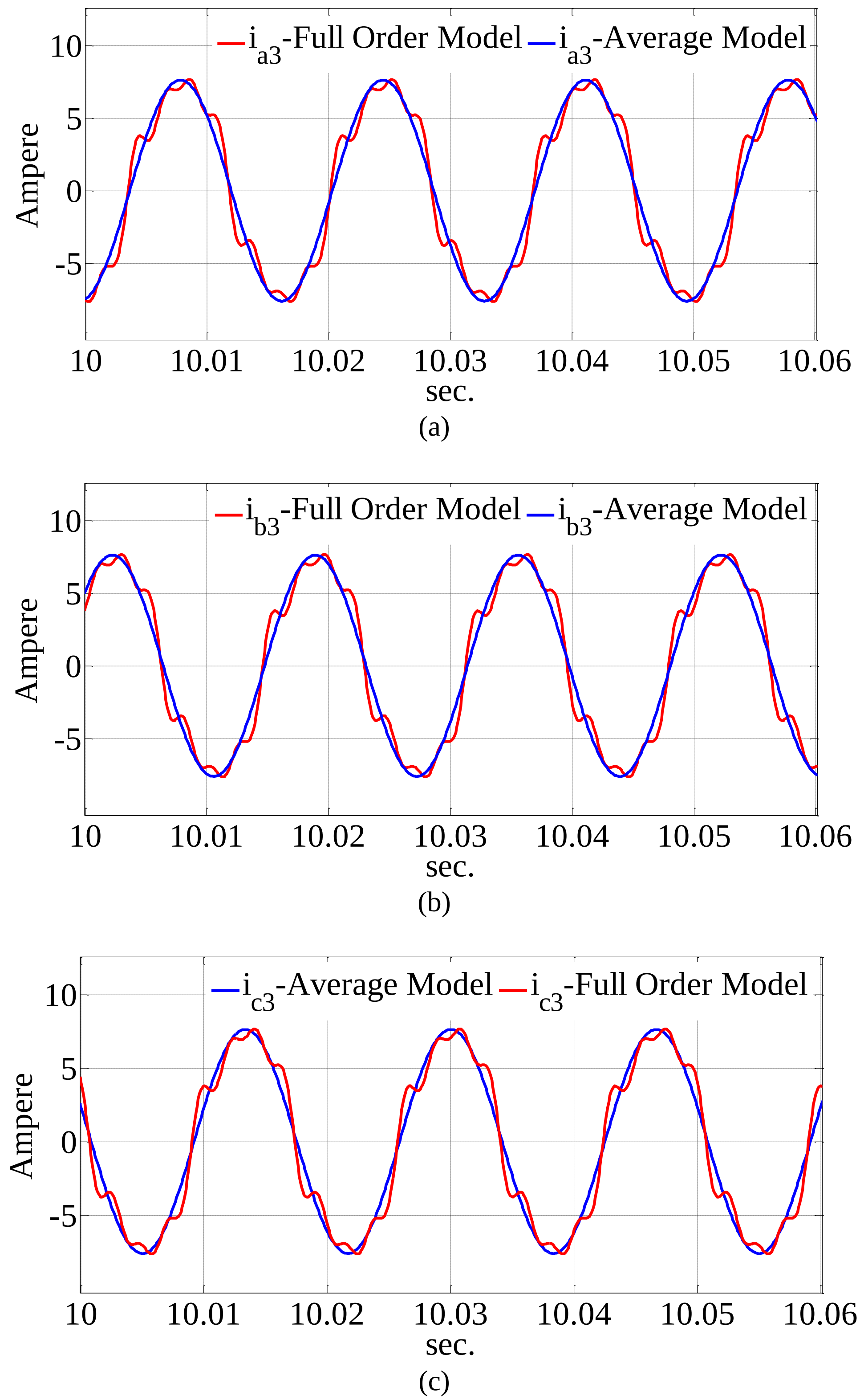


Figure 4.31: The stator currents of machine ‘3’ at steady state for average and full order model,

(a) Phase ‘a’, (b) Phase ‘b’, (c) Phase ‘c’.

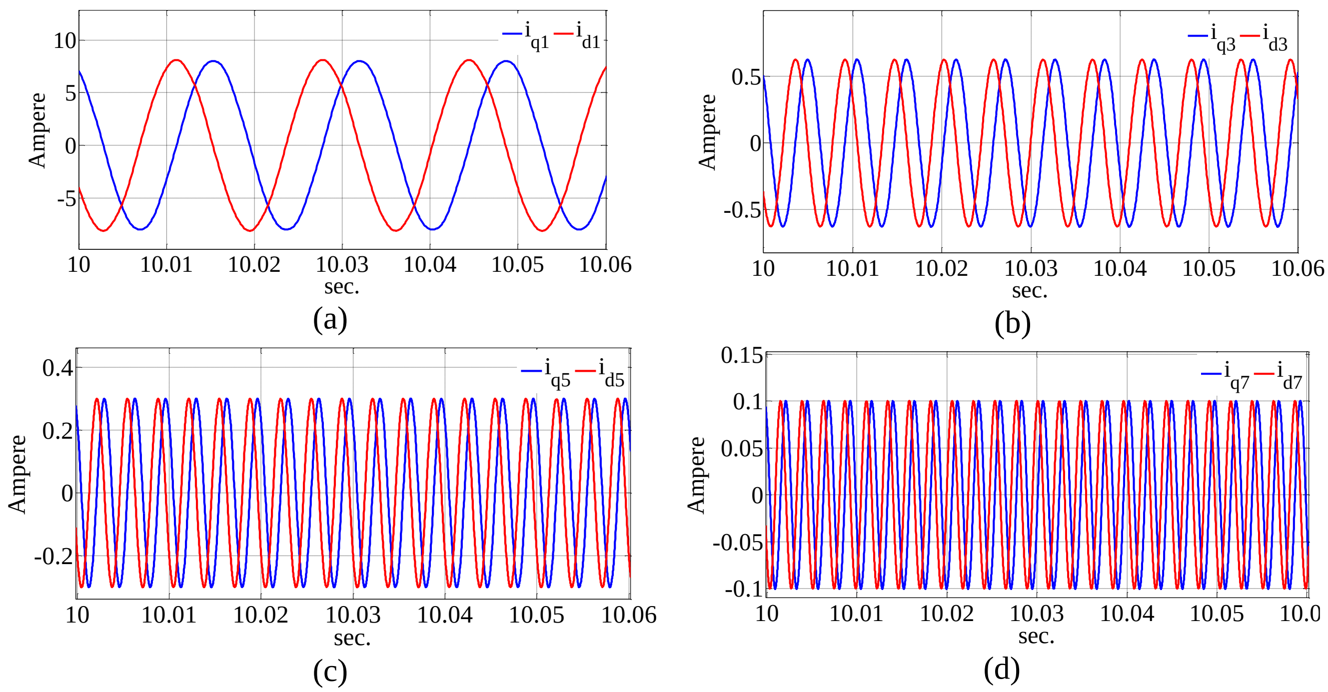


Figure 4.32: The stator currents in stationary reference frame in, (a) First sequence, (b) Third sequence, (c) Fifth sequence, (d) Seventh sequence.

The machine currents can be transformed to the stationary reference frame using the transformation matrix of equation (3.10) to obtain the different sequences of them. Figure 4.32 shows the first, third, fifth and seventh sequence of the stator currents in the stationary reference frame.

4.5 Simulation of the Asymmetrical Nine-Phase Machine

In this section the average model is simulated using MATLAB/Simulink for the asymmetrical connection. First step is to put the asymmetrical machine parameters from Tables 3.1 and 4.1 in to the general equation of inductances and substituting the resulting inductances into the voltage equations of the section 4.3. After that, the triple star IPM machine can be simulated using MATLAB/ Simulink. The machine inductances in the rotor reference frame are shown in Figures 4.33 to 4.38.

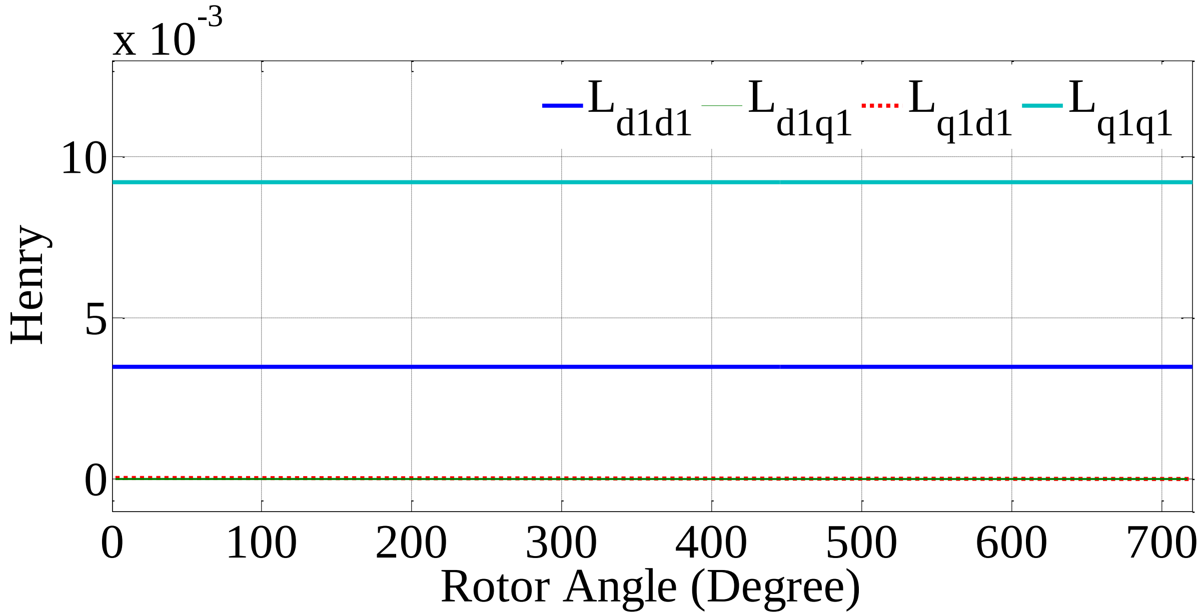


Figure 4.33: The inductances of the machine 1 in the rotor reference frame.

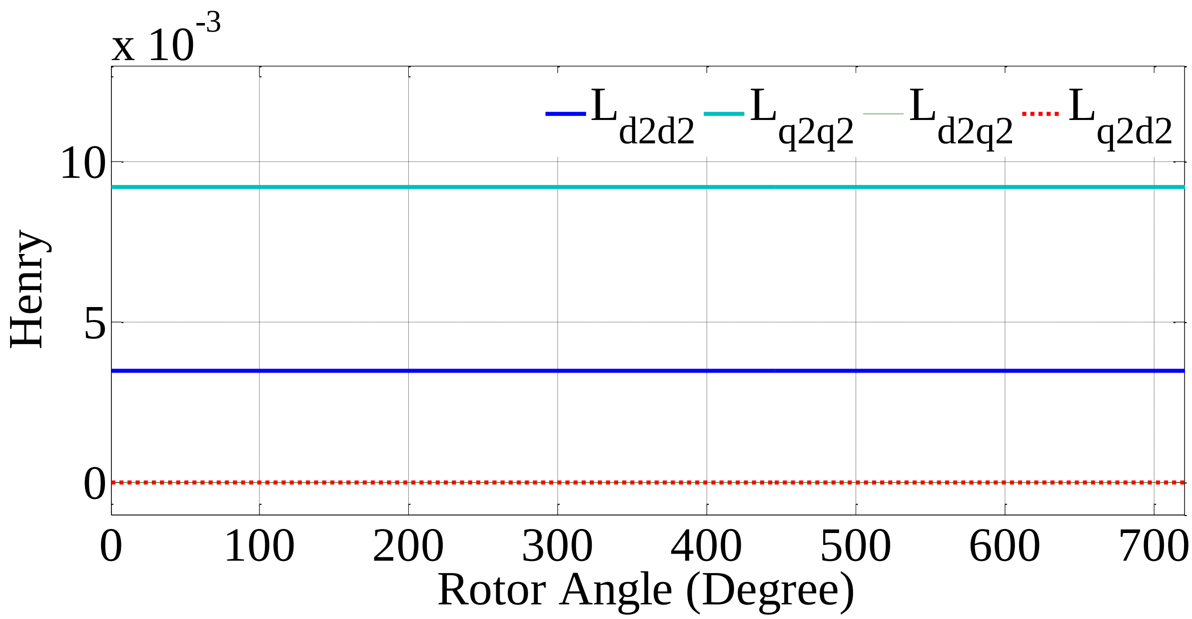


Figure 4.34: The inductances of the machine 2 in the rotor reference frame.

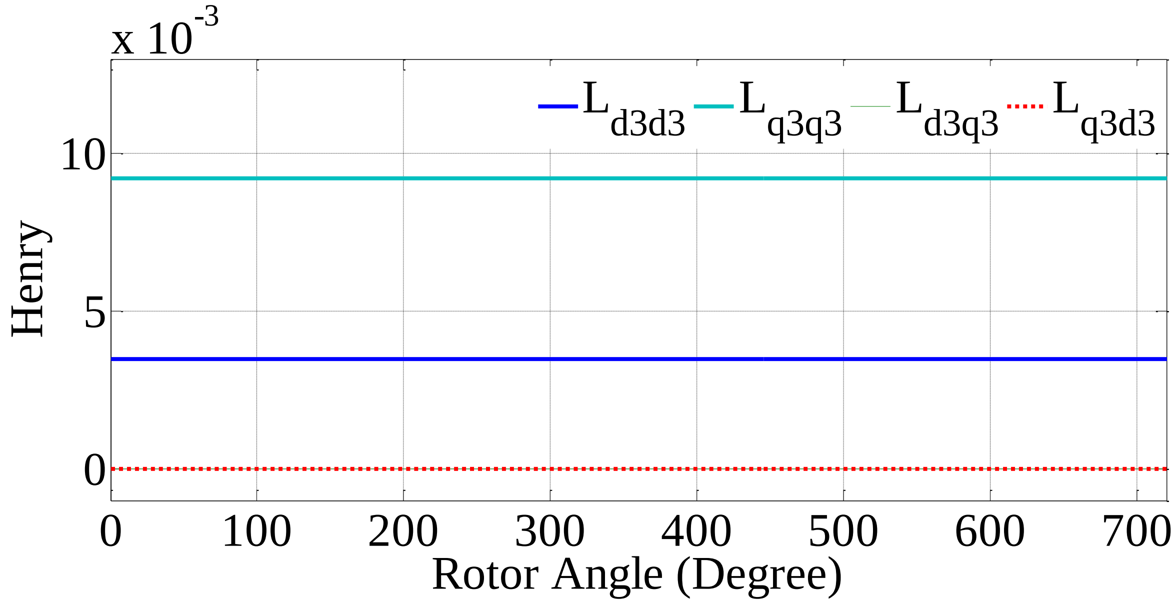


Figure 4.35: The inductances of the machine 3 in the rotor reference frame.

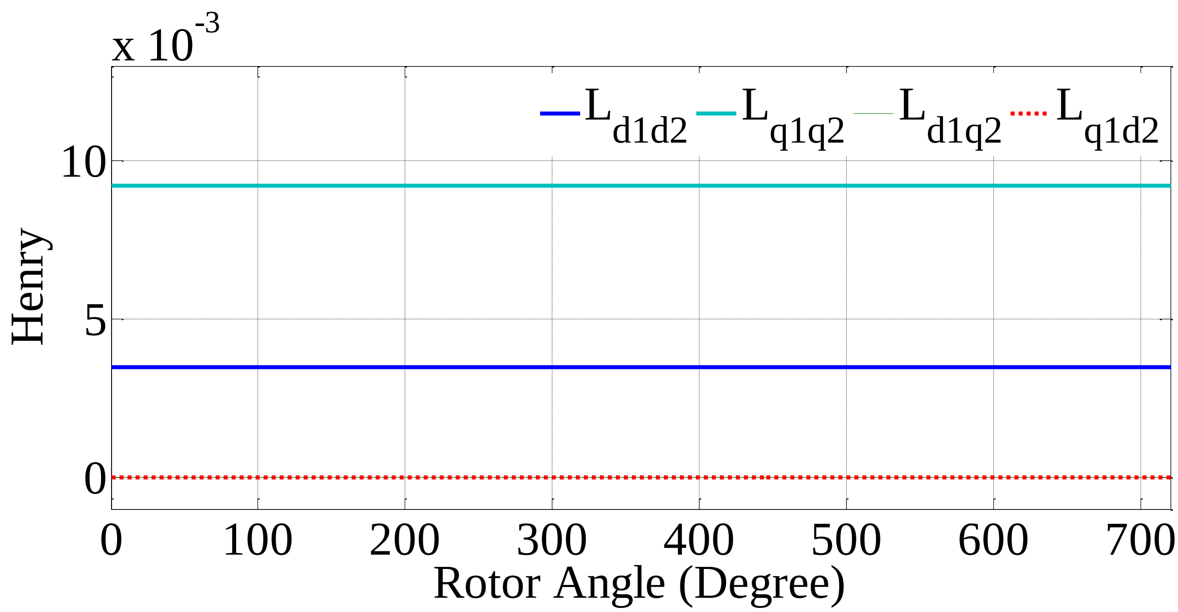


Figure 4.36: The mutual inductances between machines 1 and 2 in the rotor reference frame.

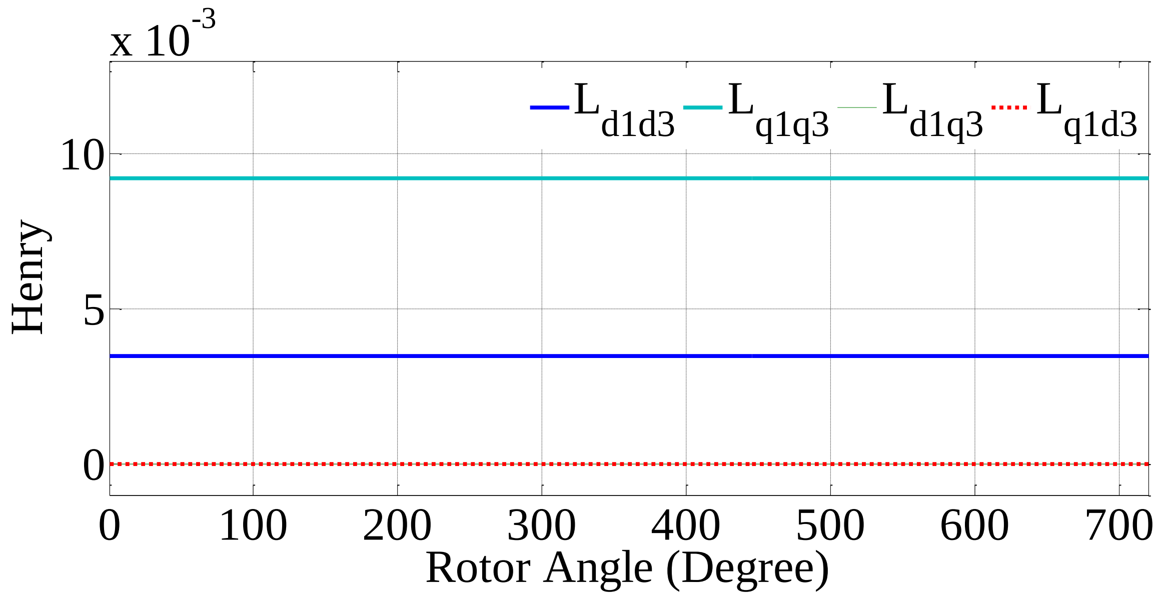


Figure 4.37: The mutual inductances between machines 1 and 3 in the rotor reference frame.

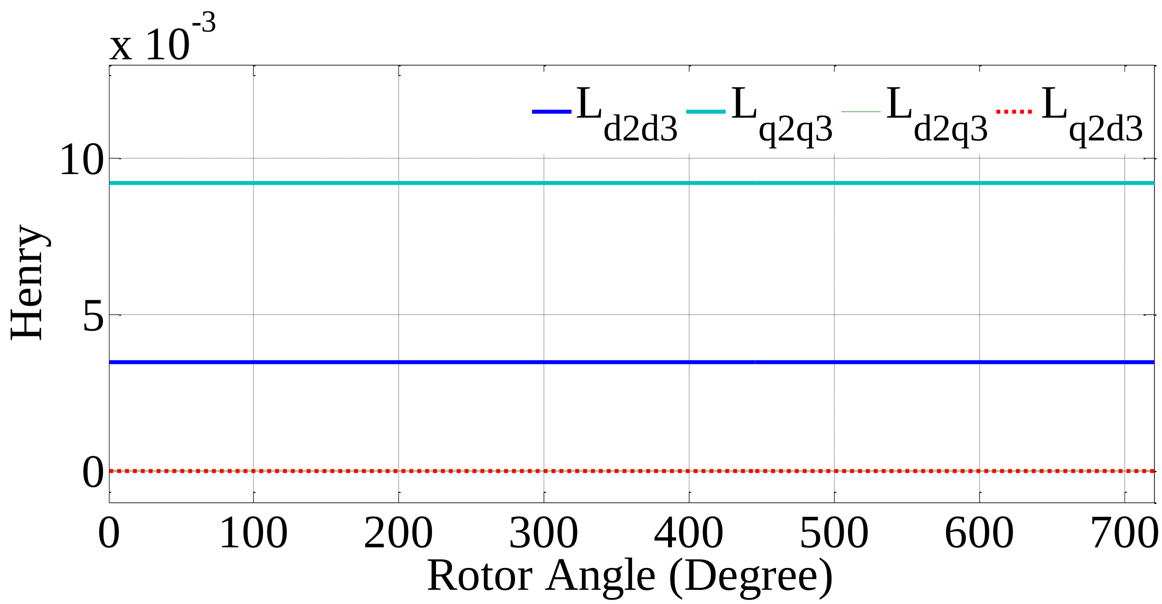


Figure 4.38: The mutual inductances between machines 2 and 3 in the rotor reference frame.

The magnetic flux linkage of the permanent magnet blocks in the rotor reference frame are also shown in Figures 4.39 to 4.41.

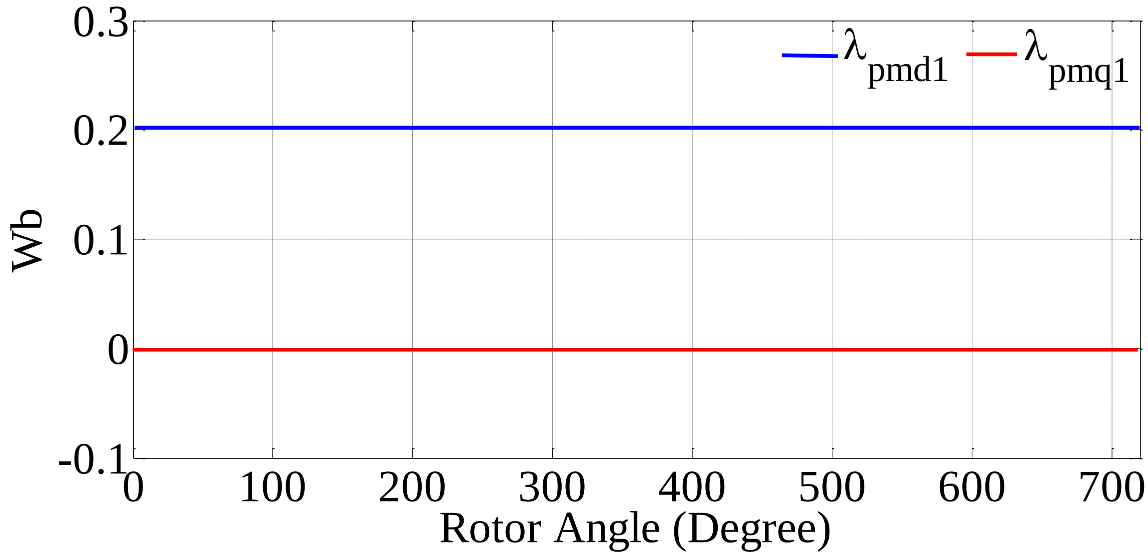


Figure 4.39: The d and q axis flux linkage due to the rotor permanent magnets of machine 1 in rotor reference frame.

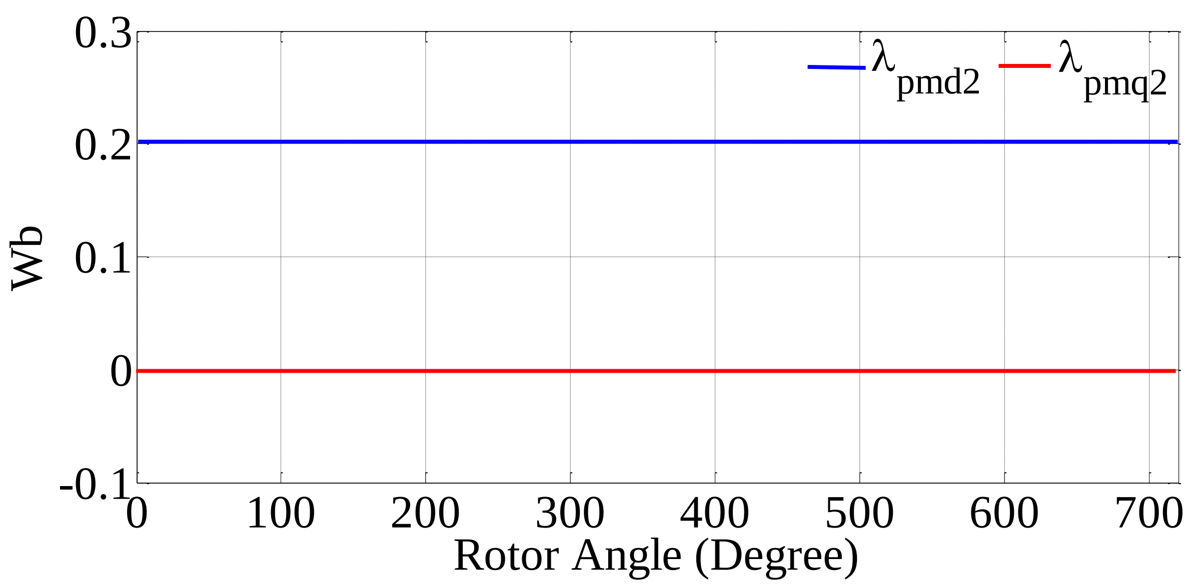


Figure 4.40: The d and q axis flux linkage due to the rotor permanent magnets of machine 2 in rotor reference frame.

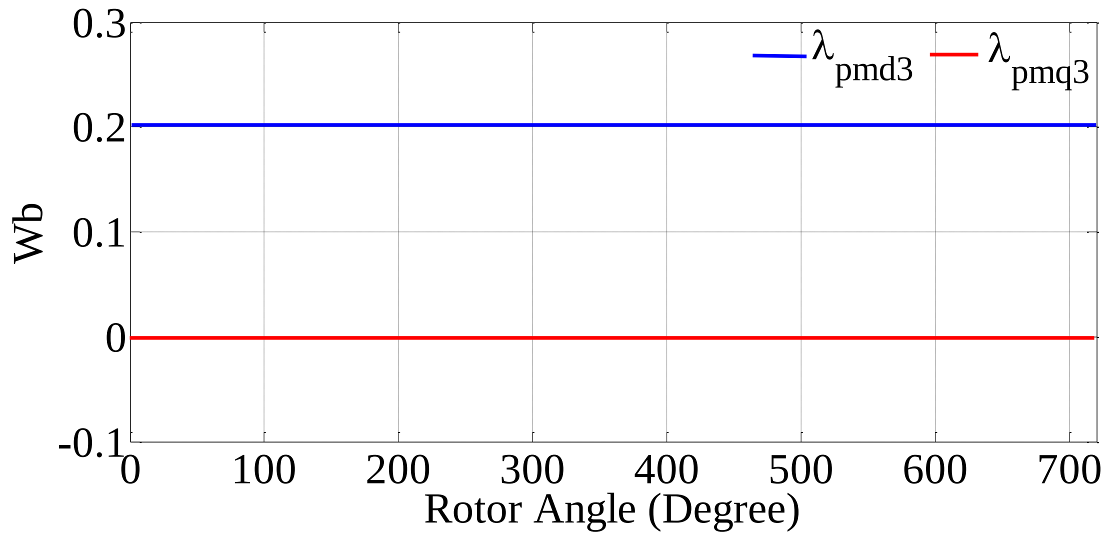


Figure 4.41: The d and q axis flux linkage due to the rotor permanent magnets of machine 3 in rotor reference frame.

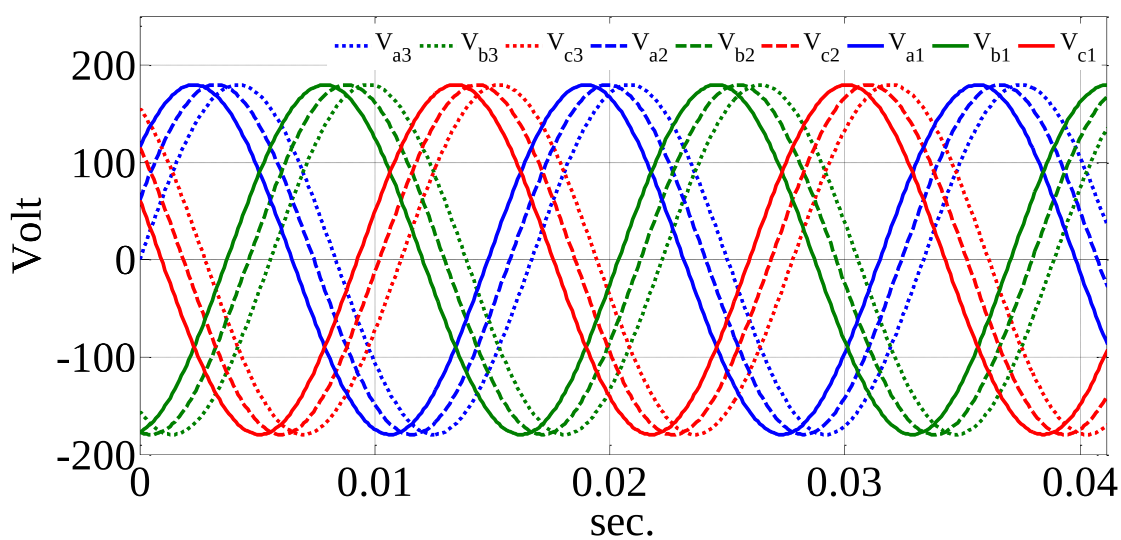


Figure 4.42: The phase voltages.

Three sets of 60 (Hz) 110 (Volts) three-phase voltages (as shown in Figure 4.42) are applied to the model while the initial rotor speed is 377 (rad/sec). When the machine passes the transients and goes to the steady state, a mechanical load torque equal to 5 N.m is applied to the machine. The simulation results are shown in the following. Figure 4.43 shows the rotor speed, the transients at the beginning and after load application can be seen on that.

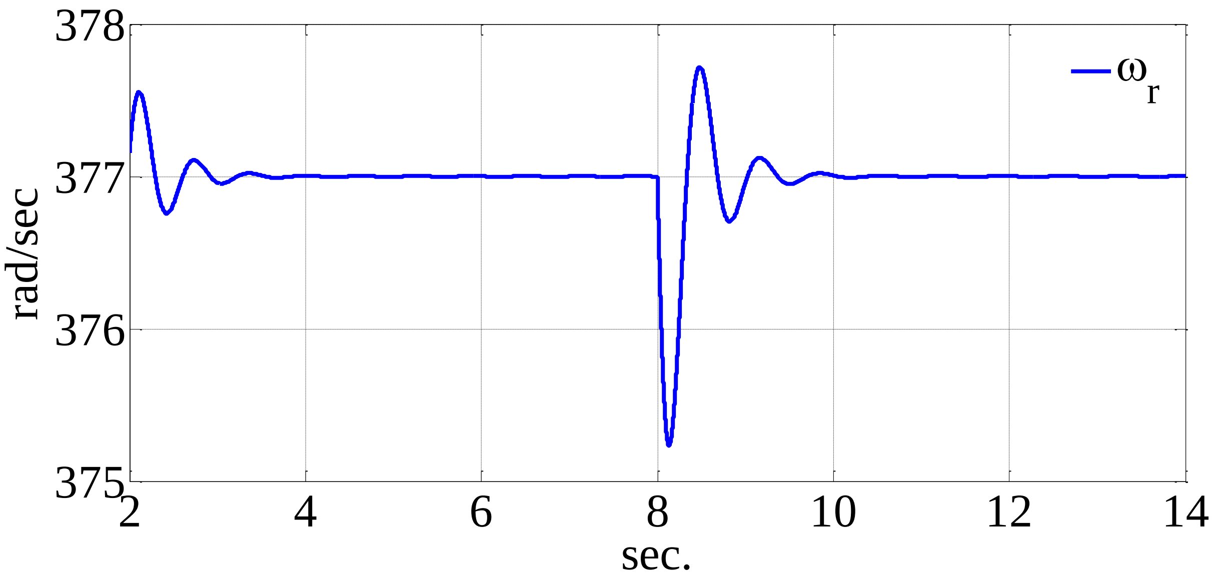


Figure 4.43: The rotor speed.

Figure 4.44 (a) shows the electromagnetic and load torque together. As it can be seen after initial transients have died the torque goes to zero. After applying the load, the machine starts generating electromagnetic torque to keep the synchronous speed. The spectrum of the electromagnetic torque is shown in the Figure 4.44 (b). The frequency of the main harmonic is zero and the rest of the higher harmonics have a relatively lower magnitude compared to the main one. The electromagnetic torque is generated by three machines and each of them shares a part of that. Figures 4.45 to 4.47 show the electromagnetic torque of each machine along with the electromagnetic torque of the same machine from full order modelling in chapter 3. Also, the spectrums of the electromagnetic torques of the average and full order model are shown in the same figures.

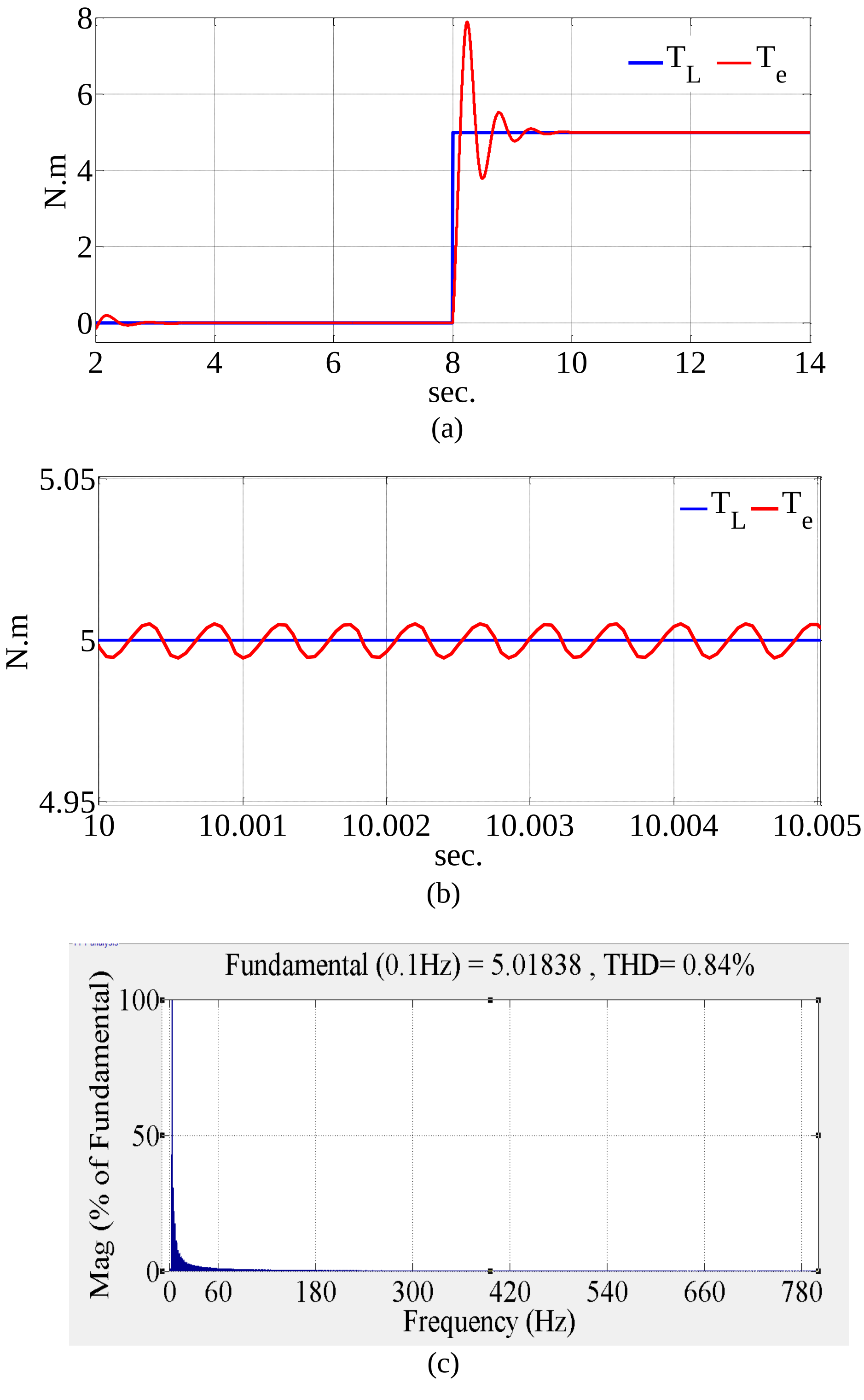


Figure 4.44: (a) The total electromagnetic torque, (b) The Zoomed view of torque at steady state, (c) The spectrum of the electromagnetic torque of the machine.

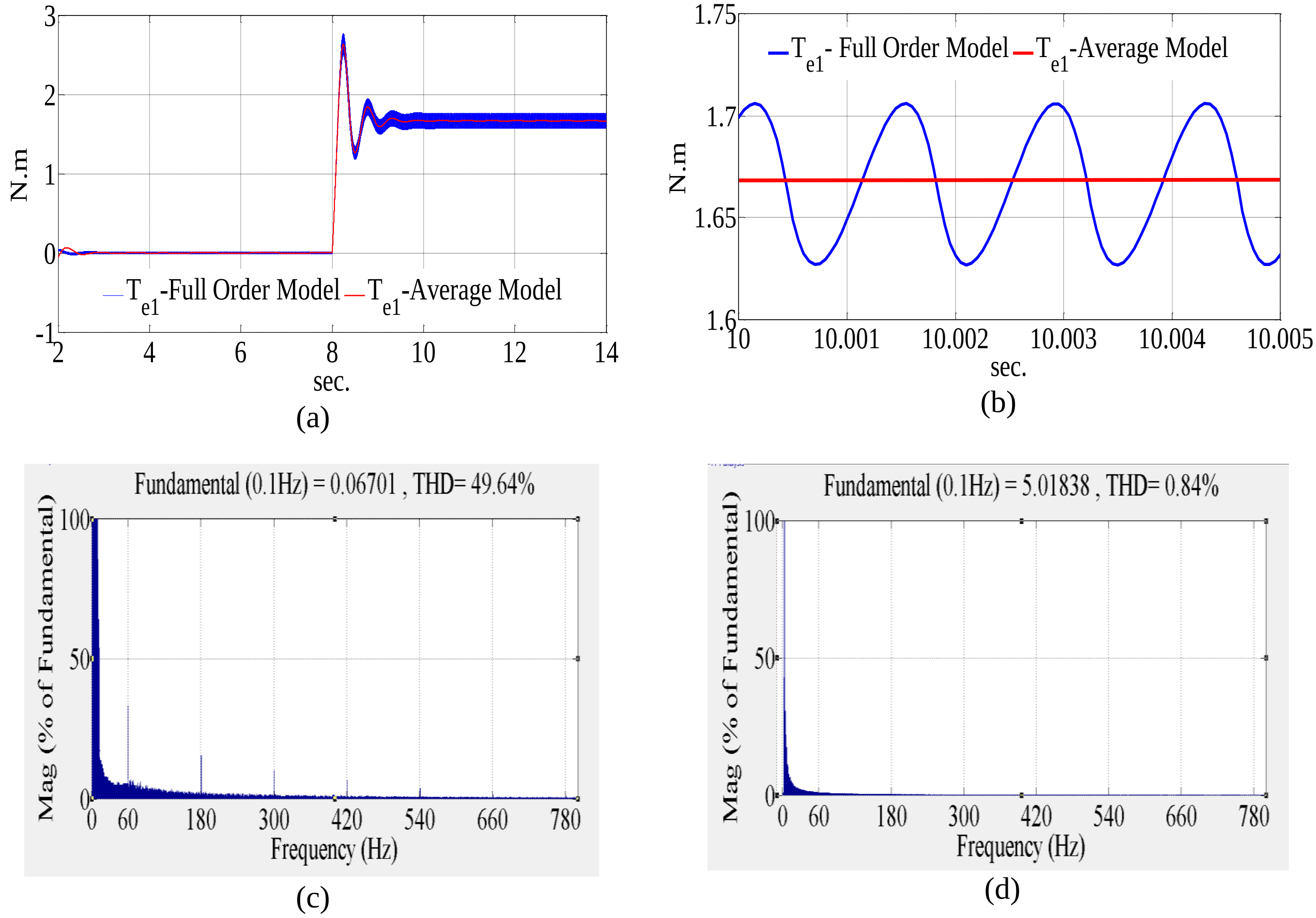


Figure 4.45: (a) The electromagnetic torques generated by machine 1 for average and full order model, (b) The zoomed view of torque at steady state, (c) The spectrum of the electromagnetic torque of the full order model, (d) The spectrum of the electromagnetic torque for the average model.

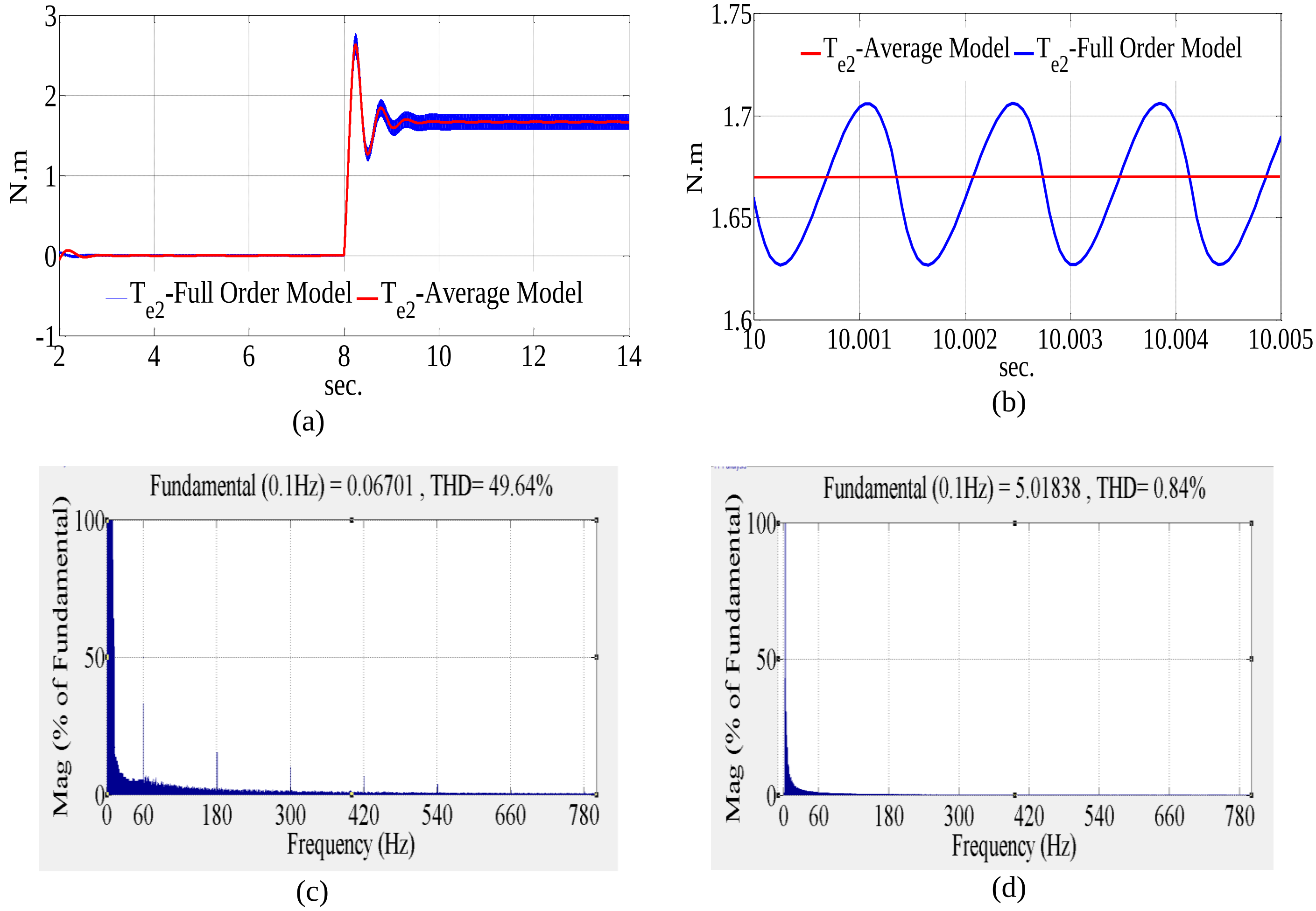


Figure 4.46: (a) The electromagnetic torque generated by machine 2 for average and full order model, (b) The zoomed view of torques at steady state, (c) The spectrum of the electromagnetic torque of the full order model, (d) The spectrum of the electromagnetic torque for the average model.

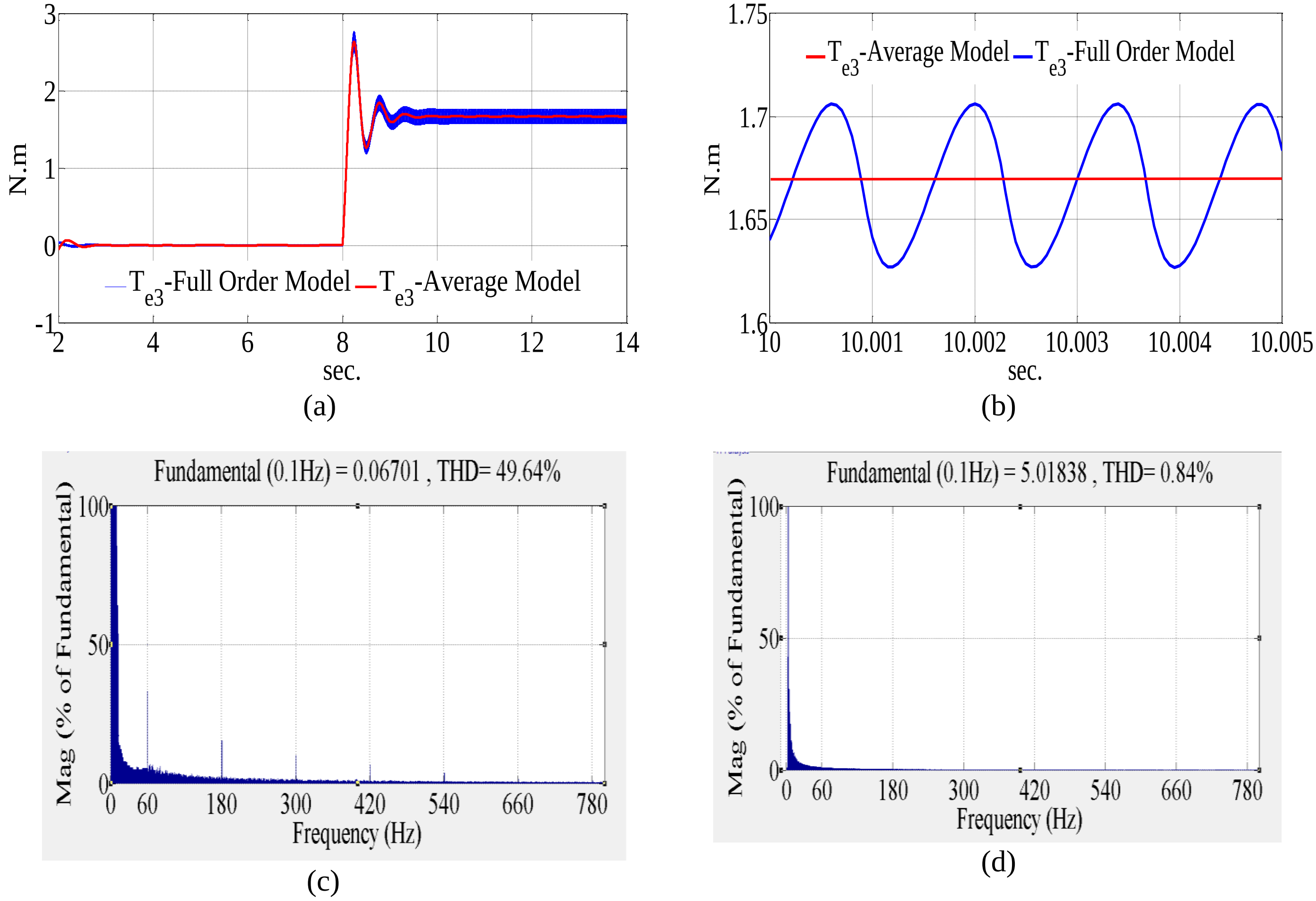


Figure 4.47: (a) The electromagnetic torque generated by machine 3 for average and full order model, (b) The zoomed view of torques at steady state, (c) The spectrum of the electromagnetic torque of the full order model, (d) The spectrum of the electromagnetic torque for the average model.

The spectrum of the airgap flux linkage for average and full order model are shown in the Figure 4.48. The main component is equal to the source frequency and the harmonics can be seen in the zoomed view of the figure. Compared to the flux linkage spectrum, shown in the Figure 3.133, the high frequency components have negligible magnitudes.

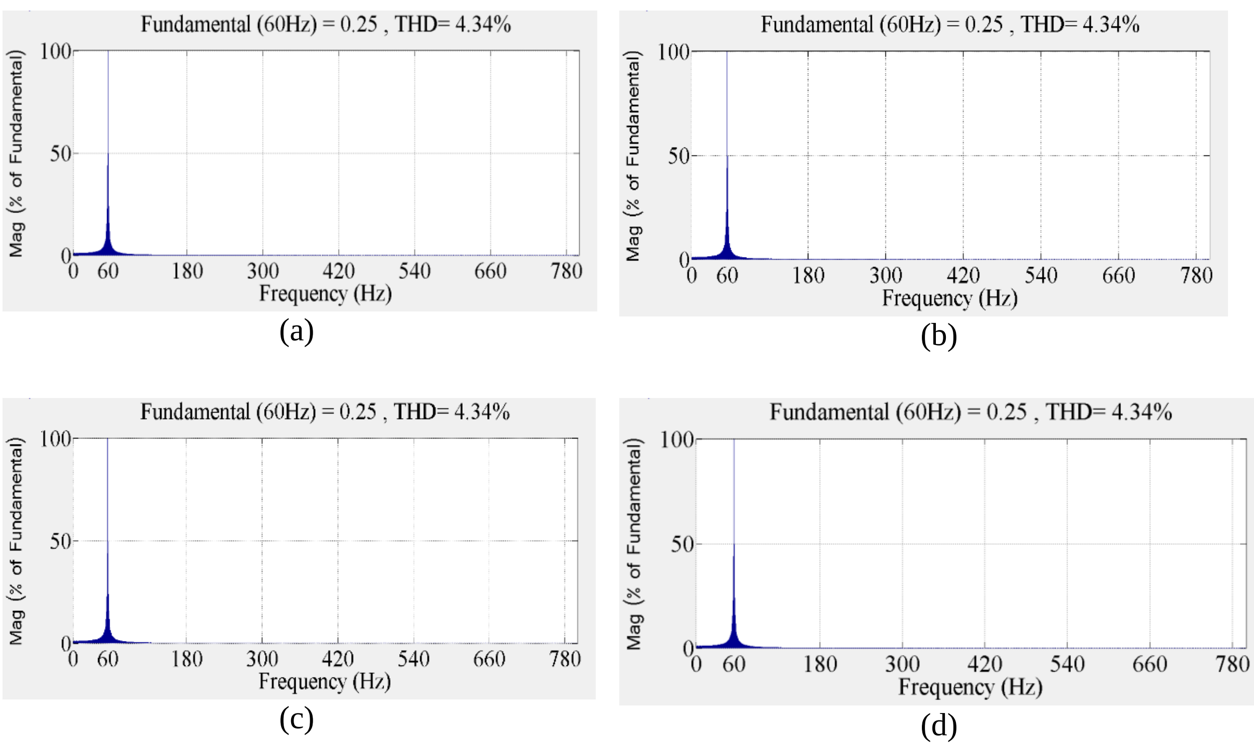
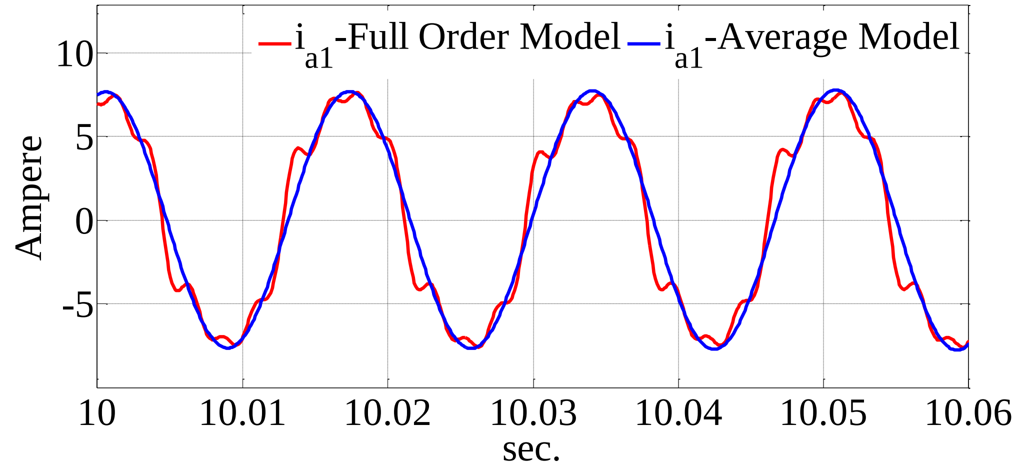
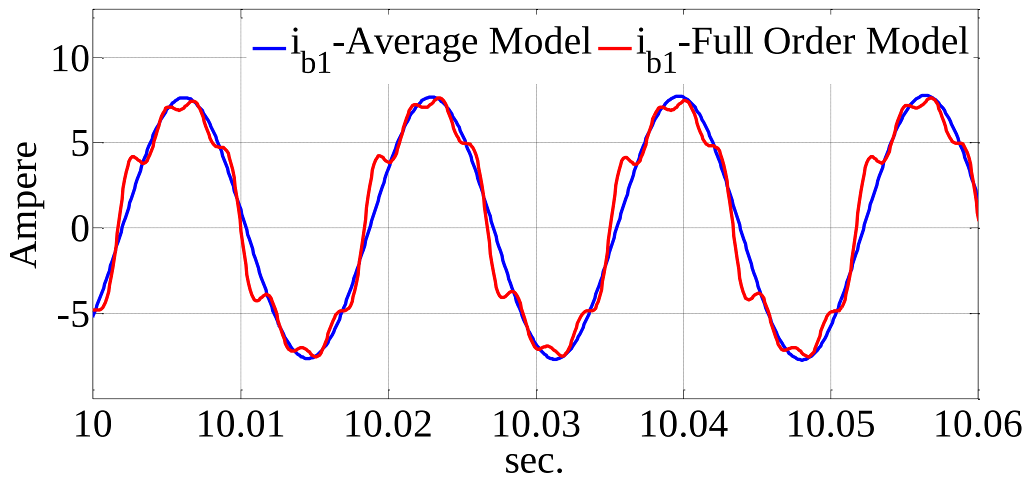


Figure 4.48: The spectrum of the airgap flux linkage of average model for, (a) Machine ‘1’, (b) Machine ‘2’, (c) Machine ‘3’, (d) The total flux linkage.

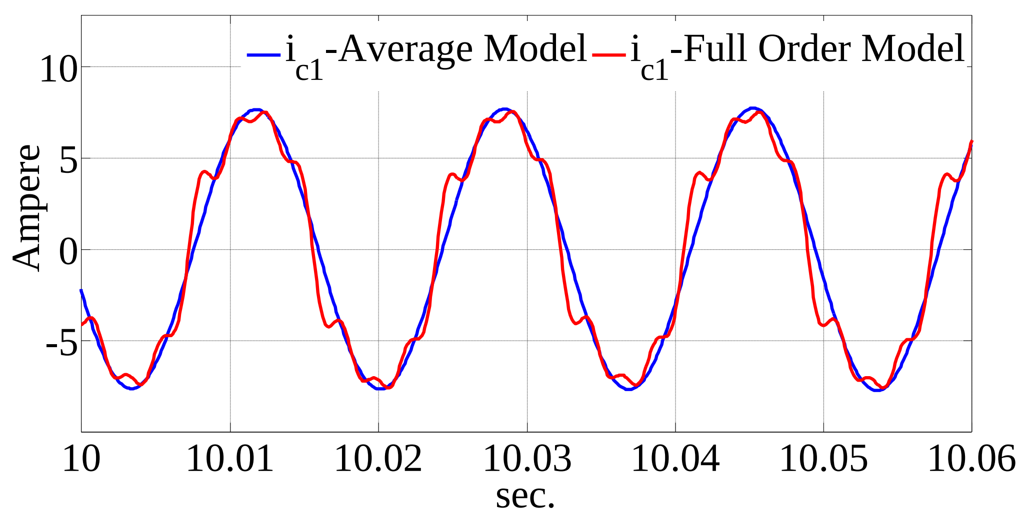
The stator currents in natural quantities are shown in the Figures 4.49 to 4.51 along with the currents of the full order modelling.



(a)



(b)



(c)

Figure 4.49: The stator currents of machine 1 at steady state (average and full order model).

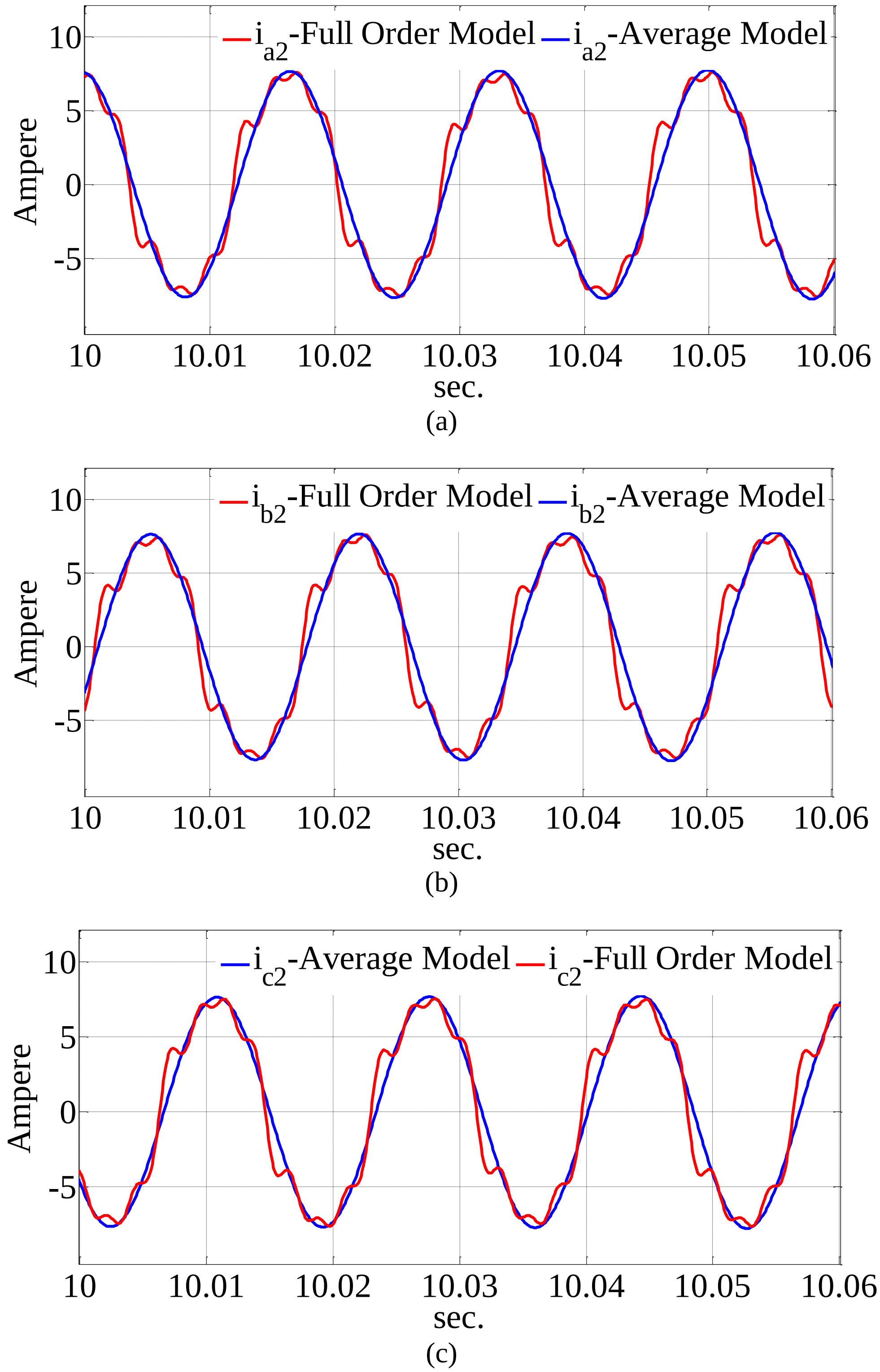


Figure 4.50: The stator currents of machine 2 at steady state (average and full order model).

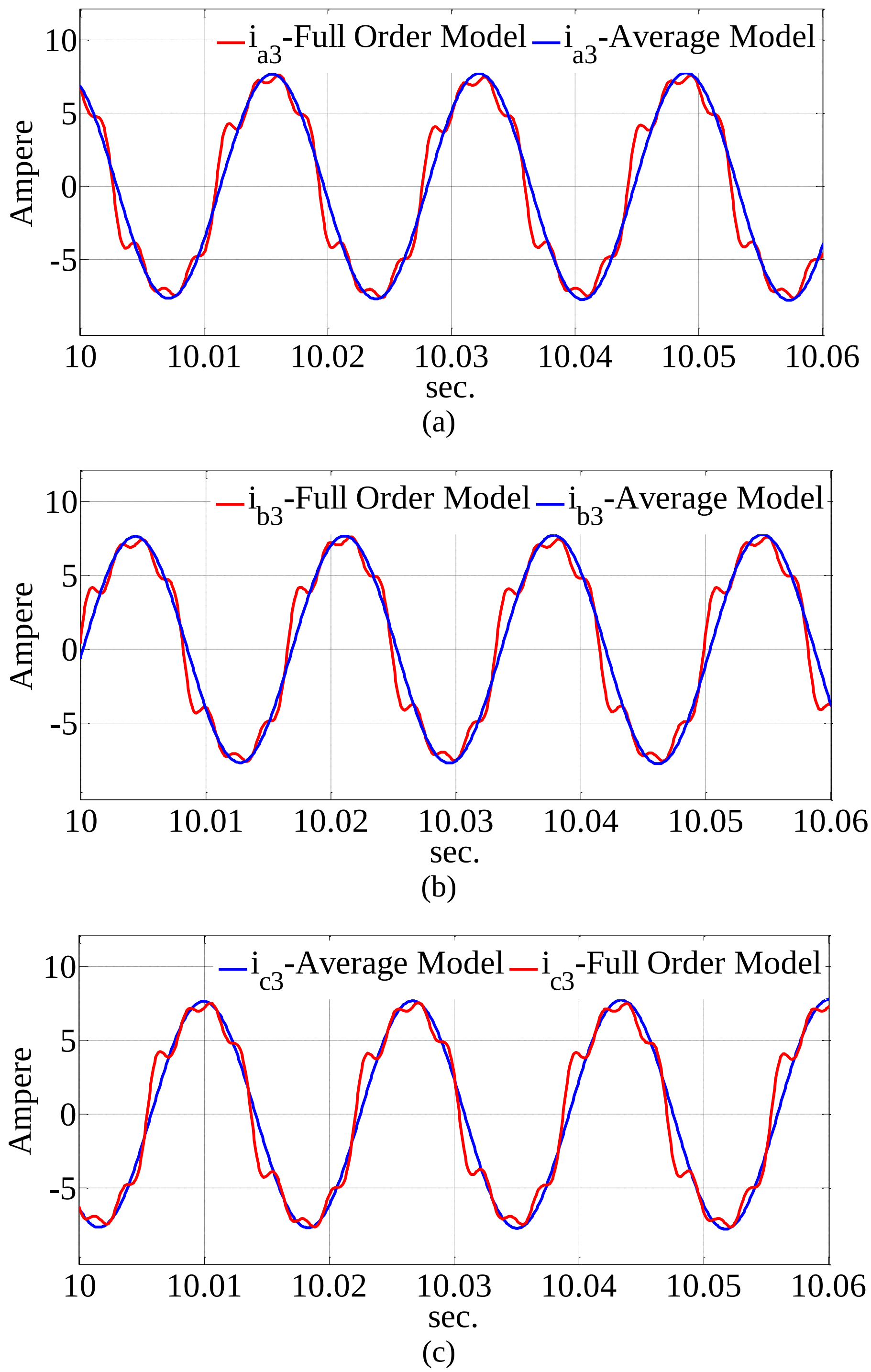


Figure 4.51: The stator currents of machine 3 at steady state (average and full order model).

The stator current in the stationary reference frame are also shown in Figure 4.52. This Figure shows the First, third, Fifth and seventh sequence currents of the stationary reference frame.

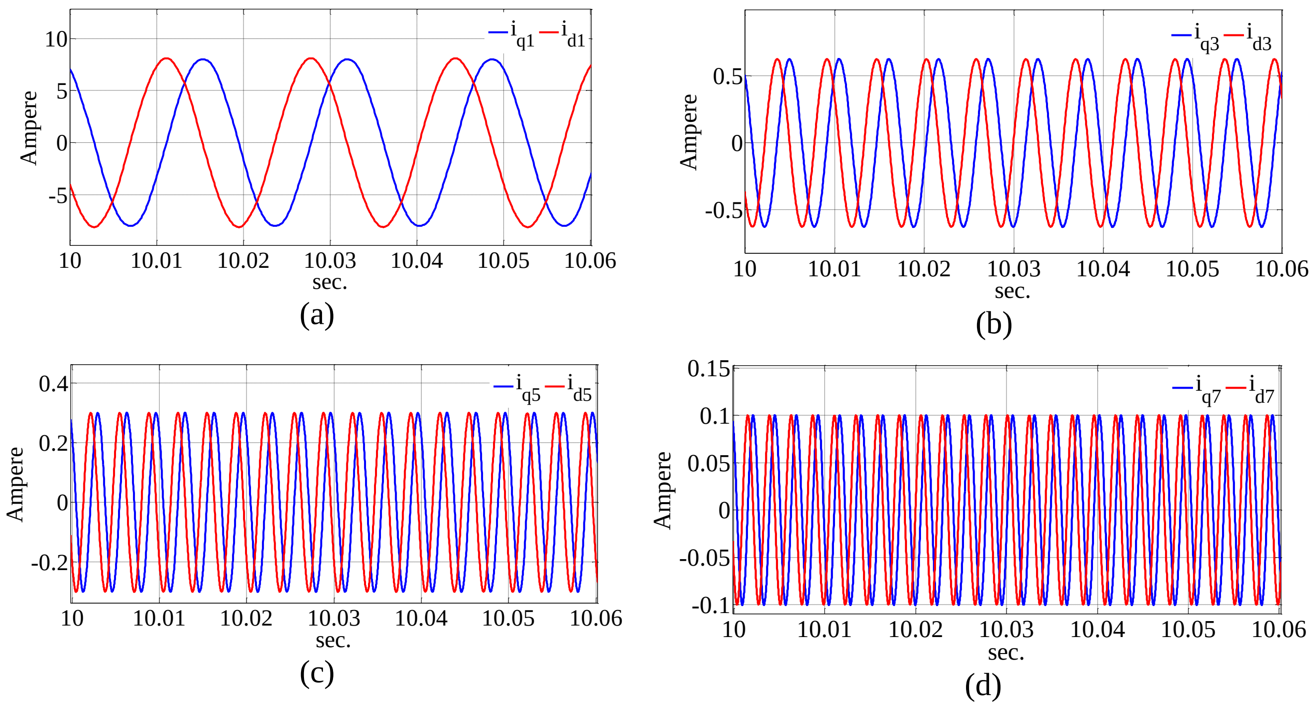


Figure 4.52: The stator currents in stationary reference frame in, (a) First sequence, (b) Third sequence, (c) Fifth sequence, (d) Seventh sequence.

4.6 Decoupling the Average Model of the Symmetrical and Asymmetrical Triple-Star IPM Machines

4.6.1 Background

Generally, a linear system can be defined as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BU(t) \\ y(t) &= Cx(t) + DU(t) \end{aligned} \tag{4.54}$$

In this equation 'A' represents the system matrix, 'B' is the input matrix, 'U' is the input of the system, 'C' is the output matrix and 'X' represents the state vector of the system [160]. The couplings between the state variables are due to the non-diagonal terms of the matrix 'A'. To decouple the state variables from each other, the system needs to be transformed to a decoupled form. The transformation to the new form is basically multiplying the system by a decoupling matrix. The decoupling matrix can be generated from the matrix ‘A’. An n×n matrix like ‘A’ with distinct eigen values is called diagonalizable if there exists an invertible matrix like P such that the $P^{-1}AP$ is diagonal [160].

By considering the eigen values of the matrix A as λ_i , $i = 1,2,3,....n$, then for each eigen value there will be an associated eigen vector given as q_i , $i = 1,2,3,....n$ such that:

$$Aq_i = \lambda_i q_i \tag{4.55}$$

The set of the eigen vectors are linearly independent, therefore they can be used as a base to represent ‘*A*’ in a new form called ‘ $\hat{A}$ ’. When ‘ $\hat{A}$ ’ is the representation of the matrix ‘A’ with the respect of the q_i , $i = 1,2,3,....n$ basis, then the first column of ‘ $\hat{A}$ ’ is the representation of $Aq_1 = \lambda_1 q_1$ with respect to the q_1 .

$$Aq_1 = \lambda_1 q_1 = [q_1 \quad q_2 \quad \dots \quad q_n] \begin{bmatrix} \lambda_1 \\ 0 \\ . \\ 0 \end{bmatrix} \tag{4.56}$$

It means the first column of the ‘ $\hat{A}$ ’ can be defined as:

$$V_1 = \begin{bmatrix} \lambda_1 \\ 0 \\ . \\ 0 \end{bmatrix} \tag{4.57}$$

By repeating the same procedure for the rest of the columns of the matrix 'A' the rest of the columns of the matrix ' $\hat{A}$ ' can be derived. The matrix ' $\hat{A}$ ' can be represented as:

$$\hat{A} = \begin{bmatrix} \lambda_1 & 0 & .. & 0 \\ 0 & \lambda_2 & .. & 0 \\ . & . & .. & . \\ 0 & 0 & .. & \lambda_n \end{bmatrix} \tag{4.58}$$

The above matrix is a diagonal matrix which means each square matrix with distinct eigen values can be represented as a diagonal one using it's eigen vectors as a basis. Based on this, if 'P' is defined as:

$$p = \begin{bmatrix} q_1 & q_2 & ... & q_n \end{bmatrix} \tag{4.59}$$

Then:

$$\hat{A} = P^{-1}AP \tag{4.60}$$

From the equation (4.60) A can be rewritten as:

$$A = P\hat{A}P^{-1} \tag{4.61}$$

Substituting this equation in equation (4.54) results in:

$$\begin{aligned} \dot{x}(t) &= P\hat{A}P^{-1}x(t) + BU(t) \\ y(t) &= Cx(t) + DU(t) \end{aligned} \tag{4.62}$$

By multiplying ' P^{-1} ' from the left hand side the equation (4.62) changes to:

$$\begin{aligned} P^{-1}\dot{x}(t) &= P^{-1}P\hat{A}P^{-1}x(t) + P^{-1}BU(t) \\ P^{-1}y(t) &= P^{-1}Cx(t) + P^{-1}DU(t) \end{aligned} \tag{4.63}$$

Now the decoupled state space equations can be defined as [160]:

$$\dot{x}'(t) = \hat{A}x'(t) + B'U(t)$$
$$y'(t) = C'\,x(t) + D'U(t) \tag{4.64}$$

Where:

$$\dot{x}'(t) = P^{-1}\dot{x}(t),\;\; B' = P^{-1}B,\;\; C' = P^{-1}C,\;\;\; D' = P^{-1}D \tag{4.65}$$

4.6.2 Decoupling the Machine Model

As it can be seen from the equation (4.37) there are some coupling terms between the different machines inductances. The coupling terms are actually the non-diagonal terms of the inductance matrix. These inductances can cause some complexity in designing the controller for the machine [140]. To remove these coupling terms, a new reference frame is needed to be presented. The transformation will be a combination of the rotor reference frame transformation and a second transformation that can make the inductance matrix of equation (4.35) diagonal. The procedure of finding the new transformation can start from diagonalzing the matrix of equation (4.35). To be able to determine the diagonal matrix of the inductances, the matrix should have distinct eigen values [143]. To have distinct Eigen values the zero sequence inductances should be removed, therefore the inductance matrix can shrink to the matrix of equation (4.66).

$$L_{qd} = \begin{bmatrix} L_{q1q1} & 0 & L_{q1q2} & 0 & L_{q1q3} & 0 \\ 0 & L_{d1d1} & 0 & L_{d1d2} & 0 & L_{d1d3} \\ L_{q2q1} & 0 & L_{q2q2} & 0 & L_{q2q3} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2} & 0 & L_{d2d3} \\ L_{q1q3} & 0 & L_{q3q2} & 0 & L_{q3q3} & 0 \\ 0 & L_{d1d3} & 0 & L_{d3d2} & 0 & L_{d3d3} \end{bmatrix} = \begin{bmatrix} L_{q} & 0 & L_{mq} & 0 & L_{mq} & 0 \\ 0 & L_{d} & 0 & L_{md} & 0 & L_{md} \\ L_{mq} & 0 & L_{q} & 0 & L_{mq} & 0 \\ 0 & L_{md} & 0 & L_{d} & 0 & L_{md} \\ L_{mq} & 0 & L_{mq} & 0 & L_{q} & 0 \\ 0 & L_{md} & 0 & L_{md} & 0 & L_{d} \end{bmatrix} \tag{4.66}$$

Generally, a matrix like L_{qd} is diagnosable if there exists an invertible matrix like P such that the $P^{-1}L_{qd}P$ is invertible [147].

$$L_{qdn} = P^{-1}\begin{bmatrix} L_{q1q1} & 0 & L_{q1q2} & 0 & L_{q1q3} & 0 \\ 0 & L_{d1d1} & 0 & L_{d1d2} & 0 & L_{d1d3} \\ L_{q2q1} & 0 & L_{q2q2} & 0 & L_{q2q3} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2} & 0 & L_{d2d3} \\ L_{q1q3} & 0 & L_{q3q2} & 0 & L_{q3q3} & 0 \\ 0 & L_{d1d3} & 0 & L_{d3d2} & 0 & L_{d3d3} \end{bmatrix} P \tag{4.67}$$

Where: *'P'*, is a matrix formed by the eigen vectors of the main matrix (L_{qd}).

$$P = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{bmatrix} \tag{4.68}$$

To obtain the eigen vectors of the matrix, the eigen values are needed. The eigen values can be calculated as:

$$(L_{qd} - \lambda I) = 0$$

$$\left(\begin{bmatrix} L_{q1q1} & 0 & L_{q1q2} & 0 & L_{q1q3} & 0 \\ 0 & L_{d1d1} & 0 & L_{d1d2} & 0 & L_{d1d3} \\ L_{q2q1} & 0 & L_{q2q2} & 0 & L_{q2q3} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2} & 0 & L_{d2d3} \\ L_{q1q3} & 0 & L_{q3q2} & 0 & L_{q3q3} & 0 \\ 0 & L_{d1d3} & 0 & L_{d3d2} & 0 & L_{d3d3} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\right) = 0 \tag{4.69}$$

The last equation is equal to:

$$\left(\begin{vmatrix} L_{q1q1}-\lambda & 0 & L_{q1q2} & 0 & L_{q1q3} & 0 \\ 0 & L_{d1d1}-\lambda & 0 & L_{d1d2} & 0 & L_{d1d3} \\ L_{q2q1} & 0 & L_{q2q2}-\lambda & 0 & L_{q2q3} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2}-\lambda & 0 & L_{d2d3} \\ L_{q1q3} & 0 & L_{q3q2} & 0 & L_{q3q3}-\lambda & 0 \\ 0 & L_{d1d3} & 0 & L_{d3d2} & 0 & L_{d3d3}-\lambda \end{vmatrix}\right) = 0 \tag{4.70}$$

Using the MATLAB (Symbolic Toolbox), the eigen values of the matrix are.

$$\lambda_1 = L_{q1q1} - L_{q1q3}$$
$$\lambda_2 = L_{d1d1} - L_{d1d3}$$

$$\begin{aligned}
\lambda_3 &= L_{q1q1} + \frac{L_{q1q3}}{2} - \frac{\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{2} \\
\lambda_4 &= L_{q1q1} + \frac{L_{q1q3}}{2} + \frac{\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{2} \\
\lambda_5 &= L_{d1d1} + \frac{L_{d1d3}}{2} + \frac{\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{2} \\
\lambda_6 &= L_{d1d1} + \frac{L_{d1d3}}{2} - \frac{\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{2}
\end{aligned} \tag{4.71}$$

Using the eigen values the eigen vectors (V_i) of the matrix can be derived as:

$$(A - \lambda_i I)V_i = 0 \tag{4.72}$$

Therefore, the eigen vectors corresponding to each of the eigen values are given in equations (4.73) to (4.77).

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ \dfrac{L_{q1q1} + 0.5L_{q1q3} - 0.5\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}} - (L_{q1q1} + L_{q1q3})}{L_{q1q2}} \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{4.73}$$

$$V_2 = \begin{bmatrix} 1 \\ 0 \\ \dfrac{L_{q1q1} + 0.5L_{q1q3} + 0.5\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}} - (L_{q1q1} + L_{q1q3})}{L_{q1q2}} \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{4.74}$$

$$V_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad V_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{4.75}$$

$$V_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{L_{d1d1} + 0.5L_{d1d3} + 0.5\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}} - \left(L_{d1d1} + L_{d1d3}\right)}{L_{d1d2}} \\ 0 \\ 1 \end{bmatrix} \tag{4.76}$$

$$V_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{L_{d1d1} + 0.5L_{d1d3} - 0.5\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}} - \left(L_{d1d1} + L_{d1d3}\right)}{L_{d1d2}} \\ 0 \\ 1 \end{bmatrix} \tag{4.77}$$

Substituting the self and mutual inductances from the equation 4.66 into the eigen vectors, they change to:

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ \dfrac{L_q + 0.5L_{mq} - 0.5\sqrt{8L^2{}_{mq} + L^2{}_{mq}} - \left(L_q + L_{mq}\right)}{L_{mq}} \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{4.78}$$

$$V_2 = \begin{bmatrix} 1 \\ 0 \\ \dfrac{L_q + 0.5L_{mq} + 0.5\sqrt{9L^2{}_{mq}} - \left(L_q + L_{mq}\right)}{L_{mq}} \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad (4.79)$$

$$V_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad V_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (4.80)$$

$$V_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{L_d + 0.5L_{md} + 0.5\sqrt{9L^2{}_{md}} - \left(L_d + L_{md}\right)}{L_{md}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \qquad (4.81)$$

$$V_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{L_d + 0.5L_{md} - 0.5\sqrt{9L^2{}_{md}} - \left(L_d + L_{md}\right)}{L_d} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} \qquad (4.82)$$

Now by arranging the vectors in the same matrix, the matrix P can be formed as:

$$P = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{4.83}$$

The inverse of the matrix P also can be defined as:

$$P^{-1} = [W_1 \quad W_2 \quad W_3 \quad W_4 \quad W_5 \quad W_6] \tag{4.84}$$

The columns of the matrix (W_i) can be defined as:

$$W_1 = \begin{bmatrix} -\frac{L_{q1q3} - \sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{4\sqrt{(8L^2{}_{q1q2} + L^2{}_{q1q3})}} \\ \frac{L_{q1q3} + \sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{4\sqrt{(8L^2{}_{q1q2} + L^2{}_{q1q3})}} \\ -\frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, W_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{2} \\ \frac{L_{d1d3} + \sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{4\sqrt{(8L^2{}_{d1d2} + L^2{}_{d1d3})}} \\ -\frac{L_{d1d3} - \sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{4\sqrt{(8L^2{}_{d1d2} + L^2{}_{d1d3})}} \end{bmatrix} \tag{4.85}$$

$$W_3 = \begin{bmatrix} \frac{-L_{q1q2}}{\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}} \\ \frac{L_{d1d2}}{\sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ W_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{L_{d1d2}}{\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}} \\ \frac{-L_{q1q2}}{\sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}} \end{bmatrix} \tag{4.86}$$

$$W_5 = \begin{bmatrix} -\dfrac{L_{q1q3} - \sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{4\sqrt{\left(8L^2{}_{q1q2} + L^2{}_{q3q3}\right)}} \\ \dfrac{L_{q1q3} + \sqrt{8L^2{}_{q1q2} + L^2{}_{q1q3}}}{4\sqrt{\left(8L^2{}_{q1q2} + L^2{}_{q1q3}\right)}} \\ \dfrac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; W_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{2} \\ \dfrac{L_{d1d3} + \sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{4\sqrt{\left(8L^2{}_{d1d2} + L^2{}_{d1d3}\right)}} \\ -\dfrac{L_{d1d3} - \sqrt{8L^2{}_{d1d2} + L^2{}_{d1d3}}}{4\sqrt{\left(8L^2{}_{d1d2} + L^2{}_{d1d3}\right)}} \end{bmatrix} \tag{4.87}$$

Substituting the self and mutual inductances from equation (4.66) into the vectors results in:

$$W_1 = \begin{bmatrix} -\dfrac{L_{mq} - \sqrt{9L^2{}_{mq}}}{4\sqrt{\left(9L^2{}_{mq}\right)}} \\ \dfrac{L_{mq} + \sqrt{9L^2{}_{mq}}}{4\sqrt{\left(9L^2{}_{mq}\right)}} \\ -\dfrac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{6} \\ \dfrac{1}{3} \\ -\dfrac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; W_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{1}{2} \\ \dfrac{L_{md} + \sqrt{9L^2{}_{md}}}{4\sqrt{9L^2{}_{md}}} \\ -\dfrac{L_{md} - \sqrt{9L^2{}_{md}}}{4\sqrt{9L^2{}_{md}}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{1}{2} \\ \dfrac{1}{3} \\ \dfrac{1}{6} \end{bmatrix} \tag{4.88}$$

$$W_3 = \begin{bmatrix} \dfrac{-L_{mq}}{\sqrt{9L^2{}_{mq}}} \\ \dfrac{L_{mq}}{\sqrt{9L^2{}_{mq}}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{3} \\ \dfrac{1}{3} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, W_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{L_{md}}{\sqrt{9L^2{}_{md}}} \\ \dfrac{-L_{md}}{\sqrt{9L^2{}_{md}}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{3} \\ -\dfrac{1}{3} \end{bmatrix} \tag{4.89}$$

$$W_5 = \begin{bmatrix} -\dfrac{L_{mq} - \sqrt{9L^2{}_{mq}}}{4\sqrt{9L^2{}_{mq}}} \\ \dfrac{L_{mq} + \sqrt{9L^2{}_{mq}}}{4\sqrt{9L^2{}_{mq}}} \\ \dfrac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{6} \\ \dfrac{1}{3} \\ \dfrac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ W_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{2} \\ \dfrac{L_{md} + \sqrt{9L^2{}_{md}}}{4\sqrt{9L^2{}_{md}}} \\ -\dfrac{L_{md} - \sqrt{9L^2{}_{md}}}{4\sqrt{9L^2{}_{md}}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{2} \\ \dfrac{1}{3} \\ \dfrac{1}{6} \end{bmatrix} \tag{4.90}$$

Therefore, by arranging the vectors in the same matrix, the matrix P^{-1} can be expressed as:

$$P^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} \end{bmatrix} \tag{4.91}$$

Now using the P and P^{-1} the inductance matrix can be diagonalized as equation (4.92).

$$L_{qdn} = P^{-1} L_{qd} P =$$

$$\begin{bmatrix} L_d - L_{md} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d + 2L_{md} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_d - L_{md} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q - L_{mq} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_q + 2L_{mq} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_q - L_{mq} \end{bmatrix} \tag{4.92}$$

By adding the leakage inductances to the self-inductances of the equations (4.26) to (4.34) and substitute them into the equation (4.92) the inductance matrix changes to:

$$L_{qdn} = P^{-1} L_{qd} P = \begin{bmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls} + 3L_{md} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} + 3L_{mq} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \quad (4.93)$$

Where: L_{ls} represents the leakage inductance of the stator phases. Now the whole model of the machine can be transformed to the new reference frame to get the decoupled model for the machine. The new transformation matrix which includes the diagonalizing matrix (after inserting the zero sequence) can be defined as:

$$T_n(\theta_r) = P^{-1}T(\theta_r) =$$

$$\begin{bmatrix} \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \frac{2}{3} \begin{pmatrix} C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r+\alpha_2) & C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r+\alpha_2) & S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r+\alpha_3) & S(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3+\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\frac{2}{3} \begin{bmatrix} \frac{C(\theta_r+\alpha_1)}{6} & \frac{C(\theta_r+\alpha_1-\gamma)}{6} & \frac{C(\theta_r+\alpha_1+\gamma)}{6} & \frac{-C(\theta_r+\alpha_2)}{3} & \frac{-C(\theta_r+\alpha_2-\gamma)}{3} & \frac{-C(\theta_r+\alpha_2+\gamma)}{3} & \frac{C(\theta_r+\alpha_3)}{6} & \frac{C(\theta_r+\alpha_3-\gamma)}{6} & \frac{C(\theta_r+\alpha_3+\gamma)}{6} \\ \frac{C(\theta_r+\alpha_1)}{3} & \frac{C(\theta_r+\alpha_1-\gamma)}{3} & \frac{C(\theta_r+\alpha_1+\gamma)}{3} & \frac{C(\theta_r+\alpha_2)}{3} & \frac{C(\theta_r+\alpha_2-\gamma)}{3} & \frac{C(\theta_r+\alpha_2+\gamma)}{3} & \frac{C(\theta_r+\alpha_3)}{3} & \frac{C(\theta_r+\alpha_3-\gamma)}{3} & \frac{C(\theta_r+\alpha_3+\gamma)}{3} \\ \frac{-C(\theta_r+\alpha_2)}{2} & \frac{-C(\theta_r+\alpha_2-\gamma)}{2} & \frac{-C(\theta_r+\alpha_2+\gamma)}{2} & 0 & 0 & 0 & \frac{C(\theta_r+\alpha_3)}{2} & \frac{C(\theta_r+\alpha_3-\gamma)}{2} & \frac{C(\theta_r+\alpha_3+\gamma)}{2} \\ \frac{-S(\theta_r+\alpha_1)}{2} & \frac{-S(\theta_r+\alpha_1-\gamma)}{2} & \frac{-S(\theta_r+\alpha_1+\gamma)}{2} & 0 & 0 & 0 & \frac{S(\theta_r+\alpha_3)}{2} & \frac{S(\theta_r+\alpha_3-\gamma)}{2} & \frac{S(\theta_r+\alpha_3+\gamma)}{2} \\ \frac{S(\theta_r+\alpha_1)}{3} & \frac{S(\theta_r+\alpha_1-\gamma)}{3} & \frac{S(\theta_r+\alpha_1+\gamma)}{3} & \frac{S(\theta_r+\alpha_2)}{3} & \frac{S(\theta_r+\alpha_2-\gamma)}{3} & \frac{S(\theta_r+\alpha_2+\gamma)}{3} & \frac{S(\theta_r+\alpha_3)}{3} & \frac{S(\theta_r+\alpha_3-\gamma)}{3} & \frac{S(\theta_r+\alpha_3+\gamma)}{3} \\ \frac{S(\theta_r+\alpha_1)}{6} & \frac{S(\theta_r+\alpha_1-\gamma)}{6} & \frac{S(\theta_r+\alpha_1+\gamma)}{6} & \frac{-S(\theta_r+\alpha_2)}{3} & \frac{-S(\theta_r+\alpha_2-\gamma)}{3} & \frac{-S(\theta_r+\alpha_2+\gamma)}{3} & \frac{S(\theta_r+\alpha_3)}{6} & \frac{S(\theta_r+\alpha_3-\gamma)}{6} & \frac{S(\theta_r+\alpha_3+\gamma)}{6} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.94}$$

And also the reverse of the transformation matrix can be defined as:

$$T_n(\theta_r)^{-1} = T(\theta_r)^{-1} P =$$

$$\begin{bmatrix} C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & C(\theta_r+\alpha_2) & S(\theta_r+\alpha_2) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & C(\theta_r+\alpha_2+\gamma) & S(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \tag{4.95}$$

$$\begin{bmatrix} C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1) & -C(\theta_r+\alpha_1) & -S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 1 & 0 & 0 \\ C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1-\gamma) & -C(\theta_r+\alpha_1-\gamma) & -S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 1 & 0 & 0 \\ C(\theta_r+\alpha_1+\gamma) & C(\theta_r+\alpha_1+\gamma) & -C(\theta_r+\alpha_1+\gamma) & -S(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 1 & 0 & 0 \\ -2C(\theta_r+\alpha_2) & C(\theta_r+\alpha_2) & 0 & 0 & S(\theta_r+\alpha_2) & -2S(\theta_r+\alpha_2) & 0 & 1 & 0 \\ -2C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2-\gamma) & 0 & 0 & S(\theta_r+\alpha_2-\gamma) & -2S(\theta_r+\alpha_2-\gamma) & 0 & 1 & 0 \\ -2C(\theta_r+\alpha_2+\gamma) & C(\theta_r+\alpha_2+\gamma) & 0 & 0 & S(\theta_r+\alpha_2+\gamma) & -2S(\theta_r+\alpha_2+\gamma) & 0 & 1 & 0 \\ C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & 0 & 0 & 1 \\ C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & 0 & 0 & 1 \\ C(\theta_r+\alpha_3+\gamma) & C(\theta_r+\alpha_3+\gamma) & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & 0 & 0 & 1 \end{bmatrix}$$

Where ‘C’ represents cos, ‘S’ represents sin, $\beta = \frac{\pi}{9}$, $\gamma = \frac{2\pi}{3}$ and α_i is defined according to Table 4.1. Now the machine equations can be transformed to the new reference frame (T_n) as:

$$
\begin{aligned}
&V_{abci} = r_s i_{abci} + p\lambda_{abci} \\
&V_{abci} = r_s T_n^{-1}(\theta_r) i_{qdn} + p T_n^{-1}(\theta_r)\lambda_{qdn} \\
&T_n(\theta_r) V_{abci} = T_n(\theta_r) r_s T_n^{-1}(\theta_r) i_{qdn} + T_n(\theta_r) p T_n^{-1}(\theta_r)\lambda_{qdn}
\end{aligned}
\qquad (4.96)
$$

The different terms of the equation (4.96) can be represented as:

$$
T_n(\theta_r) V_{abci} = T_n(\theta_r) \times \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \\ V_{a3} \\ V_{b3} \\ V_{c3} \end{bmatrix} = \begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{q3n} \\ V_{d3n} \\ V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} \qquad (4.97)
$$

$$
i_{qdn} = T_n(\theta_r) i_{abci} = T_n(\theta_r) \times \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} = \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} \qquad (4.98)
$$

The last term in the equation (4.96) which includes derivation can be expanded as:

$$T_n(\theta_r)pT_n^{-1}(\theta_r)\lambda_{qdn} = T_n(\theta_r)pT_n^{-1}(\theta_r)\lambda_{qdn} + T_n(\theta_r)T_n^{-1}(\theta_r)p\lambda_{qdn} \tag{4.99}$$

The first term of the equation (4.99) is:

$$T_n(\theta_r)pT_n^{-1}(\theta_r)\lambda_{qdn} = P^{-1}T(\theta_r)pT^{-1}(\theta_r)P\lambda_{qdn} =$$
$$\begin{bmatrix} W_1 & W_2 & W_3 & W_4 & W_5 & W_6 \end{bmatrix}\omega_r\begin{bmatrix} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{bmatrix}\lambda_{qdn} \tag{4.100}$$

Substituting the P , P^{-1} and ω_r into the equation (4.100) and inserting zero sequences, results in:

$$T_n(\theta_r)pT_n^{-1}(\theta_r)\lambda_{qdn} =$$

$$\begin{bmatrix} \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \omega_r \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{do1} \\ \lambda_{do2} \\ \lambda_{do3} \end{bmatrix} = \quad (4.101)$$

$$\omega_r \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{do1} \\ \lambda_{do2} \\ \lambda_{do3} \end{bmatrix}$$

The second part of the equation (4.100) is equal to:

$$T_n(\theta_r)T_n^{-1}(\theta_r)p\lambda_{qdn} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p\lambda_{q1n} \\ p\lambda_{d1n} \\ p\lambda_{q2n} \\ p\lambda_{d2n} \\ p\lambda_{q3n} \\ p\lambda_{d3n} \\ p\lambda_{do1} \\ p\lambda_{do2} \\ p\lambda_{do3} \end{bmatrix} \tag{4.102}$$

The resistances of the machine in the new reference frame also can be presented as:

$$r_{sn} = P^{-1}T(\theta_r)r_sT^{-1}P(\theta_r) = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s \end{bmatrix} \tag{4. 103}$$

Substituting the different terms of the machine in the decoupled reference frame in to the equation (4.96) and adding zero sequences of the machines to that, results in equation (4.104).

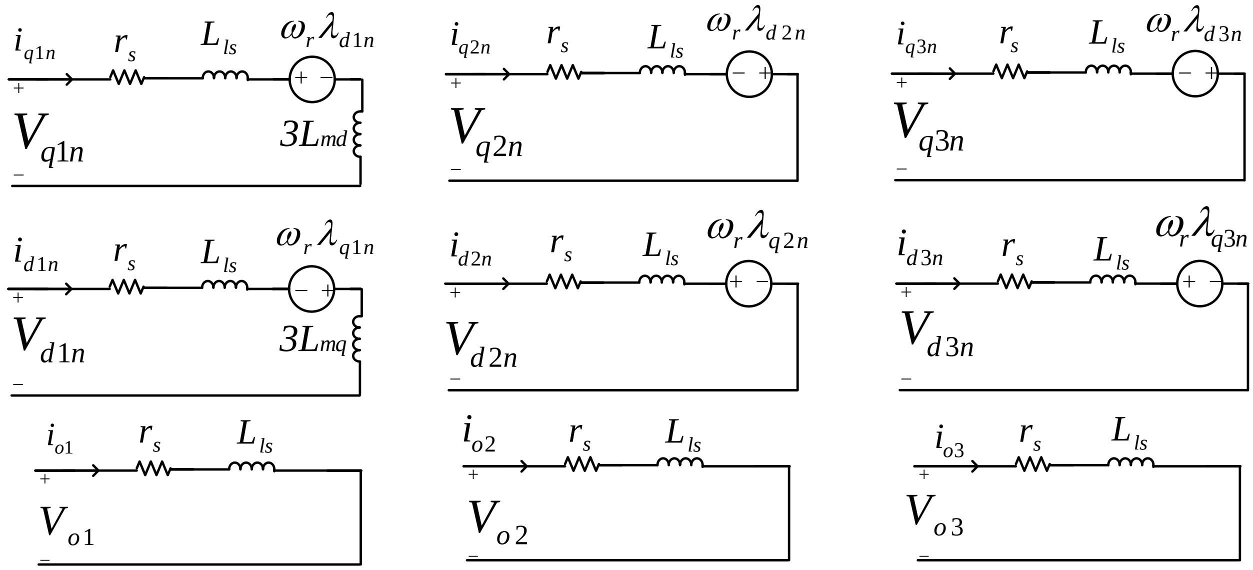


Figure 4.53: The equivalent circuit of the machine in the rotor reference frame.

$$\begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{q3n} \\ V_{d3n} \\ V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} = r_{sn} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \omega_{rn} \left(\begin{bmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \quad (4.104)$$

$$+ \begin{bmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} pi_{q1n} \\ pi_{d1n} \\ pi_{q2n} \\ pi_{d2n} \\ pi_{q3n} \\ pi_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix}$$

And finally the voltage equations of the machine can be represented as:

$$\begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{q3n} \\ V_{d3n} \\ V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} = r_{sn} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \omega_r \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_{ls} i_{q1n} \\ (L_{ls} + 3L_{md}) i_{d1n} + \lambda_{pm} \\ L_{ls} i_{q2n} \\ L_{ls} i_{d2n} \\ (L_{ls} + 3L_{mq}) i_{q3n} \\ L_{ls} i_{d3n} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} pL_{ls} i_{q1n} \\ p(L_{ls} + 3L_{md}) i_{d1n} \\ pL_{ls} i_{q2n} \\ pL_{ls} i_{d2n} \\ p(L_{ls} + 3L_{mq}) i_{q3n} \\ pL_{ls} i_{d3n} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.105)$$

The flux linkages of the machines can be represented as:

$$\begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{o1} \\ \lambda_{o2} \\ \lambda_{o3} \end{bmatrix} = \begin{bmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls} + 3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} + 3L_{mq} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.106)$$

The generated torque of the machine can be calculated using the co-energy equation. The co-energy of the machine can be presented as function of the stator currents and the flux linkages as [122]:

$$W_{co} = \frac{1}{2} I_s{}^t L_{ss} I_s + I_s{}^t \lambda_{pm} \quad (4.107)$$

Where L_{ss} and $\boldsymbol{I}_s$ are the machine inductances and current, defined as:

$$I_s = \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} \tag{4.109}$$

$$L_{ss} = \begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} & L_{a1a3} & L_{a1b3} & L_{a1c3} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} & L_{b1a3} & L_{b1b3} & L_{b1c3} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} & L_{c1a3} & L_{c1b3} & L_{c1c3} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} & L_{a2a3} & L_{a2b3} & L_{a2c3} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} & L_{b2a3} & L_{b2b3} & L_{b2c3} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} & L_{c2a3} & L_{c2b3} & L_{c2c3} \\ L_{a3a1} & L_{a3b1} & L_{a3c1} & L_{a3a2} & L_{a3b2} & L_{a3c2} & L_{a3a3} & L_{a3b3} & L_{a3c3} \\ L_{b3a1} & L_{b3b1} & L_{b3c1} & L_{b3a2} & L_{b3b2} & L_{b3c2} & L_{b3a3} & L_{b3b3} & L_{b3c3} \\ L_{c3a1} & L_{c3b1} & L_{c3c1} & L_{c3a2} & L_{c3b2} & L_{c3c2} & L_{c3a3} & L_{c3b3} & L_{c3c3} \end{pmatrix} \tag{4.108}$$

From the torque and co-energy equation, the electromagnetic torque can be derived as [122]:

$$T_e = \frac{\partial W_{co}}{\partial \theta_{rm}} \tag{4.110}$$

From the equation (4.110), the torque can be expressed as:

$$T_e = \frac{1}{2} I_s^{\,t} \frac{\partial L_{ss}}{\partial \theta_{rm}} I_s + I_s^{\,t} \frac{\partial \lambda_{pm}}{\partial \theta_{rm}} \tag{4.111}$$

Since the previous equations are in term of the electrical angle, the mechanical angle needs to be converted to the electrical angle as presented in equation (4.112).

$$\theta_r = \frac{P}{2}\theta_{rm} \tag{4.112}$$

Therefore, the torque equation changes to the equation (4.113):

$$T_e = \frac{P}{2}\frac{1}{2} I_s{}^t \frac{\partial L_{ss}}{\partial \theta_r} I_s + \frac{P}{2} I_s{}^t \frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{4.113}$$

Substituting the stator currents with their corresponding values in rotor reference frame results in:

$$T_e = \frac{3}{2}\frac{P}{2}(I_{qdn})^t\, T_n(\theta_r)\frac{\partial L_{ss}}{\partial \theta_r} T_n(\theta_r)^{-1} I_{qdn} + \frac{3}{2}\frac{P}{2}(I_{qdn})^t\, T_n(\theta_r)\frac{\partial \lambda_{pm}}{\partial \theta_r} \tag{4.114}$$

This equation can be rewritten as:

$$T_e = \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix}^t \begin{bmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \frac{3}{2}\frac{P}{2}\begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix}^t \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.115)$$

The electromagnetic torque of the machine can be represented as:

$$T_e = \frac{3P}{4}(L_{ls} + 3L_{md} - L_{ls})i_{q1n}i_{d1n} + \frac{3P}{4}(L_{ls} - L_{ls})i_{q2n}i_{d2n} + \\ \frac{3P}{4}(L_{ls} - 3L_{mq} - L_{ls})i_{q3n}i_{d3n} + \frac{3P}{4}\left(i_{q1n}\lambda_{pm}\right) + \frac{3P}{4}(L_{ls} - L_{ls})i_{o1}i_{o2} + \\ \frac{3P}{4}(L_{ls} - L_{ls})i_{o1}i_{o3} + \frac{3P}{4}(L_{ls} - L_{ls})i_{o2}i_{o3} = \\ \frac{9P}{4}\left(L_{md}i_{q1n}i_{d1n} - L_{mq}i_{q3n}i_{d3n} - i_{d1n}\lambda_{pm}\right) \tag{4.116}$$

The dynamic equation governing rotor speed can also be derived using the electromagnetic and load torque. In the equation (4.117) *'P'* is the pole pairs of the machine, 'ω_r' is the rotor speed, *'B'* is the friction coefficient and 'T_L' is the mechanical load torque applied to the machine.

$$T_e = J\left(\frac{2}{P}\right)p\omega_r + T_L + B\omega_r \tag{4.117}$$

In equation (4.104) the torque producing voltages (the voltages that have inductances bigger than the leakage) are V_{q1n} and V_{d3n}. The rest of the voltages are not able to produce electromagnetic torque. To generate a simpler model, the torque producing voltages need to be shifted to the top rows of the model matrix. The model modification can be done by modifying the diagonalizing matrix. The new diagonalizig matrix ($P^{'-1}$ and $P^{'}$) are presented in the equations (4.118) and (4.119) respectively. This matrix has been generated by exchanging the first and fifth rows of the original matrix and also multiplying the first and second row to '-1'.

$$P^{'-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} \end{bmatrix} \tag{4.118}$$

$$P^{'} = \begin{bmatrix} 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 & -2 & 0 \\ -1 & 0 & 0 & 0 & 0 & -2 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{4.119}$$

Using the new diagonalzing matrix and inserting the zero sequences, the new transformation matrix can be generated as equation (4.120).

$$T'_n(\theta_r) = P'^{-1} T'(\theta_r) =$$

$$\frac{2}{3}\begin{bmatrix} 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C(\theta_r+\alpha_2) & C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & S(\theta_r+\alpha_2) & S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3+\gamma) \\ 0 & 0 & 0 & 0 & 0 & 0 & S(\theta_r+\alpha_3) & S(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3+\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} =$$

$$\frac{2}{3}\begin{bmatrix} -\frac{S(\theta_r+\alpha_1)}{3} & -\frac{S(\theta_r+\alpha_1-\gamma)}{3} & -\frac{S(\theta_r+\alpha_1+\gamma)}{3} & -\frac{S(\theta_r+\alpha_2)}{3} & -\frac{S(\theta_r+\alpha_2-\gamma)}{3} & -\frac{S(\theta_r+\alpha_2+\gamma)}{3} & -\frac{S(\theta_r+\alpha_3-\beta)}{3} & -\frac{S(\theta_r+\alpha_3-\gamma)}{3} & -\frac{S(\theta_r+\alpha_3+\gamma)}{3} \\ -\frac{C(\theta_r+\alpha_1)}{3} & -\frac{C(\theta_r+\alpha_1-\gamma)}{3} & -\frac{C(\theta_r+\alpha_1+\gamma)}{3} & -\frac{C(\theta_r+\alpha_2)}{3} & -\frac{C(\theta_r+\alpha_2-\gamma)}{3} & -\frac{C(\theta_r+\alpha_2+\gamma)}{3} & -\frac{C(\theta_r+\alpha_3)}{3} & -\frac{C(\theta_r+\alpha_3-\gamma)}{3} & -\frac{C(\theta_r+\alpha_3+\gamma)}{3} \\ \frac{-C(\theta_r+\alpha_1)}{2} & \frac{-C(\theta_r+\alpha_1-\gamma)}{2} & \frac{-C(\theta_r+\alpha_1+\gamma)}{2} & 0 & 0 & 0 & \frac{C(\theta_r+\alpha_3)}{2} & \frac{C(\theta_r+\alpha_3-\gamma)}{2} & \frac{C(\theta_r+\alpha_3+\gamma)}{2} \\ \frac{-S(\theta_r+\alpha_1)}{2} & \frac{-S(\theta_r+\alpha_1-\gamma)}{2} & \frac{-S(\theta_r+\alpha_1+\gamma)}{2} & 0 & 0 & 0 & \frac{S(\theta_r+\alpha_3)}{2} & \frac{S(\theta_r+\alpha_3-\gamma)}{2} & \frac{S(\theta_r+\alpha_3+\gamma)}{2} \\ \frac{C(\theta_r+\alpha_1)}{6} & \frac{C(\theta_r+\alpha_1-\gamma)}{6} & \frac{C(\theta_r+\alpha_1+\gamma)}{6} & \frac{-C(\theta_r+\alpha_2)}{3} & \frac{-C(\theta_r+\alpha_2-\gamma)}{3} & \frac{-C(\theta_r+\alpha_2+\gamma)}{3} & \frac{C(\theta_r+\alpha_3)}{6} & \frac{C(\theta_r+\alpha_3-\gamma)}{6} & \frac{C(\theta_r+\alpha_3+\gamma)}{6} \\ \frac{S(\theta_r+\alpha_1)}{6} & \frac{S(\theta_r+\alpha_1-\gamma)}{6} & \frac{S(\theta_r+\alpha_1+\gamma)}{6} & \frac{-S(\theta_r+\alpha_2)}{3} & \frac{-S(\theta_r+\alpha_2-\gamma)}{3} & \frac{-S(\theta_r+\alpha_2+\gamma)}{3} & \frac{S(\theta_r+\alpha_3)}{6} & \frac{S(\theta_r+\alpha_3-\gamma)}{6} & \frac{S(\theta_r+\alpha_3+\gamma)}{6} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.120}$$

And the inverse transformation can be represented as:

$$T'^{-1}{}_{n}(\theta_r) = T'^{-1}(\theta_r)P' =$$

$$\begin{bmatrix}
C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & C(\theta_r+\alpha_2) & S(\theta_r+\alpha_2) & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & C(\theta_r+\alpha_2+\gamma) & S(\theta_r+\alpha_2+\gamma) & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} = \quad (4.121)$$

$$\begin{bmatrix}
-S(\theta_r+\alpha_1) & -C(\theta_r+\alpha_1) & -C(\theta_r+\alpha_1) & -S(\theta_r+\alpha_1) & C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 1 & 0 & 0 \\
-S(\theta_r+\alpha_1-\gamma) & -C(\theta_r+\alpha_1-\gamma) & -C(\theta_r+\alpha_1-\gamma) & -S(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 1 & 0 & 0 \\
-S(\theta_r+\alpha_1+\gamma) & -C(\theta_r+\alpha_1+\gamma) & -C(\theta_r+\alpha_1+\gamma) & -S(\theta_r+\alpha_1+\gamma) & C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 1 & 0 & 0 \\
-S(\theta_r+\alpha_2) & -C(\theta_r+\alpha_2) & 0 & 0 & -2C(\theta_r+\alpha_2) & -2S(\theta_r+\alpha_2) & 0 & 1 & 0 \\
S(\theta_r+\alpha_2-\gamma) & -C(\theta_r+\alpha_2-\gamma) & 0 & 0 & -2C(\theta_r+\alpha_2-\gamma) & -2S(\theta_r+\alpha_2-\gamma) & 0 & 1 & 0 \\
-S(\theta_r+\alpha_2+\gamma) & -C(\theta_r+\alpha_2+\gamma) & 0 & 0 & -2C(\theta_r+\alpha_2+\gamma) & -2S(\theta_r+\alpha_2+\gamma) & 0 & 1 & 0 \\
-S(\theta_r+\alpha_3) & -C(\theta_r+\alpha_3) & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & C(\theta_r+\alpha_3) & S(\theta_r+\alpha_3) & 0 & 0 & 1 \\
-S(\theta_r+\alpha_3-\gamma) & -C(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & C(\theta_r+\alpha_3-\gamma) & S(\theta_r+\alpha_3-\gamma) & 0 & 0 & 1 \\
-S(\theta_r+\alpha_3+\gamma) & -C(\theta_r+\alpha_3+\gamma) & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & C(\theta_r+\alpha_3+\gamma) & S(\theta_r+\alpha_3+\gamma) & 0 & 0 & 1
\end{bmatrix}$$

Now the machine voltage equations can be transformed to the new reference frame using the new transformation matrix. The machine equations in natural quantities can be expressed as:

$$V_{abci} = r_s i_{abci} + p\lambda_{abci} \tag{4.122}$$

Substituting the currents and flux linkages by their equivalent in the decoupled reference frame results in:

$$V_{abci} = r_s T'_n{}^{-1}(\theta_r) i_{qdn} + pT'_n{}^{-1}(\theta_r)\lambda_{qdn} \tag{4.123}$$

Multiplying the $T'_n(\theta_r)$ from the left hand side, the equation changes to:

$$T'_n(\theta_r)V_{abci} = T'_n(\theta_r) r_s T'_n{}^{-1}(\theta_r) i_{qdn} + T'_n(\theta_r) pT'_n{}^{-1}(\theta_r)\lambda_{qdn} \tag{4.124}$$

The different components of the equation (4.124) can be represented as:

$$T_n(\theta_r)V_{abci} = T_n(\theta_r) \times \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \\ V_{a3} \\ V_{b3} \\ V_{c3} \end{bmatrix} = \begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{q3n} \\ V_{d3n} \\ V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} \tag{4.125}$$

$$i_{qdn} = T_n(\theta_r) i_{abci} = T_n(\theta_r) \times \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \\ i_{a3} \\ i_{b3} \\ i_{c3} \end{bmatrix} = \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} \tag{4.126}$$

The derivative part also is equal to:

$$T'_n(\theta_r)pT'^{-1}_n(\theta_r)\lambda_{qdn} = T'_n(\theta_r)pT'^{-1}_n(\theta_r)\lambda_{qdn} + T'_n(\theta_r)T'^{-1}_n(\theta_r)p\lambda_{qdn} \tag{4.127}$$

The first part of the equation (4.127) is equal to:

$$T'_n(\theta_r)pT'^{-1}_n(\theta_r)\lambda_{qdn} = P'^{-1}T(\theta_r)pT^{-1}(\theta_r)P'\lambda_{qdn} = P'^{-1}\omega_r P'\lambda_{qdn} \tag{4.128}$$

Substituting the P', P'^{-1} and ω_r into the equation (4.128) results in:

$$T_n(\theta_r)pT_n^{-1}(\theta_r)\lambda_{qdn} =$$

$$\begin{bmatrix} 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \omega_r \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{do1} \\ \lambda_{do2} \\ \lambda_{do3} \end{bmatrix} = \tag{4.129}$$

$$\omega_r \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{do1} \\ \lambda_{do2} \\ \lambda_{do3} \end{bmatrix}$$

The second part of the equation (4.127) is equal to:

$$T_n(\theta_r)T_n^{-1}(\theta_r)p\lambda_{qdn} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p\lambda_{q1n} \\ p\lambda_{d1n} \\ p\lambda_{q2n} \\ p\lambda_{d2n} \\ p\lambda_{q3n} \\ p\lambda_{d3n} \\ p\lambda_{do1} \\ p\lambda_{do2} \\ p\lambda_{do3} \end{bmatrix} \tag{4.130}$$

The flux linkages in the decoupled reference frame can be explained as:

$$p\lambda_{dqin} = T'_n(\theta_r) \times (L_{ss} i_{abci} + \lambda_{Pmabci}), \quad i = 1,2,3 \tag{4.131}$$

Substituting the i_{abc_i} by their equivalents in the decoupled reference frame, the flux linkage equation can change to:

$$p\lambda_{dqin} = T'_n(\theta_r) L_{ss} T'^{-1}_n(\theta_r) i_{qdn} + T'_n(\theta_r)\lambda_{Pmabci} \tag{4.132}$$

The first term of the equation (4.132) is equal to equation (4.133).

$$T'_n(\theta_r)L_{ss}T'^{-1}_n(\theta_r)i_{qdn} = P'^{-1}\underbrace{T(\theta_r)L_{ss}T^{-1}(\theta_r)}_{L_{qd}}P'i_{qdn} = P'^{-1}L_{qd}P'i_{qdn} =$$

$$\begin{bmatrix} 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_q & 0 & L_{mq} & 0 & L_{mq} & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & L_{md} & 0 & L_{md} & 0 & 0 & 0 \\ L_{mq} & 0 & L_q & 0 & L_{mq} & 0 & 0 & 0 & 0 \\ 0 & L_{md} & 0 & L_d & 0 & L_{md} & 0 & 0 & 0 \\ L_{mq} & 0 & L_{mq} & 0 & L_q & 0 & 0 & 0 & 0 \\ 0 & L_{md} & 0 & L_{md} & 0 & L_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \times \tag{4.133}$$

$$\begin{bmatrix} 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} = \begin{bmatrix} L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix}$$

The second part of the equation (4.132) is equal to:

$$\lambda_{pmqdn} = T^{'}(\theta_r)\begin{bmatrix}\lambda_{pma1}\\ \lambda_{pmb1}\\ \lambda_{pmc1}\\ \lambda_{pma2}\\ \lambda_{pmb2}\\ \lambda_{pmc2}\\ \lambda_{pma3}\\ \lambda_{pmb3}\\ \lambda_{pmc3}\end{bmatrix} = P^{'-1}\lambda_{pmqd} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0\\ -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0\\ -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0\\ 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0\\ \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0 & 0\\ 0 & \frac{1}{6} & 0 & -\frac{1}{3} & 0 & \frac{1}{6} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix}\lambda_{pm}\\ 0\\ \lambda_{pm}\\ 0\\ \lambda_{pm}\\ 0\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}0\\ -\lambda_{pm}\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix} \qquad (4.134)$$

The resistive term also can be transformed to decoupled reference frame as:

$$r_{sn} = P^{'-1}T^{'}(\theta_r)r_s T^{'-1}P^{'}(\theta_r) = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_s \end{bmatrix} \qquad (4.135)$$

Using the generated parts and by adding the zero sequence circuits to the model, the voltage equation of the machine in the decoupled reference frame is presented in equation (4.136).

$$\begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{q3n} \\ V_{d3n} \\ V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} = r_{sn} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \omega_{rn} \left(\begin{bmatrix} L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \tag{4.136}$$

$$+ \begin{bmatrix} L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} pi_{q1n} \\ pi_{d1n} \\ pi_{q2n} \\ pi_{d2n} \\ pi_{q3n} \\ pi_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix}$$

Where the term ω_{rn} is defined as:

$$\omega_{rn} = \omega_r \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.137}$$

The flux linkages are also defined as:

$$\begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \\ \lambda_{q3n} \\ \lambda_{d3n} \\ \lambda_{o1} \\ \lambda_{o2} \\ \lambda_{o3} \end{bmatrix} = \begin{bmatrix} L_{ls}+3L_{md} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls}+3L_{mq} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{q3n} \\ i_{d3n} \\ i_{o1} \\ i_{o2} \\ i_{o3} \end{bmatrix} + \begin{bmatrix} 0 \\ -\lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.138}$$

The equation for electromagnetic torque can be generated using the power balance between the mechanical output power and the electrical input power. The equation (4.139) shows the relationship between the input and the output power.

$$T_e\left(\frac{2}{P}\right)\omega_r = \sum_{i=1}^{3}\frac{3}{2}\left(V_{qin}i_{qin} + V_{din}i_{din}\right) = \tag{4.139}$$

$$\frac{3}{2}\sum_{i=1}^{3}\left(r_s\left(i^2{}_{qin} + i^2{}_{din}\right) + \left(\lambda_{din}i_{qin} - \lambda_{qin}i_{din}\right)\omega_r + \left(p\lambda_{qin}i_{qin} + p\lambda_{din}i_{din}\right)\right)$$

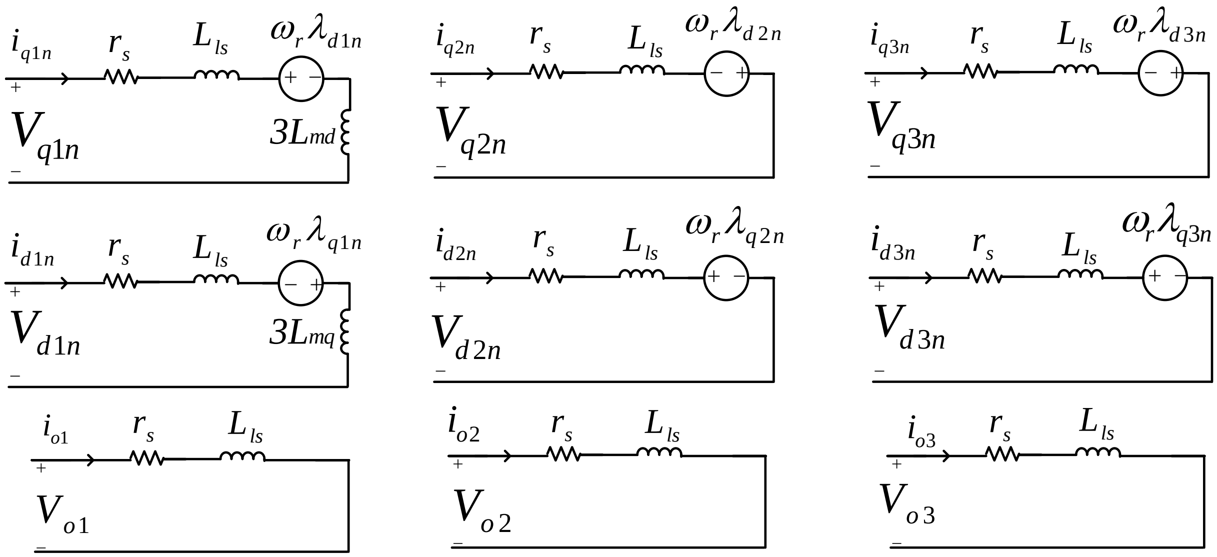


Figure 4.54: The equivalent circuit of the machine in the decoupled reference frame.

The first term is the power loss and the derivative part is the changes of the stored energy which are not effective on the electromagnetic torque. Therefore, using the average stored energy, the power balance can be expressed equation (4.140).

$$\omega_r T_e = \sum_{i=1}^{3} \frac{3}{2}\left(V_{qin} i_{qin} + V_{din} i_{din}\right) = \frac{3}{2}\left(\frac{P}{2}\right)\sum_{i=1}^{3} \omega_r \left(-\lambda_{din} i_{qin} + \lambda_{qin} i_{din}\right) \tag{4.140}$$

By removing the term ' ω_r ' from the both sides of the equation (4.141) the torque equating can be presented as:

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)\left(-\left(L_{ls}+3L_{mq}\right) i_{d1n} i_{q1n} + \lambda_{pm}\, i_{q1n} + \left(L_{ls}+3L_{md}\right) i_{q1n} i_{d1n}\right)$$

$$+\frac{3}{2}\left(\frac{P}{2}\right)\left(L_{ls} i_{d2n} i_{q2n} - L_{ls} i_{d2n} i_{q2n}\right) + \frac{3}{2}\left(\frac{P}{2}\right)\left(L_{ls} i_{d3n} i_{q3n} - L_{ls} i_{d3n} i_{q3n}\right) = \tag{4.141}$$

$$\frac{3P}{4}\left(3\left(L_{md}-L_{mq}\right) i_{d1n} i_{q1n} + \lambda_{pm} i_{q1n}\right)$$

4.7 Average Model of the Asymmetrical Double-Star Six-Phase IPM

In this section a double star asymmetrical six-phase IPM is modeled using the Fourier series of the machine parameters. The modelling starts with the turn functions of the machine phases and after generating the Fourier series of the turn function, winding function and the airgap function, the Fourier series of the machine inductances are generated and the generated inductances are used to derive the model of the machine in the rotor reference frame. Finally, the machine model is decoupled to remove the coupling terms between different machines. The decoupling matrix and the new transformation to the decoupled reference frame are generated and presented in this chapter.

4.7.1 Generating the Inductances of Six-Phase Double-Star IPM Machine

To generate the inductances of the machine the clock diagram is needed. The clock diagram of the machine can be generated using the winding design method. The six-phase machine is composed of two sets of three phase machines [149]. The machine totally has 24 slots and each machine covers 12 slots. Since the machine has four poles, the slot angular pitch can be calculated as:

$$\gamma = \frac{180 \times P}{24} = \frac{180 \times 4}{24} = 30 \ \ (Degree) \tag{4.142}$$

The slot between phases for each set can be calculated as:

$$SBPH = \frac{120}{\gamma} = \frac{120}{30} = 4 \tag{4.143}$$

The full coil pitch is:

$$FCP = \frac{24}{P} = \frac{24}{2} = 12 \tag{4.144}$$

Since the machine has concentrated windings then the belt is equal to 1. And also since the machine is an asymmetrical one the slot between two adjacent machines is:

$$SM = \frac{4}{2M} = 1 \tag{4.145}$$

Where 'M' is the number of the machines sets. For the machine 1 the winding scheme is:

Table 4.3 The winding connections of set 1.

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
1	7	5	11	9	15

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
7	13	11	17	15	21

$A1^+$	$A1^-$	$B1^+$	$B1^-$	$C1^+$	$C1^-$
13	19	17	23	21	3

$A1_-$	$A1_+$	$B1_-$	$B1_+$	$C1_-$	$C1_+$
7	13	11	17	15	21

The machine 2 has 1 slots shift from the machine 1, therefor the winding scheme for the machine 2 is:

Table 4.4 The winding connections of set 2.

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
2	8	6	12	10	16

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
8	14	12	18	16	22

$A2^+$	$A2^-$	$B2^+$	$B2^-$	$C2^+$	$C2^-$
14	20	18	24	10	16

$A2_-$	$A2_+$	$B2_-$	$B2_+$	$C2_-$	$C2_+$
20	2	24	6	16	22

Now using the Tables 4.3 and 4.4 the machine clock diagram can be presented as Figure 4.55. The modeling can start from the turn functions of the machines. The turn functions for the machine 1 and 2 phases and the airgap function of the rotor are shown in the Figures 4.56, 4.57 respectively.

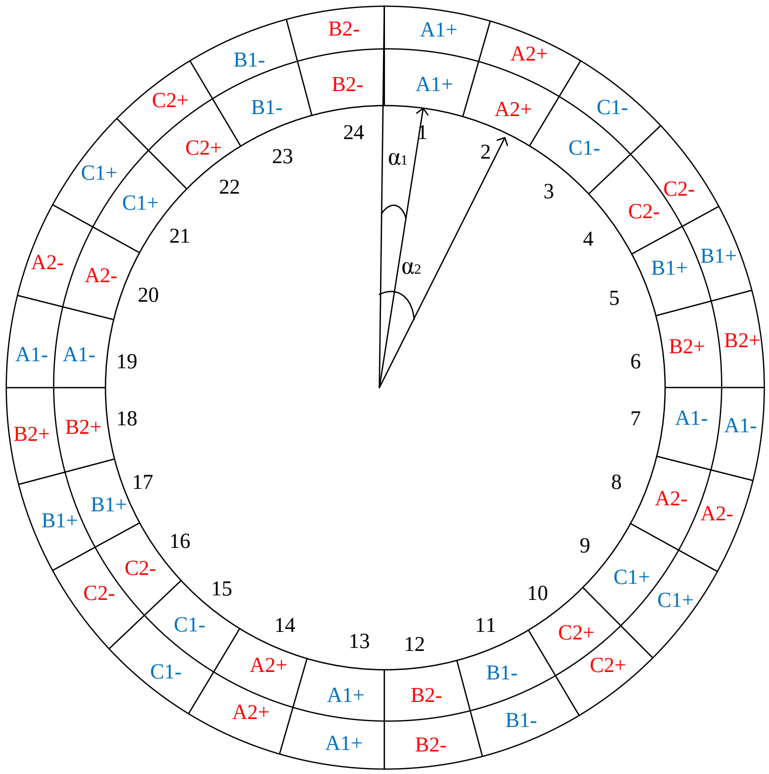


Figure 4.55: The clock diagram of the asymmetrical double star machine.

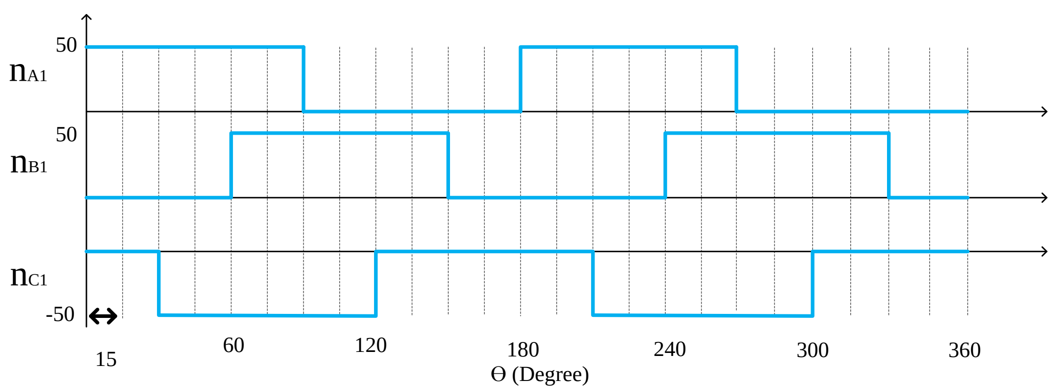


Figure 4.56: The turn functions of the machine 1 phases.

Using the Figures 4.56 and 4.57 the Fourier series of the turn function can be derived as [153]:

$$n_x = N_0 + N_1 \sin(\theta - k\beta) + N_3 \sin 3(\theta - k\beta) + N_5 \sin 5(\theta - k\beta) + N_7 \sin 7(\theta - k\beta)$$

$$x = a_i, b_i, c_i,\ i = 1,\ 2,\ \beta = \frac{\pi}{6},\quad N_n = \frac{4N_x}{n\pi} \tag{4.146}$$

Where k is defined in the Table 4.5.

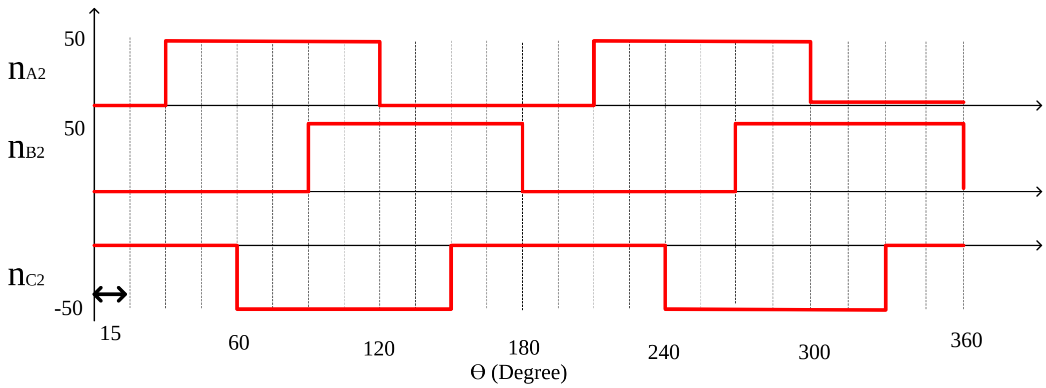


Figure 4.57: The turn functions of the machine 2 phases.

Table 4.5 The corresponding k for the phases.

x	k	x	k
a_1	0	a_2	1
b_1	4	b_2	5
c_1	8	c_2	9

The machine has the same rotor shape as the nine phase one therefore the air gap function and the Fourier series of the inverse of the airgap function will be the same. The equation (4.3) which shows the Fourier series of the inverse airgap function is repeated here [83].

$$g^{-1}(\theta,\theta_r) = a_0 + a_1\cos 2(\theta-\theta_r) + a_2\cos 6(\theta-\theta_r) + a_3\cos 10(\theta-\theta_r) + a_4\cos 14(\theta-\theta_r) \quad (4.147)$$

Where: a_0, a_1, a_2, a_3 and a_4 are the Fourier series amplitude of the inverse air gap function. They are given as:

$$a_0 = a,\ a_1 = -b,\ a_2 = -\frac{b}{3},\ a_3 = -\frac{b}{5},\ a_4 = -\frac{b}{7}$$
$$a = \frac{1}{2}\left(\frac{1}{g_b} + \frac{1}{g_b}\right),\ b = \frac{1}{2}\left(\frac{1}{g_b} - \frac{1}{g_a}\right) \quad (4.148)$$

Using the Fourier series of the turn functions and the airgap function and equation (4.149) [74] the winding function of each phase can be calculated as:

$$N_w(\theta) = n_w(\theta) - \frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} \tag{4.149}$$

The different parts of the equation (4.149) can be expressed as:

$$\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta = \int_0^{2\pi} \begin{pmatrix} a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) \\ + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r) \end{pmatrix} d\theta = 2\pi a_0 \tag{4.150}$$

The nominator integrator also is:

$$\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta = \int_0^{2\pi} \begin{array}{l} (N_0 + N_1 \sin(\theta-k\beta) + N_3 \sin 3(\theta-k\beta) + N_5 \sin 5(\theta-k\beta) + N_7 \sin 7(\theta-k\beta)) \times \\ (a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) d\theta \end{array} =$$

$$\int_0^{2\pi} \begin{pmatrix} N_0 (a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \\ + N_1 \sin(\theta-k\beta)(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \\ + N_3 \sin 3(\theta-k\beta)(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \\ + N_5 \sin 5(\theta-k\beta)(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \\ + N_7 \sin 7(\theta-k\beta)(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)) \end{pmatrix} d\theta = \tag{4.151}$$

$$\int_0^{2\pi} (N_0(a_0)) d\theta = 2\pi N_0 a_0$$

The equations (4.149) and (4.151) result in:

$$\frac{\int_0^{2\pi} \frac{n_w(\theta)}{g(\theta,\theta_r)} d\theta}{\int_0^{2\pi} \frac{1}{g(\theta,\theta_r)} d\theta} = \frac{2\pi N_0 a_0}{2\pi a_0} = N_0 = \frac{N_1}{2} \tag{4.152}$$

Substituting the equation (4.152) into the equation (4.146) results in:

$$\begin{aligned} N_w(\theta) &= N_0 + N_1 \sin(\theta-k_w\beta) + N_3 \sin 3(\theta-k_w\beta) + N_5 \sin 5(\theta-k_w\beta) + N_7 \sin 7(\theta-k_w\beta) - N_0 \\ &= N_1 \sin(\theta-k_w\beta) + N_3 \sin 3(\theta-k_w\beta) + N_5 \sin 5(\theta-k_w\beta) + N_7 \sin 7(\theta-k_w\beta) \end{aligned} \tag{4.153}$$

Using the generated winding function, airgap function and turn function the mutual inductances between each couple of the windings can be calculated according to the equation (4.154).

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \times \\ \left(N_1 \sin(\theta - k_j\beta) + N_3 \sin 3(\theta - k_j\beta) + N_5 \sin 5(\theta - k_j\beta) + N_7 \sin 7(\theta - k_j\beta)\right) \times \\ \left(N_0 + N_1 \sin(\theta - k_i\beta) + N_3 \sin 3(\theta - k_i\beta) + N_5 \sin 5(\theta - k_i\beta) + N_7 \sin 7(\theta - k_i\beta)\right) \end{pmatrix} d\theta \quad (4.154)$$

The equation (4.153) can be simplified according the following procedure:

The winding function terms can be multiplied to the turn function according to equation (4.1155).

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \times \\ \begin{pmatrix} N_1 \sin(\theta - k_j\beta) \begin{pmatrix} N_0 + N_1 \sin(\theta - k_i\beta) + N_3 \sin 3(\theta - k_i\beta) + \\ N_5 \sin 5(\theta - k_i\beta) + N_7 \sin 7(\theta - k_i\beta) \end{pmatrix} + \\ N_3 \sin 3(\theta - k_j\beta) \begin{pmatrix} N_0 + N_1 \sin(\theta - k_i\beta) + N_3 \sin 3(\theta - k_i\beta) + \\ N_5 \sin 5(\theta - k_i\beta) + N_7 \sin 7(\theta - k_i\beta) \end{pmatrix} + \\ N_5 \sin 5(\theta - k_j\beta) \begin{pmatrix} N_0 + N_1 \sin(\theta - k_i\beta) + N_3 \sin 3(\theta - k_i\beta) + \\ N_5 \sin 5(\theta - k_i\beta) + N_7 \sin 7(\theta - k_i\beta) \end{pmatrix} + \\ N_7 \sin 7(\theta - k_j\beta) \begin{pmatrix} N_0 + N_1 \sin(\theta - k_i\beta) + N_3 \sin 3(\theta - k_i\beta) + \\ N_5 \sin 5(\theta - k_i\beta) + N_7 \sin 7(\theta - k_i\beta) \end{pmatrix} \end{pmatrix} \end{pmatrix} d\theta \quad (4.155)$$

The non-zero terms are:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1 \cos 2(\theta-\theta_r) + a_2 \cos 6(\theta-\theta_r) + a_3 \cos 10(\theta-\theta_r) + a_4 \cos 14(\theta-\theta_r)\right) \times \\ \begin{pmatrix} \left(N_0 N_1 \sin(\theta - k_j\beta) + N_1 N_1 \sin(\theta - k_j\beta) \sin(\theta - k_i\beta)\right) + \\ \left(N_0 N_3 \sin 3(\theta - k_j\beta) + N_3 N_3 \sin 3(\theta - k_j\beta) \sin 3(\theta - k_i\beta) + \right) + \\ \left(N_0 N_5 \sin 5(\theta - k_j\beta) + N_5 N_5 \sin 5(\theta - k_j\beta) \sin 5(\theta - k_i\beta)\right) + \\ \left(N_0 N_7 \sin 7(\theta - k_j\beta) + N_7 N_7 \sin 7(\theta - k_j\beta) \sin 7(\theta - k_i\beta)\right) \end{pmatrix} \end{pmatrix} d\theta \quad (4.156)$$

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d\theta =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1\cos 2(\theta-\theta_r) + a_2\cos 6(\theta-\theta_r) + a_3\cos 10(\theta-\theta_r) + a_4\cos 14(\theta-\theta_r)\right)\times \\ \begin{pmatrix} \left(N_0N_1\sin(\theta-k_j\beta) + \frac{N_1^{\ 2}}{2}\left(-\cos\left(2\theta-(k_j+k_i)\beta\right)+\cos\left((-k_j+k_i)\beta\right)\right)\right)+ \\ \left(N_0N_3\sin 3(\theta-k_j\beta) + \frac{N_3^{\ 2}}{2}\left(-\cos 3\left(2\theta-(k_j+k_i)\beta\right)+\cos 3\left((-k_j+k_i)\beta\right)\right)\right)+ \\ \left(N_0N_5\sin 5(\theta-k_j\beta) + \frac{N_5^{\ 2}}{2}\left(-\cos 5\left(2\theta-(k_j+k_i)\beta\right)+\cos 5\left((-k_j+k_i)\beta\right)\right)\right)+ \\ \left(N_0N_7\sin 7(\theta-k_j\beta) + \frac{N_7^{\ 2}}{2}\left(-\cos 7\left(2\theta-(k_j+k_i)\beta\right)+\cos 7\left((-k_j+k_i)\beta\right)\right)\right) \end{pmatrix} \end{pmatrix} d\theta \quad (4.157)$$

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} \left(a_0 + a_1\cos 2(\theta-\theta_r) + a_2\cos 6(\theta-\theta_r) + a_3\cos 10(\theta-\theta_r) + a_4\cos 14(\theta-\theta_r)\right)\times \\ \begin{pmatrix} \left(N_0N_1\sin(\theta-k_j\beta) + \frac{N_1^{\ 2}}{2}\left(-\cos\left(2\theta-(k_j+k_i)\beta\right)+\cos\left((k_i-k_j)\beta\right)\right)\right)+ \\ \left(N_0N_3\sin 3(\theta-k_j\beta) + \frac{N_3^{\ 2}}{2}\left(-\cos 3\left(2\theta-(k_j+k_i)\beta\right)+\cos 3\left((k_i-k_j)\beta\right)\right)\right)+ \\ \left(N_0N_5\sin 5(\theta-k_j\beta) + \frac{N_5^{\ 2}}{2}\left(-\cos 5\left(2\theta-(k_j+k_i)\beta\right)+\cos 5\left((k_i-k_j)\beta\right)\right)\right)+ \\ \left(N_0N_7\sin 7(\theta-k_j\beta) + \frac{N_7^{\ 2}}{2}\left(-\cos 7\left(2\theta-(k_j+k_i)\beta\right)+\cos 7\left((k_i-k_j)\beta\right)\right)\right) \end{pmatrix} \end{pmatrix} d\theta \quad (4.158)$$

Again the non-zero terms are:

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} a_0\left(\frac{N_1{}^2}{2}\cos\left(\left(k_i - k_j\right)\beta\right) + \frac{N_3{}^2}{2}\cos 3\left(\left(k_i - k_j\right)\beta\right) + \frac{N_5{}^2}{2}\cos 5\left(\left(k_i - k_j\right)\beta\right) + \frac{N_7{}^2}{2}\cos 7\left(\left(k_i - k_j\right)\beta\right)\right) \\ - a_1 \frac{N_1{}^2}{2}\cos\left(2\theta - \left(k_j + k_i\right)\beta\right)\cos 2(\theta - \theta_r) \\ - a_2 \frac{N_3{}^2}{2}\cos 3\left(2\theta - \left(k_j + k_i\right)\beta\right)\cos 6(\theta - \theta_r) \\ - a_3 \frac{N_5{}^2}{2}\cos 5\left(2\theta - \left(k_j + k_i\right)\beta\right)\cos 10(\theta - \theta_r) \\ - a_4 \frac{N_7{}^2}{2}\cos 7\left(2\theta - \left(k_j + k_i\right)\beta\right)\cos 14(\theta - \theta_r) \end{pmatrix} d\theta \qquad (4.159)$$

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$\mu_o rl \int_0^{2\pi} \begin{pmatrix} a_0\left(\frac{N_1{}^2}{2}\cos\left(\left(k_i - k_j\right)\beta\right) + \frac{N_3{}^2}{2}\cos 3\left(\left(k_i - k_j\right)\beta\right) + \frac{N_5{}^2}{2}\cos 5\left(\left(k_i - k_j\right)\beta\right) + \frac{N_7{}^2}{2}\cos 7\left(\left(k_i - k_j\right)\beta\right)\right) \\ - a_1 \frac{N_1{}^2}{4}\left(\cos\left(4\theta - 2\theta_r - \left(k_j + k_i\right)\beta\right) + \cos\left(2\theta_r - \left(k_j + k_i\right)\beta\right)\right) \\ - a_2 \frac{N_3{}^2}{4}\left(\cos\left(12\theta - 6\theta_r - 3\left(k_j + k_i\right)\beta\right) + \cos\left(6\theta_r - 3\left(k_j + k_i\right)\beta\right)\right) \\ - a_3 \frac{N_5{}^2}{4}\left(\cos\left(20\theta - 10\theta_r - 5\left(k_j + k_i\right)\beta\right) + \cos\left(10\theta_r - 5\left(k_j + k_i\right)\beta\right)\right) \\ - a_4 \frac{N_7{}^2}{4}\left(\cos\left(28\theta - 14\theta_r - 7\left(k_j + k_i\right)\beta\right) + \cos\left(14\theta_r - 7\left(k_j + k_i\right)\beta\right)\right) \end{pmatrix} d\theta \qquad (4.160)$$

$$L_{ji} = \mu_o rl \int_0^{2\pi} \frac{1}{g(\varphi,\theta_r)} n_j(\theta) N_i(\theta) d(\theta) =$$

$$2\pi\mu_o rl \begin{pmatrix} a_0\left(\frac{N_1{}^2}{2}\cos\left(\left(k_i - k_j\right)\beta\right) + \frac{N_3{}^2}{2}\cos 3\left(\left(k_i - k_j\right)\beta\right) + \frac{N_5{}^2}{2}\cos 5\left(\left(k_i - k_j\right)\beta\right) + \frac{N_7{}^2}{2}\cos 7\left(\left(k_i - k_j\right)\beta\right)\right) \\ - a_1 \frac{N_1{}^2}{4}\cos\left(2\theta_r - \left(k_j + k_i\right)\beta\right) - a_2 \frac{N_3{}^2}{4}\cos\left(6\theta_r - 3\left(k_j + k_i\right)\beta\right) \\ - a_3 \frac{N_5{}^2}{4}\cos\left(10\theta_r - 5\left(k_j + k_i\right)\beta\right) - a_4 \frac{N_7{}^2}{4}\cos\left(14\theta_r - 7\left(k_j + k_i\right)\beta\right) \end{pmatrix} \qquad (4.161)$$

In the equation (4.161) the terms k_i and k_j can be defined as:

$$k_{i,j} = Corresponding\ Slot\ Number - 1 \qquad (4.162)$$

Now using the transformation matrix of equation (4.163), the inductances can be transformed to the rotor reference frame. The self-inductances of the machine 1 can be transformed to the rotor reference frame as:

$$L_{qd11} = T_1(\theta_r)L_1T_1(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{bmatrix} C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1+\gamma) \\ S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1+\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} \end{bmatrix}\begin{bmatrix} C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 1 \\ C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 1 \\ C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 1 \end{bmatrix}$$

$$\begin{bmatrix} L_{q1q1} & L_{q1d1} & L_{q101} \\ L_{d1q1} & L_{d1d1} & L_{d101} \\ L_{01q1} & L_{01d1} & L_{0101} \end{bmatrix} = \frac{3}{2}\begin{bmatrix} L_0C(\alpha_1-\alpha_1)+L_2C(\alpha_1+\alpha_1) & -L_0S(\alpha_1-\alpha_1)-L_2S(\alpha_1+\alpha_1) & 0 \\ L_0S(\alpha_1-\alpha_1)-L_2S(\alpha_1+\alpha_1) & L_0C(\alpha_1-\alpha_1)-L_2C(\alpha_1+\alpha_1) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.163)$$

$$= \frac{3\pi\mu_o rlN_1^{\,2}}{2}\begin{bmatrix} a_0C(\alpha_1-\alpha_1)+a_2C(\alpha_1+\alpha_1) & -a_0S(\alpha_1-\alpha_1)-a_2S(\alpha_1+\alpha_1) & 0 \\ a_0S(\alpha_1-\alpha_1)-a_2S(\alpha_1+\alpha_1) & a_0C(\alpha_1-\alpha_1)-a_2C(\alpha_1+\alpha_1) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Similarly, the self-inductances of the machine 2 can be transformed to the rotor reference frame as:

$$L_{qd22} = T_2(\theta_r)L_2T_2(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{bmatrix} C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2-\gamma) \\ S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_{a2a2} & L_{a2b2} & L_{a2c2} \\ L_{b2a2} & L_{b2b2} & L_{b2c2} \\ L_{c2a2} & L_{c2b2} & L_{c2c2} \end{bmatrix}\begin{bmatrix} C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 1 \\ C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 1 \\ C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 1 \end{bmatrix} =$$

$$\begin{bmatrix} L_{q2q2} & L_{q2d2} & L_{q202} \\ L_{d2q2} & L_{d2d2} & L_{d202} \\ L_{02q2} & L_{02d2} & L_{0202} \end{bmatrix} = \frac{3}{2}\begin{bmatrix} L_0C(\alpha_2-\alpha_2)+L_2C(\alpha_2+\alpha_2) & -L_0S(\alpha_2-\alpha_2)-L_2S(\alpha_2+\alpha_2) & 0 \\ L_0S(\alpha_2-\alpha_2)-L_2S(\alpha_2+\alpha_2) & L_0C(\alpha_2-\alpha_2)-L_2C(\alpha_2+\alpha_2) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.164)$$

$$= \frac{3\pi\mu_o rlN_1^{\,2}}{2}\begin{bmatrix} a_0C(\alpha_2-\alpha_2)+a_2C(\alpha_2+\alpha_2) & -a_0S(\alpha_2-\alpha_2)-a_2S(\alpha_2+\alpha_2) & 0 \\ a_0S(\alpha_2-\alpha_2)-a_2S(\alpha_2+\alpha_2) & a_0C(\alpha_2-\alpha_2)-a_2C(\alpha_2+\alpha_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Also, the mutual inductances between the machine 1 and 2 can be transformed to the rotor reference frame as:

$$L_{qd12} = T_1(\theta_r) L_{12} T_2(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{bmatrix} C(\theta_r+\alpha_1) & C(\theta_r+\alpha_1-\gamma) & C(\theta_r+\alpha_1+\gamma) \\ S(\theta_r+\alpha_1) & S(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1+\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a2} & L_{c1b2} & L_{c1c2} \end{bmatrix}\begin{bmatrix} C(\theta_r+\alpha_2) & S(\theta_r+\alpha_2) & 1 \\ C(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2-\gamma) & 1 \\ C(\theta_r+\alpha_2+\gamma) & S(\theta_r+\alpha_2+\gamma) & 1 \end{bmatrix} =$$

$$\begin{bmatrix} L_{q1q2} & L_{q1d2} & L_{q102} \\ L_{d1q2} & L_{d1d2} & L_{d102} \\ L_{01q2} & L_{01d2} & L_{0102} \end{bmatrix} = \frac{3}{2}\begin{bmatrix} L_0 C\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)+L_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & -L_0 S\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-L_2 S\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ L_0 S\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-L_2 S\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & L_0 C\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-L_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.165)$$

$$= \frac{3\pi\mu_o r l N_1^{\ 2}}{2}\begin{bmatrix} a_0 C\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)+a_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & -a_0 S\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-a_2 S\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ a_0 S\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-a_2 S\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & a_0 C\left(\alpha_1-\alpha_2+\frac{\pi}{6}\right)-a_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L_{qd21} = T_2(\theta_r) L_{21} T_1(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{bmatrix} C(\theta_r+\alpha_2) & C(\theta_r+\alpha_2-\gamma) & C(\theta_r+\alpha_2+\gamma) \\ S(\theta_r+\alpha_2) & S(\theta_r+\alpha_2-\gamma) & S(\theta_r+\alpha_2+\gamma) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_{a2a1} & L_{a2b1} & L_{a2c1} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} \end{bmatrix}\begin{bmatrix} C(\theta_r+\alpha_1) & S(\theta_r+\alpha_1) & 1 \\ C(\theta_r+\alpha_1-\gamma) & S(\theta_r+\alpha_1-\gamma) & 1 \\ C(\theta_r+\alpha_1+\gamma) & S(\theta_r+\alpha_1+\gamma) & 1 \end{bmatrix} =$$

$$\begin{bmatrix} L_{q2q1} & L_{q2d1} & L_{q201} \\ L_{d2q1} & L_{d2d1} & L_{d201} \\ L_{02q1} & L_{02d1} & L_{0201} \end{bmatrix} = \frac{3}{2}\begin{bmatrix} L_0 C\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)+L_2 C\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & -L_0 S\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-L_2 S\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & 0 \\ L_0 S\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-L_2 S\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & L_0 C\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-L_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.166)$$

$$= \frac{3\pi\mu_o r l N_1^{\ 2}}{2}\begin{bmatrix} a_0 C\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)+a_2 C\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & -a_0 S\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-a_2 S\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & 0 \\ a_0 S\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-a_2 S\left(\alpha_2+\alpha_1-\frac{\pi}{6}\right) & a_0 C\left(\alpha_2-\alpha_1-\frac{\pi}{6}\right)-a_2 C\left(\alpha_1+\alpha_2-\frac{\pi}{6}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To remove the mutual between q and d axis the non-diagonal terms of the inductance matrixes in the rotor reference frame should be zero. By setting the non-diagonal terms of the matrixes in the equations (4.165) and (4.166) to zero, the initial angels of the transformations can be calculated as:

$$\left.\begin{array}{l} -a_0 S\left(\alpha_2-\alpha_1-\dfrac{\pi}{6}\right)-a_2 S\left(\alpha_2+\alpha_1-\dfrac{\pi}{6}\right)=0 \\ \\ -a_0 S\left(\alpha_1-\alpha_2+\dfrac{\pi}{6}\right)-a_2 S\left(\alpha_1+\alpha_2-\dfrac{\pi}{6}\right)=0 \end{array}\right\} \Rightarrow \alpha_1=0, \alpha_1=\frac{\pi}{6} \tag{4.167}$$

Substituting the initial angles of the equation (4.167) in to the equations (4.163) to (4.166), the final transformation to rotor reference frame can be expressed as:

$$T(\theta_r)=$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r+\frac{\pi}{6}\right) & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r+\frac{\pi}{6}\right) & Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.168}$$

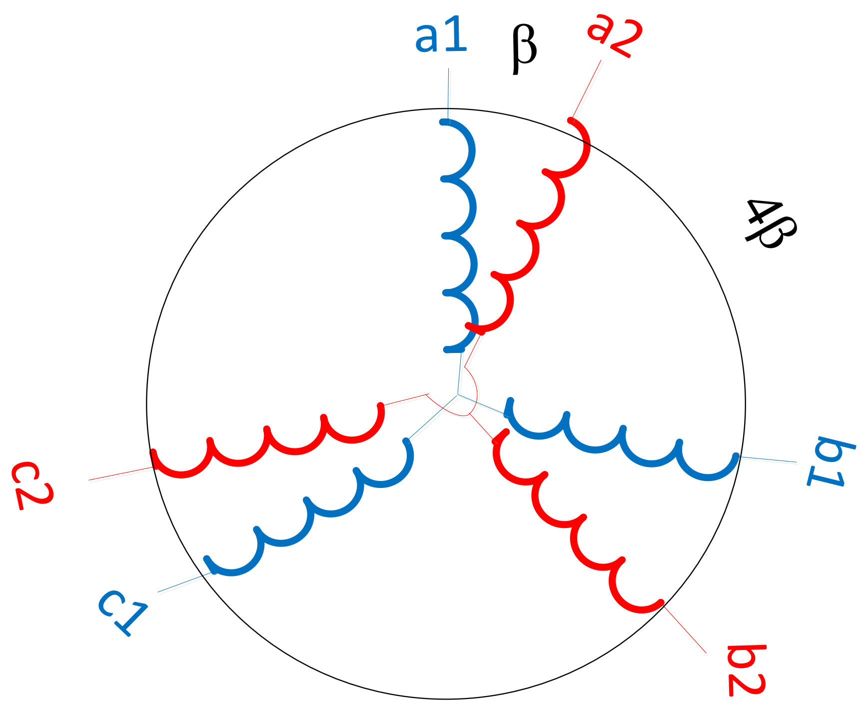


Figure 4.58: The asymmetrical double star machine connection.

$$T^{-1}(\theta_r)=\begin{bmatrix} Cos(\theta_r) & Sin(\theta_r) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r-\frac{2\pi}{3}\right) & Sin\left(\theta_r-\frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r+\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r+\frac{\pi}{6}\right) & Sin\left(\theta_r+\frac{\pi}{6}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & 1 \end{bmatrix} \quad (4.169)$$

By adding the leakage inductances to the self-inductances of the machine, the terms of the matrix in the equation (4.169) can be defined as:

$$L_{d1d1} = \frac{3}{2}(L_0 - L_2) + L_{ls} = L_d + L_{ls} \qquad L_{d1d2} = \frac{3}{2}(L_0 - L_2) = L_d$$
$$L_{q1q1} = \frac{3}{2}(L_0 + L_2) + L_{ls} = L_q + L_{ls} \quad , \quad L_{q1q2} = \frac{3}{2}(L_0 + L_2) = L_q$$
$$L_{d2d2} = \frac{3}{2}(L_0 - L_2) + L_{ls} = L_d + L_{ls} \qquad L_{d2d1} = \frac{3}{2}(L_0 - L_2) = L_d \tag{4.170}$$
$$L_{q2q2} = \frac{3}{2}(L_0 + L_2) + L_{ls} = L_q + L_{ls} \qquad L_{q2q1} = \frac{3}{2}(L_0 + L_2) = L_q$$

$$L_{qd} = \begin{bmatrix} L_{q1q1} & 0 & 0 & L_{q1q2} & 0 & 0 \\ 0 & L_{d1d1} & 0 & 0 & L_{d1d2} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ L_{q2q1} & 0 & 0 & L_{q2q2} & 0 & 0 \\ 0 & L_{d2d1} & 0 & 0 & L_{d2d2} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \tag{4.171}$$

4.7.2 Modelling of the Asymmetrical Six Phase Double Star IPM Machine

Now, the inductances and the transformation are generated so the model of the machine can be generated. The modeling can start from the voltage equations of the stator. According to the Figure 4.58, the voltage Equations of the stator can be expressed as equation (4.172).

In this equation 'I_{xi}' is the current of phase 'x' and '$p\lambda_x$' is the derivation of the flux linkage seen from the phase 'x' of the machine 'i' and the term 'r_s' represents the stator resistance for each phase of the stator.

$$V_{ai} = r_s i_{ai} + p\lambda_{ai}$$
$$V_{bi} = r_s i_{bi} + p\lambda_{bi} \quad , i = 1,2 \tag{4.172}$$
$$V_{ci} = r_s i_{ci} + p\lambda_{ci}$$

Therefore, for machine 1 the voltage equations are presented as equation (4.173).

$$V_{a1} = r_s i_{a1} + p\lambda_{a1}$$

$$V_{b1} = r_s i_{b1} + p\lambda_{b1} \tag{4.173}$$

$$V_{c1} = r_s i_{c1} + p\lambda_{c1}$$

And for machine 2, the voltage equations are:

$$V_{a2} = r_s i_{a2} + p\lambda_{a2}$$

$$V_{b2} = r_s i_{b2} + p\lambda_{b2} \tag{4.174}$$

$$V_{c2} = r_s i_{c2} + p\lambda_{c2}$$

The flux linkages of each phase of the stator can be expressed as:

$$\lambda_{a1} = L_{a1a1}i_{a1} + L_{a1b1}i_{b1} + L_{a1c1}i_{c1} + L_{a1a2}i_{a2} + L_{a1b2}i_{b2} + L_{a1c2}i_{c2} + \lambda_{pma1}$$
$$\lambda_{b1} = L_{b1a1}i_{a1} + L_{b1b1}i_{b1} + L_{b1c1}i_{c1} + L_{b1a2}i_{a2} + L_{b1b2}i_{b2} + L_{b1c2}i_{c2} + \lambda_{pmb1} \tag{4.175}$$
$$\lambda_{c1} = L_{c1a1}i_{a1} + L_{c1b1}i_{b1} + L_{c1c1}i_{c1} + L_{c1a2}i_{a2} + L_{c1b2}i_{b2} + L_{c1c2}i_{c2} + \lambda_{pmc1}$$

And for the machine 2, the flux linkages are:

$$\lambda_{a2} = L_{a2a1}i_{a1} + L_{a2b1}i_{b1} + L_{a2c1}i_{c1} + L_{a2a2}i_{a2} + L_{a2b2}i_{b2} + L_{a2c2}i_{c2} + \lambda_{pma2}$$
$$\lambda_{b2} = L_{b2a1}i_{a1} + L_{b2b1}i_{b1} + L_{b2c1}i_{c1} + L_{b2b2}i_{b2} + L_{b2a2}i_{a2} + L_{b2c2}i_{c2} + \lambda_{pmb2} \tag{4.176}$$
$$\lambda_{c2} = L_{c2a1}i_{a1} + L_{c2b1}i_{b1} + L_{c2c1}i_{c1} + L_{c2a2}i_{a2} + L_{c2b2}i_{b2} + L_{c2c2}i_{c2} + \lambda_{pmc2}$$

The flux linkage for each phase has four components:

1- The flux linkage due to it’s current.

2- The flux linkage due to the mutual inductances between the phase of the same machine.

3- The flux linkage due to the mutual inductances between the phases of one machine and the phases of the other machine.

4- The flux linkage due to the permanent magnets of the rotor.

In this equations the terms 'L_{xixi}' represents the self-inductance of the phase *'x'* of the machine *'i'* and 'L_{xiyj}' represents the mutual inductance between phase 'x' and of the machine *'i'* and phase *'y'* of the machine *'j'*. Also the term 'λ_{pmxi}' represents the flux linkage due to the permanent magnets of the rotor seen from the phase *'x'* of the machine *'i'*. The flux linkages can be represented in the matrix form as:

$$\lambda_{abci} = L_{ss} i_{abci} + \lambda_{pmabci} =$$

$$\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} \end{pmatrix} \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} + \begin{bmatrix} \lambda_{pma1} \\ \lambda_{pmb1} \\ \lambda_{pmc1} \\ \lambda_{pma2} \\ \lambda_{pmb2} \\ \lambda_{pmc2} \end{bmatrix} \quad (4.177)$$

The equation (4.177) can be transformed to the rotor reference frame according to the below procedure:

$$V_{abci} = r_s i_{abci} + p\lambda_{abci} \quad , i = 1,2 \quad (4.178)$$

The abc currents and the flux linkage of the equation (4.178) can be replaced by their corresponding values in the rotor reference frame to get the equation (4.179).

$$V_{abci} = r_s T^{-1}(\theta_r) i_{qdoi} + pT^{-1}(\theta_r)\lambda_{qdoi}, \ i = 1,2 \quad (4.179)$$

By multiplying the $T(\theta_r)$ from the left side of the equation the equation (4.179) changes to:

$$T(\theta_r)V_{abci} = T(\theta_r) r_s T^{-1}(\theta_r) i_{qdoi} + T(\theta_r) pT^{-1}(\theta_r)\lambda_{qdoi}, i = 1,2 \quad (4.180)$$

The different parts of the equation (4.180) can be represented as:

$T(\theta_r)\, V_{abci} =$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \tag{4.181}$$

$$\begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} V_{q1} \\ V_{d1} \\ V_{o1} \\ V_{q2} \\ V_{d2} \\ V_{o2} \end{bmatrix}$$

The resistive part is:

$$T(\theta_r) r_s T^{-1}(\theta_r) = r_s \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_s \tag{4.182}$$

The term with derivation can be expanded as:

$$T(\theta_r) p T^{-1}(\theta_r) \lambda_{qdoi} = T(\theta_r) p T^{-1}(\theta_r) \lambda_{qdoi} + T(\theta_r) T^{-1}(\theta_r) p \lambda_{qdoi} \tag{4.183}$$

The first part of the equation (4.184) is expanded as:

$$T(\theta_r) p T^{-1}(\theta_r) \lambda_{qdoi} =$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times$$

$$p\begin{bmatrix} Cos(\theta_r) & Sin(\theta_r) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \lambda_{qdoi} \tag{4.184}$$

By applying the derivative part and simplifying equation (4.184), it results in equation (4.185).

$$T(\theta_r)pT^{-1}(\theta_r)\lambda_{qdoi} =$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \tag{4.185}$$

$$\omega_r \begin{bmatrix} -Sin(\theta_r) & Cos(\theta_r) & 0 & 0 & 0 & 0 \\ -Sin\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & 0 & 0 & 0 & 0 \\ -Sin\left(\theta_r + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Sin\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6}\right) & 0 \\ 0 & 0 & 0 & -Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & 0 \\ 0 & 0 & 0 & -Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & 0 \end{bmatrix} \lambda_{qdoi} = \omega_r \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \lambda_{qdoi}$$

And the second part of the equation (4.183) is expanded as equation (4.186).

$$T(\theta_r)T^{-1}(\theta_r)p\lambda_{qdoi} =$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \tag{4.186}$$

$$\begin{bmatrix} Cos(\theta_r) & Sin(\theta_r) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & 1 \end{bmatrix} p\lambda_{qdoi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} p\lambda_{qdoi}$$

The flux linkage of the machine can also be presented as below.

$$\lambda_{qdoi} = T(\theta_r) L_{ss} i_{abci} + T(\theta_r) \lambda_{pm\_abci} \tag{4.187}$$

Substituting the currents by their corresponding currents in the rotor reference frame results in:

$$\lambda_{qdoi} = T(\theta_r) L_{ss} T^{-1}(\theta_r) i_{qdo} + T(\theta_r) \lambda_{pm\_abci} \tag{4.188}$$

The flux linkage due to the permanent magnet of the machine in the stator phases can be transformed to the rotor reference frame according to equation (4.189). In this equation since the d axis of the machine 1 and 2 are aligned with the flux linkage of the permanent magnet seen from the machine 1 and 2, the flux linkage of the permanent magnet in the q axis will be zero.

$$T(\theta_r)\lambda_{pm\_abci} =$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \tag{4.189}$$

$$\begin{bmatrix} \lambda_{pma1} \\ \lambda_{pmb1} \\ \lambda_{pmc1} \\ \lambda_{pma2} \\ \lambda_{pmb2} \\ \lambda_{pmc2} \end{bmatrix} = \begin{bmatrix} \lambda_{pmq1} \\ \lambda_{pmd1} \\ \lambda_{pmo} \\ \lambda_{pmq2} \\ \lambda_{pmd2} \\ \lambda_{pmo} \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ \lambda_{pm} \\ 0 \end{bmatrix}$$

The equation (4.188) also has a term which includes 'L_{SS}' and represents the inductances of the stator of the machine. The inductances can be transformed to the rotor reference frame as equation (4.190).

$$T(\theta_r)L_{ss}T(\theta_r)^{-1} =$$

$$\frac{2}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times$$

$$\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} \end{pmatrix} \times \qquad (4.190)$$

$$\begin{bmatrix} Cos(\theta_r) & Sin(\theta_r) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ Cos\left(\theta_r + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & 1 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & 1 \end{bmatrix} =$$

$$\begin{bmatrix} L_{q1q1} & 0 & 0 & L_{q1q2} & 0 & 0 \\ 0 & L_{d1d1} & 0 & 0 & L_{d1d2} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ L_{q2q1} & 0 & 0 & L_{q2q2} & 0 & 0 \\ 0 & L_{d2d1} & 0 & 0 & L_{d2d2} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix}$$

Then, using the permanent magnet flux linkages and the inductances of the machine in the rotor reference frame, the flux linkages of the machines in equation (4.188), can be represented in rotor reference frame as below:

$$
\begin{aligned}
&\lambda_{q1} = L_{q1q1} i_{q1} + L_{q1q2} i_{q2} \\
&\lambda_{d1} = L_{d1d1} i_{d1} + L_{d1d2} i_{d2} + \lambda_{pmd1} \\
&\lambda_{o1} = L_{ls} i_{o1} \\
&\lambda_{q2} = L_{q2q2} i_{q2} + L_{q2q1} i_{q1} \\
&\lambda_{d2} = L_{d2d2} i_{d2} + L_{d2d1} i_{d1} + \lambda_{pmd2} \\
&\lambda_{o2} = L_{ls} i_{o2}
\end{aligned}
\tag{4.191}
$$

Also the voltages and the currents of the machines can be transformed to the rotor reference frame as:

$$
\begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \end{bmatrix} = T(\theta_r)\, i_{s12} = T(\theta_r) \times \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} \tag{4.192}
$$

Using the different components of the machines the voltage equations of the machine can be presented as equation (4.193).

$$\begin{bmatrix} V_{q1} \\ V_{d1} \\ V_{o1} \\ V_{q2} \\ V_{d2} \\ V_{o2} \end{bmatrix} = r_s \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \end{bmatrix} + \omega_r \left(\begin{bmatrix} L_{q1q1} & 0 & 0 & L_{q1q2} & 0 & 0 \\ 0 & L_{d1d1} & 0 & 0 & L_{d1d2} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ L_{q2q1} & 0 & 0 & L_{q2q2} & 0 & 0 \\ 0 & L_{d2d1} & 0 & 0 & L_{d2d2} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ \lambda_{pm} \\ 0 \end{bmatrix} \right) +$$

$$p \left(\begin{bmatrix} L_{q1q1} & 0 & 0 & L_{q1q2} & 0 & 0 \\ 0 & L_{d1d1} & 0 & 0 & L_{d1d2} & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ L_{q2q1} & 0 & 0 & L_{q2q2} & 0 & 0 \\ 0 & L_{d2d1} & 0 & 0 & L_{d2d2} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1} \\ i_{d1} \\ i_{o1} \\ i_{q2} \\ i_{d2} \\ i_{o2} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ \lambda_{pm} \\ 0 \end{bmatrix} \right) \quad (4.193)$$

The equivalent circuit if the machine for different axis are shown in the Figures 4.59 to 4.61.

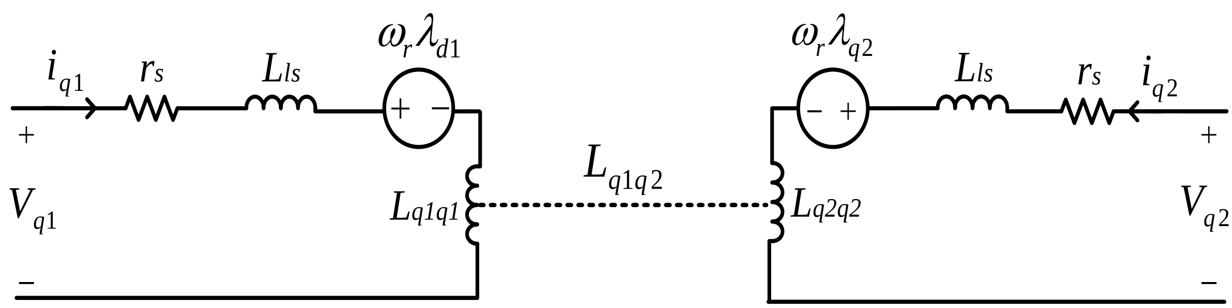


Figure 4.59: The equivalent circuit of the q axis.

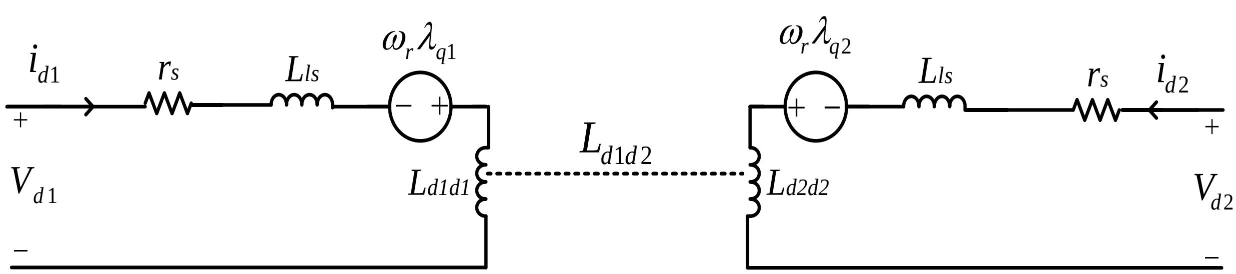


Figure 4.60: The equivalent circuit of the d axis.

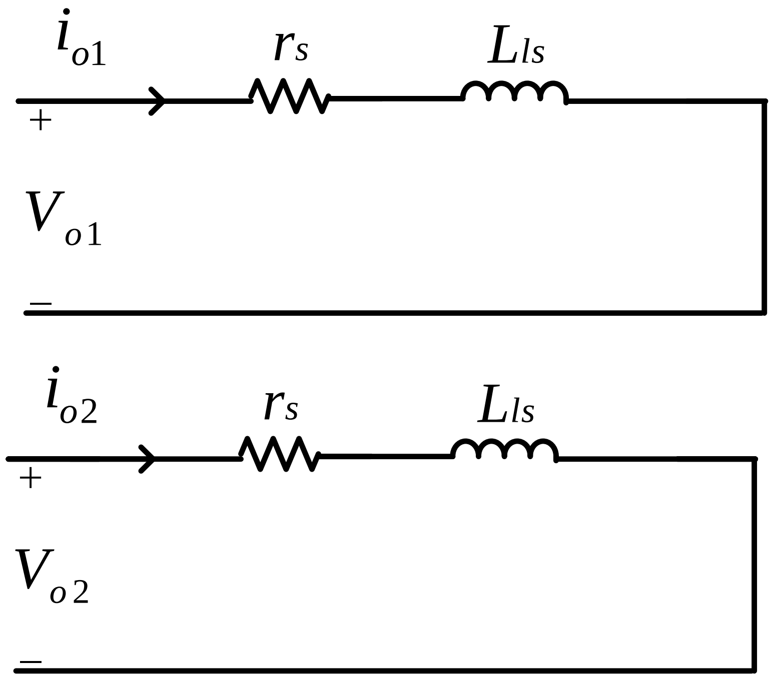


Figure 4.61: The equivalent circuit of the zero sequence.

The equation for electromagnetic torque can be generated using the power balance between the mechanical output power and the electrical input power. The equation (4.194) shows the relationship between the input and the output power [152].

$$T_e\left(\frac{2}{P}\right)\omega_r = \sum_{i=1}^{2}\frac{3}{2}\left(V_{qi}i_{qi} + V_{di}i_{di}\right) = \quad (4.194)$$

$$\frac{3}{2}\sum_{i=1}^{2}\left(r_s\left(i^2{}_{qi} + i^2{}_{di}\right) + \left(\lambda_{di}i_{qi} - \lambda_{qi}i_{di}\right)\omega_r + \left(p\lambda_{qi}i_{qi} + p\lambda_{di}i_{di}\right)\right)$$

The first term is the power loss and the derivative part is the changings of the stored energy which are not effective on the electromagnetic torque, therefore using the average stored energy, the torque can be expressed as equation (4.195).

$$T_e = \sum_{i=1}^{2} \frac{3}{2}\left(V_{qi} i_{qi} + V_{di} i_{di}\right) = \frac{3}{2}\left(\frac{P}{2}\right)\sum_{i=1}^{2}\left(\lambda_{di} i_{qi} - \lambda_{qi} i_{di}\right) =$$

$$\frac{3}{2}\left(\frac{P}{2}\right)\left(\left(L_{d1d1} i_{d1} + L_{d1d2} i_{d2} + \lambda_{pm}\right) i_{q1} - \left(L_{q1q1} i_{q1} + L_{q1q2} i_{q2}\right) i_{d1}\right) +$$
$$\frac{3}{2}\left(\frac{P}{2}\right)\left(\left(L_{d2d2} i_{d2} + L_{d2d1} i_{d1} + \lambda_{pm}\right) i_{q2} - \left(L_{q2q2} i_{q2} + L_{q2q1} i_{q1}\right) i_{d3}\right) = \tag{4.195}$$

$$\frac{3}{2}\left(\frac{P}{2}\right)\begin{pmatrix} \left(L_{d1d1} i_{d1} + L_{d1d2} i_{d2} + \lambda_{pm}\right) i_{q1} - \left(L_{q1q1} i_{q1} + L_{q1q2} i_{q2}\right) i_{d1} + \\ \left(L_{d2d2} i_{d2} + L_{d2d1} i_{d1} + \lambda_{pm}\right) i_{q2} - \left(L_{q2q2} i_{q2} + L_{q2q1} i_{q1}\right) i_{d3} \end{pmatrix}$$

4.8 Decoupling the Model

From equation (4.193) it can be seen that there are coupling terms between different machines q and d axis (The off-diagonal terms are not zero). To remove the couplings, a new transformation is needed. The new transformation is a combination of two different transformations, the rotor reference frame and decoupled reference frame transformation. The decoupled transformation can be derived by diagonalzing the matrix of the machine inductances in the rotor reference frame. The diagonalizing procedure can be done by finding the matrix P such that the equation (4.196) is diagonal. It should be noted that the zero sequence inductances need to be neglected to let the inductance matrix have distinct eigen values to be diagonalizable [144].

$$L_{qdn} = P^{-1} \underbrace{T(\theta_r) L_{ss} T(\theta_r)^{-1}}_{L_{qd}} P = P^{-1} \begin{bmatrix} L_{q1q1} & 0 & L_{q1q2} & 0 \\ 0 & L_{d1d1} & 0 & L_{d1d2} \\ L_{q2q1} & 0 & L_{q2q2} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2} \end{bmatrix} P \tag{4.196}$$

The matrix P is built from the eigen vectors of the inductance matrix.

$$P = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix} \tag{4.197}$$

To obtain the eigen vectors, the eigen values are needed, the eigen values can be calculated according to the equation (4.198).

$$\left(L_{qd}-\lambda I\right)=0\Rightarrow$$

$$\left(\begin{bmatrix} L_{q1q1} & 0 & L_{q1q2} & 0 \\ 0 & L_{d1d1} & 0 & L_{d1d2} \\ L_{q2q1} & 0 & L_{q2q2} & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d2} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right) = 0 \Rightarrow \tag{4.198}$$

$$\begin{bmatrix} L_{q1q1}-\lambda & 0 & L_{q1q2} & 0 \\ 0 & L_{d1d1}-\lambda & 0 & L_{d1d2} \\ L_{q1q1} & 0 & L_{q1q1}-\lambda & 0 \\ 0 & L_{d2d1} & 0 & L_{d2d1}-\lambda \end{bmatrix} = 0$$

The eigen values of the last matrix in the equation (4.198) are equal to:

$$\begin{aligned} \lambda_1 &= L_{q1q1}+L_{q1q2}=\frac{3}{2}\left(L_0+L_2\right)+\frac{3}{2}\left(L_0+L_2\right)=3\left(L_0+L_2\right)=2L_{q1q1}+L_{ls} \\ \lambda_2 &= L_{d1d1}+L_{d1d2}=\frac{3}{2}\left(L_0-L_2\right)+\frac{3}{2}\left(L_0-L_2\right)=3\left(L_0-L_2\right)=2L_{d1d1}+L_{ls} \\ \lambda_3 &= L_{q1q1}-L_{q1q2}=\frac{3}{2}\left(L_0+L_2\right)-\frac{3}{2}\left(L_0+L_2\right)=L_{ls} \\ \lambda_4 &= L_{d1d1}-L_{d1d2}=\frac{3}{2}\left(L_0-L_2\right)-\frac{3}{2}\left(L_0-L_2\right)=L_{ls} \end{aligned} \tag{4.199}$$

Using the eigen values, the eigen vectors of the matrix can be obtained as:

$$\left(A-\lambda_i I\right)V_i=0 \tag{4.200}$$

Therefore, the eigenvectors corresponding to each of the eigen values are presented in equation (4.201).

$$V_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} V_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} V_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} V_4 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{4.201}$$

Using the eigen vectors, the matrix P can be formed as:

$$P = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{4.202}$$

From the *P* the matrix, P^{-1} also can be obtained as:

$$P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \tag{4.203}$$

Now using the generated P, the new transformation matrix can be developed as equation (4.204).

$$T_n(\theta_r) = P^{-1}T(\theta_r) =$$

$$\frac{1}{3}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \end{bmatrix} = \tag{4.204}$$

$$\frac{1}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & -Sin\left(\theta_r + \frac{\pi}{6}\right) & -Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & -Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \\ -Cos(\theta_r) & -Cos\left(\theta_r - \frac{2\pi}{3}\right) & -Cos\left(\theta_r + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$T^{-1}{}_n(\theta_r) = T(\theta_r)^{-1} P =$$

$$\begin{bmatrix} Cos(\theta_r) & Sin(\theta_r) & 0 & 0 \\ Cos\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & 0 & 0 \\ Cos\left(\theta_r + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 \\ 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) \\ 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) \\ 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \tag{4.205}$$

$$\begin{bmatrix} Sin(\theta_r) & Cos(\theta_r) & -Sin(\theta_r) & -Cos(\theta_r) \\ Sin\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & -Sin\left(\theta_r - \frac{2\pi}{3}\right) & -Cos\left(\theta_r - \frac{2\pi}{3}\right) \\ Sin\left(\theta_r + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & -Sin\left(\theta_r + \frac{2\pi}{3}\right) & -Cos\left(\theta_r + \frac{2\pi}{3}\right) \\ Sin\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6}\right) \\ Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) \\ Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) \end{bmatrix}$$

Now using the new transformation, the machine voltage equation can be transformed to the decoupled reference frame. The machine equations are presented in equation (4.172). By substituting currents and flux linkages by their corresponding values in the decoupled reference frame (equation (4.206)), the voltage equation changes to the equation (4.207).

$$i_{abci} = T^{-1}{}_n(\theta_r) i_{qdn} \ , \ \lambda_{abci} = T^{-1}{}_n(\theta_r) \lambda_{qdn} \tag{4.206}$$

$$V_{abci} = r_s T_n^{-1}(\theta_r) i_{qdn} + p T_n^{-1}(\theta_r) \lambda_{qdn} \quad , i = 1,2 \tag{4.207}$$

By multiplying $T_n(\theta_r)$ from the left side of the equation (4.207), this equation changes to:

$$T_n(\theta_r) V_{abci} = T_n(\theta_r) r_s T_n^{-1}(\theta_r) i_{qdn} + T_n(\theta_r) p T_{dn}^{-1}(\theta_r) \lambda_{qdn} \tag{4.208}$$

The different parts of the equation (4.208) can be expanded following:

The voltages in decoupled reference frame are:

$$T_n(\theta_r) V_{abci} = T_n(\theta_r) \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \\ V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \end{bmatrix} \tag{4.209}$$

The currents in decoupled reference frame are:

$$T_n(\theta_r) i_{abci} = T_n(\theta_r) \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} = \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \end{bmatrix} \tag{4.210}$$

The resistive part of the equation (4.208) presented in equation (4.211).

$$r_{sn} = T_n(\theta_r) r_s T_n^{-1}(\theta_r) = P^{-1} T(\theta_r) r_s T^{-1} P(\theta_r) = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_s \end{bmatrix} \tag{4.211}$$

The last term in the equation (4.208), which includes derivation, can be expanded as:

$$T_n(\theta_r) p T_n^{-1}(\theta_r) \lambda_{qdn} = T_n(\theta_r) p T_n^{-1}(\theta_r) \lambda_{qdn} + T_n(\theta_r) T_n^{-1}(\theta_r) p \lambda_{qdn} \tag{4.212}$$

The first term of the equation (4.212) is:

$$T_n(\theta_r) p T_n^{-1}(\theta_r) \lambda_{qdn} = P^{-1} \underbrace{T(\theta_r) p T^{-1}(\theta_r)}_{\omega_r} \lambda_{qdn} =$$

$$\frac{\omega_r}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \lambda_{qdn} = \tag{4.213}$$

$$\omega_r \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \lambda_{qdn}$$

Also, the second term of the equation (4.212) is:

$$T_n(\theta_r) T_n^{-1}(\theta_r) p \lambda_{qdn} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} p \lambda_{qdn} \tag{4.214}$$

The flux linkage in the decoupled reference frame can be obtain according to equation (4.215).

$$\lambda_{qdn} = T_n(\theta_r)\lambda_{abci} = T_n(\theta_r)(L_{ss}i_{abci} + \lambda_{Pmabci}) \tag{4.215}$$

By substituting the currents from the equation (4.214) in to the equation (4.215) flux linkages can be expressed as:

$$\lambda_{qdn} = T_n(\theta_r)\lambda_{abci} = T_n(\theta_r)L_{ss}T^{-1}{}_n(\theta_r)i_{qdn} + T_n(\theta_r)\lambda_{Pmabci} \tag{4.216}$$

The first term of the equation (4.216) is equal to:

$$T_n(\theta_r)L_{ss}T^{-1}{}_n(\theta_r)i_{qdn} =$$

$$T_n(\theta_r)\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} \end{pmatrix} T^{-1}{}_n(\theta_r)i_{qdn} \tag{4.217}$$

The inductance matrix in the decoupled reference frame is equal to:

$$T_n(\theta_r)\begin{pmatrix} L_{a1a1} & L_{a1b1} & L_{a1c1} & L_{a1a2} & L_{a1b2} & L_{a1c2} \\ L_{b1a1} & L_{b1b1} & L_{b1c1} & L_{b1a2} & L_{b1b2} & L_{b1c2} \\ L_{c1a1} & L_{c1b1} & L_{c1c1} & L_{c1a2} & L_{c1b2} & L_{c1c2} \\ L_{a2a1} & L_{a2b1} & L_{a2c1} & L_{a2a2} & L_{a2b2} & L_{a2c2} \\ L_{b2a1} & L_{b2b1} & L_{b2c1} & L_{b2a2} & L_{b2b2} & L_{b2c2} \\ L_{c2a1} & L_{c2b1} & L_{c2c1} & L_{c2a2} & L_{c2b2} & L_{c2c2} \end{pmatrix} T^{-1}{}_n(\theta_r) =$$

$$\begin{bmatrix} 2L_{q1q1} + L_{ls} & 0 & 0 & 0 \\ 0 & 2L_{d1d1} + L_{ls} & 0 & 0 \\ 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & L_{ls} \end{bmatrix} \tag{4.218}$$

The second part of equation (4.216) (permanent magnet flux linkages seen from the stator phases, transformed to the decoupled reference frame) can be expanded as:

$$\lambda_{pmn} = T_n(\theta_r)\begin{bmatrix}\lambda_{pma1}\\ \lambda_{pmb1}\\ \lambda_{pmc1}\\ \lambda_{pma2}\\ \lambda_{pmb2}\\ \lambda_{pmc2}\end{bmatrix} = P^{-1}T(\theta_r)\begin{bmatrix}\lambda_{pma1}\\ \lambda_{pmb1}\\ \lambda_{pmc1}\\ \lambda_{pma2}\\ \lambda_{pmb2}\\ \lambda_{pmc2}\end{bmatrix} =$$

$$\frac{1}{3}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & -1\\ -1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}Cos(\theta_r) & Cos\left(\theta_r - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0\\ Sin(\theta_r) & Sin\left(\theta_r - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{2\pi}{3}\right) & 0 & 0 & 0\\ 0 & 0 & 0 & Cos\left(\theta_r + \frac{\pi}{6}\right) & Cos\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Cos\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right)\\ 0 & 0 & 0 & Sin\left(\theta_r + \frac{\pi}{6}\right) & Sin\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) & Sin\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right)\end{bmatrix}\begin{bmatrix}\lambda_{pma1}\\ \lambda_{pmb1}\\ \lambda_{pmc1}\\ \lambda_{pma2}\\ \lambda_{pmb2}\\ \lambda_{pmc2}\end{bmatrix} = \quad (4.219)$$

$$\frac{1}{2}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & -1\\ -1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\lambda_{pmq1}\\ \lambda_{pmd1}\\ \lambda_{pmq2}\\ \lambda_{pmd2}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & -1\\ -1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\ \lambda_{pm}\\ 0\\ \lambda_{pm}\end{bmatrix} = \begin{bmatrix}0\\ \lambda_{pm}\\ 0\\ 0\end{bmatrix}$$

Therefore, the total flux linkage of the machines in the decoupled reference frame can be represented as:

$$\begin{bmatrix} \lambda_{q1n} \\ \lambda_{d1n} \\ \lambda_{q2n} \\ \lambda_{d2n} \end{bmatrix} = \begin{bmatrix} 2L_{q1q1}+L_{ls} & 0 & 0 & 0 \\ 0 & 2L_{d1d1}+L_{ls} & 0 & 0 \\ 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \end{bmatrix} \tag{4.220}$$

Substituting the generated parts into the machine voltage equation and inserting the zero sequence circuits to that results in:

$$\begin{bmatrix} V_{q1n} \\ V_{d1n} \\ V_{q2n} \\ V_{d2n} \\ V_{o1} \\ V_{o2} \end{bmatrix} = r_{sn} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{o1} \\ i_{o2} \end{bmatrix} +$$

$$\omega_r \left(\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2L_{q1q1}+L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2L_{d1d1}+L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{q1n} \\ i_{d1n} \\ i_{q2n} \\ i_{d2n} \\ i_{o1} \\ i_{o2} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_{pm} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) \tag{4.221}$$

$$+ \begin{bmatrix} 2L_{q1q1}+L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2L_{d1d1}+L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{ls} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{ls} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} pi_{q1n} \\ pi_{d1n} \\ pi_{q2n} \\ pi_{d2n} \\ pi_{o1} \\ pi_{o2} \end{bmatrix}$$

The equivalent circuit of the machine in the decoupled reference frame is shown in Figure 4.62.

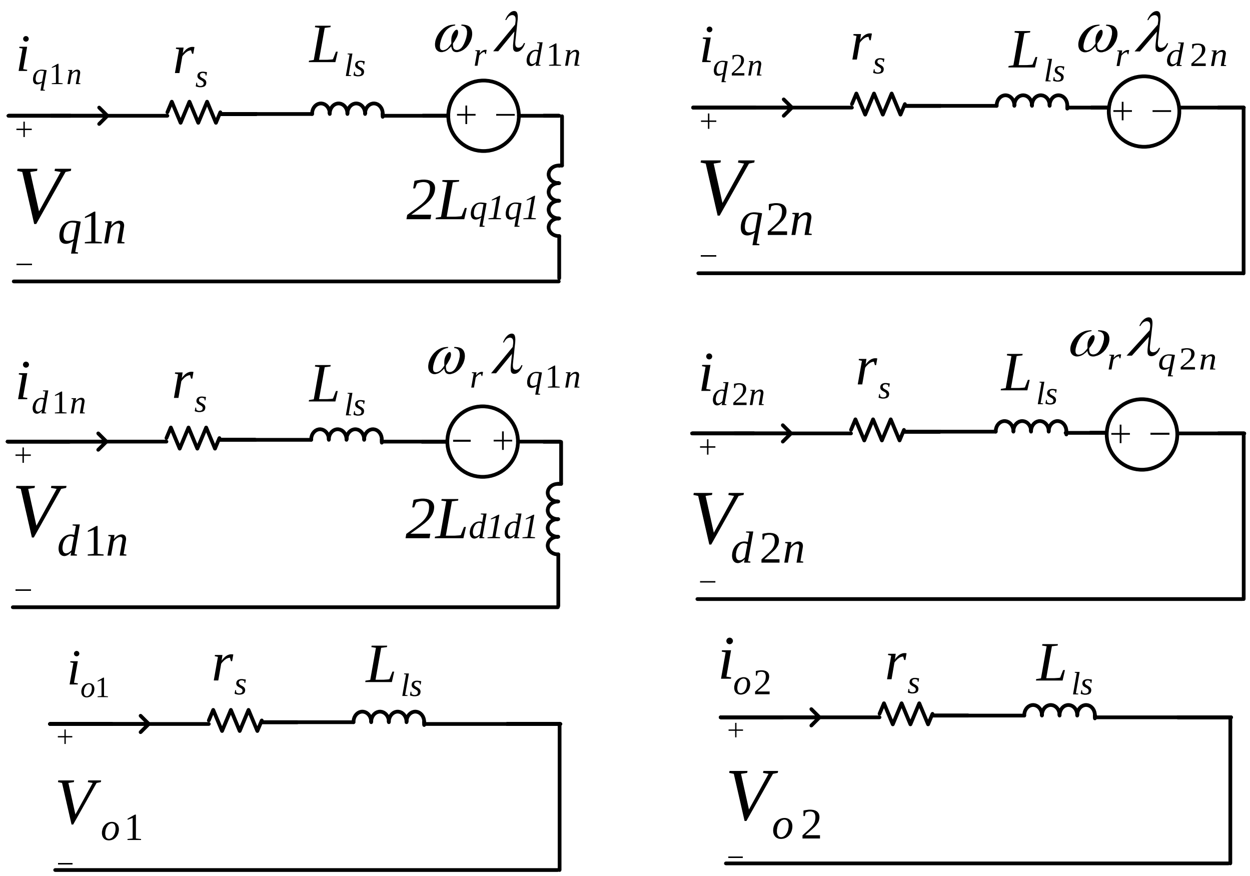


Figure 4.62: The equivalent circuit of the six phase machine in decoupled reference frame.

The equation for electromagnetic torque can be generated using the power balance between the mechanical output power and the electrical input power. The bellow equation shows the relationship between the input and the output power.

$$T_e\left(\frac{2}{P}\right)\omega_r = \sum_{i=1}^{2}\left(V_{qin}i_{qin} + V_{din}i_{din}\right) = \tag{4.222}$$

$$\sum_{i=1}^{2}\left(r_s\left(i^2{}_{qin} + i^2{}_{din}\right) + \left(\lambda_{din}i_{qin} - \lambda_{qin}i_{din}\right)\omega_r + \left(p\lambda_{qin}i_{qin} + p\lambda_{din}i_{din}\right)\right)$$

The first term is the power loss and the derivative part is the changings of the stored energy which are not effective on the electromagnetic torque generation, therefore, using the average stored energy, the torque can be expressed as:

$$T_e = \left(\frac{P}{2}\right)\sum_{i=1}^{2}\omega_r\left(\lambda_{din}i_{qin} - \lambda_{qin}i_{din}\right) =$$

$$\left(\frac{P}{2}\right)\left(\left(L_{ls} + 2L_{d1d1}\right)i_{q1n}i_{d1n} + \lambda_{pm}i_{q1n} - \left(L_{ls} + 2L_{q1q1}\right)i_{d1n}i_{q1n}\right) \tag{4.223}$$

$$+\left(\frac{P}{2}\right)\left(L_{ls}i_{d2n}i_{q2n} - L_{ls}i_{d2n}i_{q2n}\right) = P\left(\left(L_{d1d1} - L_{q1q1}\right)i_{d1n}i_{q1n} + \lambda_{pm}i_{q1n}\right)$$

New transformation matrix for the new reference frame is presented as equation (4.224).

$$T_n(\theta_r)=\frac{1}{3}\begin{bmatrix} Cos(\theta_r) & Cos\left(\theta_r-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}\right) & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ Sin(\theta_r) & Sin\left(\theta_r-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}\right) & Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ Sin(\theta_r) & Sin\left(\theta_r-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{2\pi}{3}\right) & -Sin\left(\theta_r+\frac{\pi}{6}\right) & -Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & -Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ -Cos(\theta_r) & -Cos\left(\theta_r-\frac{2\pi}{3}\right) & -Cos\left(\theta_r+\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}\right) & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (4.224)$$

The inverse of the transformation matrix is also presented as:

$$T^{-1}{}_n(\theta_r)=\begin{bmatrix} Sin(\theta_r) & Cos(\theta_r) & -Sin(\theta_r) & -Cos(\theta_r) & 1 & 0 \\ Sin\left(\theta_r-\frac{2\pi}{3}\right) & Cos\left(\theta_r-\frac{2\pi}{3}\right) & -Sin\left(\theta_r-\frac{2\pi}{3}\right) & -Cos\left(\theta_r-\frac{2\pi}{3}\right) & 1 & 0 \\ Sin\left(\theta_r+\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{2\pi}{3}\right) & -Sin\left(\theta_r+\frac{2\pi}{3}\right) & -Cos\left(\theta_r+\frac{2\pi}{3}\right) & 1 & 0 \\ Sin\left(\theta_r+\frac{\pi}{6}\right) & Cos\left(\theta_r+\frac{\pi}{6}\right) & Sin\left(\theta_r+\frac{\pi}{6}\right) & Cos\left(\theta_r+\frac{\pi}{6}\right) & 0 & 1 \\ Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}-\frac{2\pi}{3}\right) & 0 & 1 \\ Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & Sin\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & Cos\left(\theta_r+\frac{\pi}{6}+\frac{2\pi}{3}\right) & 0 & 1 \end{bmatrix} \quad (4.225)$$

4.9 Conclusion

In this chapter first an average model for a triple-star nine phase IPM is derived for the symmetrical and asymmetrical connections. In this model the Fourier series of the machines parameters are used to generate the Fourier series of the machines inductances. After calculating the inductances of the machine, the model is simulated using MATLAB Simulink and the simulation results are presented. Finally, the model is decoupled to remove the coupling terms between the q and d axis of the different machines. The decoupled model is also derived and essential transformations are presented for the both symmetrical and asymmetrical machines. Finally, an asymmetrical six phase machine is modeled using the Fourier series of the machine parameters. The average model of the machine is presented and the model is decoupled to remove the coupling terms between different sets of three phase machines. The essential transformation matrixes for the new transformation are also presented in this chapter. The major contribution of this chapter is the generation of decoupled models that can be used for designing controllers for the multiple star machines without facing the complexities that are raised by the coupling between different machine sets.