CHAPTER 6

CONTROL OF FOUR LEGGED VOLTAGE SOURCE INVERTER WITH UNBALANCED AND NONLINEAR LOADS.

6.1 Modeling of 4- legged Inverter.

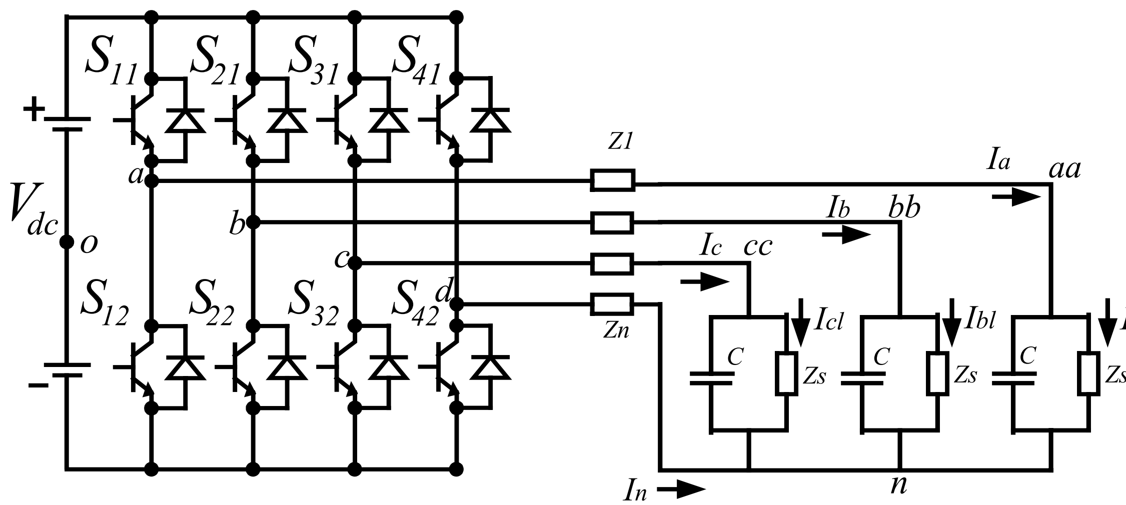


Figure 6.1 Four-leg inverter feeding a three-phase load

In Figure 6.1 is shown the four-leg inverter feeding a three-phase load which may be balanced, unbalanced, and/or nonlinear in nature. The phase loads are shunted with filtering capacitors C with the neutral point of the load connected to the fourth leg (d) of the inverter through an inductor L_n.

In the mentioned figure in obeying KVL and KCL

$$S_{11} + S_{12} = 1 \text{ , } S_{21} + S_{22} = 1, \; S_{31} + S_{32} = 1, \; S_{41} + S_{42} = 1. \qquad (6.1)$$

As shown in the figure, writing the KVL equations across the loop from point ‘a, b, c, and d’, the output voltages can be expressed as

$$V_{ao} = V_{an} + V_{no}$$

$$V_{bo} = V_{bn} + V_{no}$$

$$V_{co} = V_{cn} + V_{no}$$

$$V_{do} = V_{dn} + V_{no} \qquad (6.2)$$

where

$$V_{ao} = \frac{V_{dc}}{2}(S_{11} - S_{12})$$

$$V_{bo} = \frac{V_{dc}}{2}(S_{21} - S_{22})$$

$$V_{co} = \frac{V_{dc}}{2}(S_{31} - S_{32})$$

$$V_{do} = \frac{V_{dc}}{2}(S_{41} - S_{42})\ . \qquad (6.3)$$

Now, the voltage loop equation is given as

$$V_{an} = Z_1 I_a + V_{aan}$$

$$V_{bn} = Z_1 I_b + V_{bbn}$$

$$V_{cn} = Z_1 I_c + V_{ccn}$$

$$V_{dn} = Z_n I_n\ . \qquad (6.5)$$

Substituting (6.3) and (6.4) in (6.2) the expression is written as

$$\frac{V_{dc}}{2}(S_{11} - S_{12}) = Z_1 I_a + V_{aan} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{21} - S_{22}) = Z_1 I_b + V_{bbn} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{31} - S_{32}) = Z_1 I_c + V_{ccn} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{41} - S_{42}) = Z_n I_n + V_{no} \quad (6.6)$$

or $V_{no} = \frac{V_{dc}}{2}(S_{41} - S_{42}) - Z_n I_n$, thus substituting V_{no} in (6.5)

$$\frac{V_{dc}}{2}(S_{11} - S_{12}) = Z_1 I_a + V_{aan} + \frac{V_{dc}}{2}(S_{41} - S_{42}) - Z_n I_n$$

$$\frac{V_{dc}}{2}(S_{21} - S_{22}) = Z_1 I_b + V_{bbn} + \frac{V_{dc}}{2}(S_{41} - S_{42}) - Z_n I_n$$

$$\frac{V_{dc}}{2}(S_{31} - S_{32}) = Z_1 I_c + V_{ccn} + \frac{V_{dc}}{2}(S_{41} - S_{42}) - Z_n I_n . \quad (6.7)$$

Substituting (6.1) in (6.7) and simplifying we have

$$V_{aan} = V_{dc}(S_{11} - S_{41}) - Z_1 I_a + Z_n I_n$$

$$V_{bbn} = V_{dc}(S_{21} - S_{41}) - Z_1 I_b + Z_n I_n$$

$$V_{ccn} = V_{dc}(S_{31} - S_{41}) - Z_1 I_c + Z_n I_n \quad (6.8)$$

Setting $Z_1 = r_s + L_s p$ and $Z_n = L_n p$ where $p = \frac{d}{dt}$, the output voltage equations can be written as

$$V_{aan} = V_{dc}(S_{11} - S_{41}) - r_s I_a - L_s pI_a + L_n pI_n$$

$$V_{bbn} = V_{dc}(S_{21} - S_{41}) - r_s I_b - L_s pI_b + L_n pI_n$$

$$V_{ccn} = V_{dc}(S_{31} - S_{41}) - r_s I_c - L_s pI_c + L_n pI_n \quad (6.9)$$

Define $S_a = S_{11} - S_{41}$, $S_b = S_{21} - S_{41}$, $S_c = S_{31} - S_{41}$. (6.10)

The output voltage equations can be written as

$$V_{aan} = V_{dc}S_a - r_s I_a - L_s pI_a + L_n pI_n$$

$$V_{bbn} = V_{dc}S_b - r_s I_b - L_s pI_b + L_n pI_n$$

$$V_{ccn} = V_{dc}S_c - r_s I_c - L_s pI_c + L_n pI_n \,. \qquad (6.11)$$

The DC link current flowing into the inverter is given as

$$I_d = S_{11}I_a + S_{21}I_b + S_{31}I_c + S_{41}I_n \qquad (6.12)$$

but

$$I_a + I_b + I_c + I_n = 0 \text{ or } I_n = -(I_a + I_b + I_c)\,. \qquad (6.13)$$

Substituting (6.13) in (6.12)

$$I_d = (S_{11} - S_{41})I_a + (S_{21} - S_{41})I_b + (S_{31} - S_{41})I_c \,. \qquad (6.14)$$

The output capacitor currents are given as

$$CpV_{aan} = I_a - I_{al}$$

$$CpV_{bbn} = I_b - I_{bl}$$

$$CpV_{ccn} = I_c - I_{cl} \,. \qquad (6.15)$$

The neutral current is given by (6.13).

Hence from (6.15) and (6.13) the neutral current can be written as

$$I_n = -[CpV_{aan} + CpV_{bbn} + CpV_{ccn} + I_{al} + I_{bl} + I_{cl}]\,. \qquad (6.17)$$

Thus having the voltage equations as given by (6.11) and current equations as given by (6.14), (6.15), and (6.16) the derived model can be transformed into q-d-o synchronous reference frame for designing the controller.

6.2 Modeling in q-d-o Synchronous Reference Frame

The q-d-o modeling of the four legged inverter in synchronous reference frame is done by applying the transformation on Equations (6.11), (6.14), (6.15), and (6.16). Consider (6.11);

$$\begin{bmatrix} V_{aan} \\ V_{bbn} \\ V_{ccn} \end{bmatrix} = V_{dc} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} + r_s \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + L_s p \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + L_n p \begin{bmatrix} I_n \\ I_n \\ I_n \end{bmatrix}. \tag{6.18}$$

The q-d-o quantities can be obtained from the a-b-c coordinates through following relationship.

$$\lfloor f_{qdos} \rfloor = T(\theta)[f_{abc}] \text{ where} \tag{6.19}$$

$$T(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ \sin(\theta) & \sin(\theta-\beta) & \sin(\theta+\beta) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \text{ and } \beta = \frac{2\pi}{3}.$$

The relations that will be used during the transformation are derived as below Using (6.18) and (6.19)

$$\lfloor V_{qdo} \rfloor = T(\theta)[V_{aabbccn}], \tag{6.20}$$

$$\lfloor I_{qdos} \rfloor = T(\theta)[I_{abc}], \tag{6.21}$$

$$\lfloor S_{qdos} \rfloor = T(\theta)[S_{abc}], \tag{6.22}$$

$$\lfloor I_{qdon} \rfloor = T(\theta)[I_{nnn}]. \tag{6.23}$$

Also the inverse transformation can be applied as

$$f_{abc} = T^{-1}(\theta) f_{qdo} \tag{6.24}$$

$$T^{-1}(\theta) = \begin{bmatrix} \sin(\theta) & \sin(\theta-\beta) & \sin(\theta+\beta) \\ \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ 0 & 0 & 0 \end{bmatrix}.$$

Consider the term $T(\theta)pI_{abc}$ this can be solved as explained below.

$$T(\theta)pf_{abc} = T(\theta)\left[pT^{-1}(\theta)f_{qdos}\right]$$

$$T(\theta)pf_{abc} = T(\theta)\left[pT^{-1}(\theta)f_{qdos} + T(\theta)T^{-1}(\theta)pf_{qdos}\right]$$

Consider the first term, $T(\theta)pT^{-1}(\theta)f_{qdos}$.

Now define $\theta = \int w dt + \theta_o$ where $\theta_o = 0$ thus $\frac{d\theta}{dt} = w$.

And $\frac{d}{dt}\cos(\theta) = \frac{d}{d\theta}(\cos(\theta))\frac{d\theta}{dt} = -\sin(\theta)w$.

$$\frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ \sin(\theta) & \sin(\theta-\beta) & \sin(\theta+\beta) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \omega \begin{bmatrix} -\sin(\theta) & -\sin(\theta-\beta) & -\sin(\theta+\beta) \\ \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ 0 & 0 & 0 \end{bmatrix} f_{qdos}$$

$$T(\theta)p\left[f_{abc}\right] = w\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{qdos} = \begin{bmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{qdos}. \tag{6.25}$$

Thus $T(\theta)p\left[I_{abc}\right] = \begin{bmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\left[I_{qdos}\right].$ (6.26)

Consider the term $T(\theta)pI_{nnn}$ this can be solved as explained below.

$$T(\theta)pf_{nnn} = [T(\theta)pT^{-1}(\theta)]f_{qdon} + T(\theta)T^{-1}(\theta)pf_{qdon}$$

Now

$$T(\theta)f_{nnn} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ \sin(\theta) & \sin(\theta-\beta) & \sin(\theta+\beta) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_{qn} \\ f_{dn} \\ f_{on} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3f_{on} \end{bmatrix} = 0 .$$

But

$T(\theta)T^{-1}(\theta)pf_{qdon} = \begin{bmatrix} 0 & 0 & 3pf_{on} \end{bmatrix}$ but I_{on} is the zero sequence current which is same as I_{os}.

$$T(\theta)I_{nnn} = \begin{bmatrix} 0 & 0 & 3pI_{os} \end{bmatrix} \tag{6.27}$$

Thus using (6.20) through (6.23) and (6.26) and (6.27), the q-d-o transform in synchronous reference frame of (6.18) is given by the following equation.

$$V_{qs} = V_{dc}S_{qs} - r_sI_{qs} - L_spI_{qs} - \omega L_sI_{ds}$$

$$V_{ds} = V_{dc}S_{ds} - r_sI_{ds} - L_spI_{ds} + \omega L_sI_{qs}$$

$$V_{os} = V_{dc}S_o - r_sI_{os} - L_spI_{os} + 3L_npI_{os} \tag{6.28}$$

Now consider 6.14

$$I_d = \begin{bmatrix} I_a & I_b & I_c \end{bmatrix}\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix}. \tag{6.29}$$

Applying transformation

$$[T^{-1}(\theta)]^{'}[I_{qdos}]^{'}[S_{qdos}]^{'}T^{-1}(\theta)[S_{qdos}]$$

$$[T^{-1}(\theta)^{'}T^{-1}(\theta)][I_{qdos}]^{'}[S_{qdos}]$$

$$= \begin{bmatrix} c^2(\theta)+c^2(\theta-\beta)+c^2(\theta+\beta) & c(\theta)s(\theta)+c(\theta-\beta)s(\theta-\beta)+c(\theta+\beta)s(\theta+\beta) & c(\theta)+c(\theta-\beta)+c(\theta+\beta) \\ c(\theta)+c(\theta-\beta)s(\theta-\beta)+c(\theta+\beta)s(\theta+\beta) & s^2(\theta)+s^2(\theta-\beta)+s^2(\theta+\beta) & s^2(\theta)+s^2(\theta-\beta)+s^2(\theta+\beta) \\ c(\theta)+c(\theta-\beta)+c(\theta+\beta) & s(\theta)+s(\theta-\beta)+s(\theta+\beta) & 3 \end{bmatrix} [I_{qds}][S_{qdo}]$$

$$= \begin{bmatrix} \cos(\theta) & \cos(\theta-\beta) & \cos(\theta+\beta) \\ \sin(\theta) & \sin(\theta-\beta) & \sin(\theta+\beta) \\ 1 & 1 & 1 \end{bmatrix} w \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta-\beta) & \sin(\theta-\beta) & 1 \\ \cos(\theta+\beta) & \sin(\theta+\beta) & 1 \end{bmatrix} \left[I_{qdos}\right]^{'} \left[S_{qdo}\right]$$

$$= \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \left[I_{qdos}\right]^{'} \left[S_{qdos}\right]$$

$$I_d = \frac{3}{2}[I_{qs}S_{qs} + I_{ds}S_{ds} + 2I_oS_o]. \tag{6.30}$$

Thus output capacitor currents transformation using (6.27) and (6.15) gives

$$CpV_{qs} + \omega CV_{ds} = I_{qs} - I_{ql}$$

$$CpV_{ds} - \omega CV_{qs} = I_{ds} - I_{dl}$$

$$CpV_o = I_{os} - I_{ol}. \tag{6.31}$$

6.3 Controller Design for Voltage Regulation of Four Legged Inverter

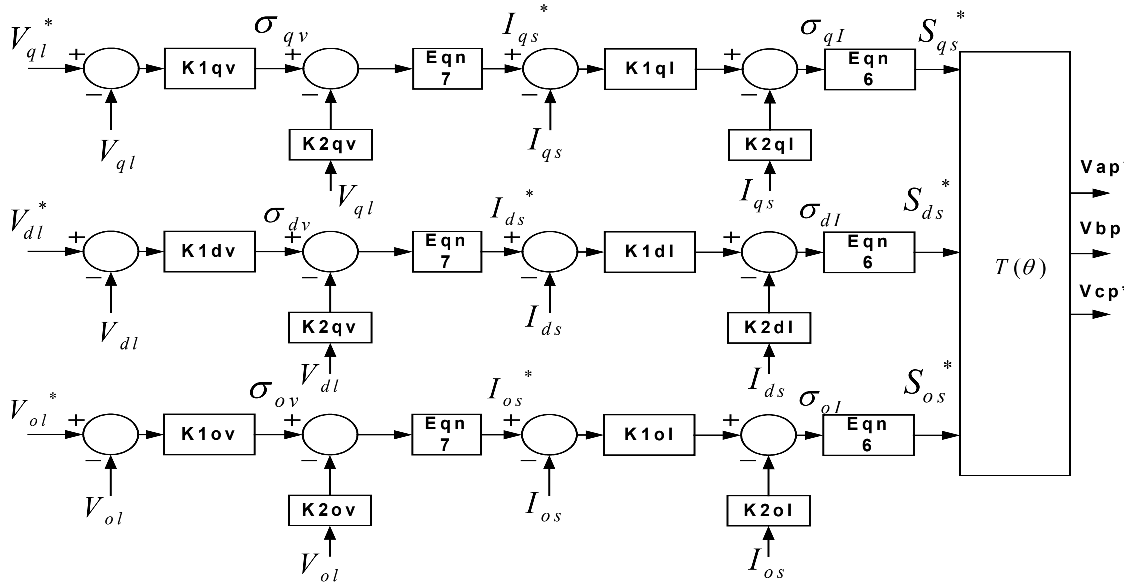


Figure 6.2 Controller structure for the four-leg inverter.

The controller objective is to regulate the three-phase voltages of the load both in magnitude and phase in-spite of its nature - balanced, unbalanced, and/or nonlinear. This control objective translates to the individual regulation of the three qdo voltages V_{ql}, V_{dl}, V_{ol} in the synchronous reference frame. Although the system equations in the synchronous reference frame become linear but coupled, a input-output linearization with decoupling control methodology is adopted [9-10]. This method linearizes the control equations, ensures system stability and high performance at all anticipated operating points given sufficient inverter DC voltage margin. The input-output linearization and decoupling are achieved by the following definitions and manipulations. The states available from modeling equations are pI_{qdos} and pV_{qdos}. Thus we can control either output voltage or current.

6.3.1 Feedback Linearization Control [A.3]

This control scheme is a type of nonlinear control scheme for Inverter.

The design based on exact linearization consists of two steps:

1. A nonlinear compensation, which cancels the nonlinearities included in the Inverter, is implemented as an inner feedback loop.
2. A controller, which ensures stability and some predefined performance, is designed based on the conventional theory; this linear controller is implemented as an outer feedback loop.

The application selected here is utility inverter wherein the output voltage of the inverter should remain constant irrespective of the load.

The IP controller is designed using the following state equations as given in (6.28) and (6.31).

$$V_{qs} + r_s I_{qs} + L_s pI_{qs} + \omega L_s I_{ds} = V_{dc} S_{qs}$$

$$V_{ds} + r_s I_{ds} + L_s pI_{ds} - \omega L_s I_{qs} = V_{dc} S_{ds}$$

$$V_{os} + r_s I_{os} + (L_s - 3L_n) pI_{os} = V_{dc} S_o \quad (6.32)$$

and

$$CpV_{qs} = I_{qs} - I_{ql} - \omega C V_{ds}$$

$$CpV_{ds} = I_{ds} - I_{dl} + \omega C V_{qs}$$

$$CpV_o = I_{os} - I_{ol} \quad (6.33)$$

The control law is derived as follows.

Let $CpV_{qs} = \sigma_q = I_{qs} - I_{ql} - \omega CV_{ds}$ (6.34)

$$I_{qs}^* = \sigma_q + I_{ql} + \omega CV_{ds} \quad (6.35)$$

and $CpV_{ds} = \sigma_d = I_{ds} - I_{dl} + \omega CV_{qs}$ hence (6.36)

$$I_{ds}^* = \sigma_d + I_{dl} - \omega CV_{qs} \quad (6.37)$$

and $CpV_o = \sigma_o = I_{os} - I_{ol}$ hence (6.38)

$$I_{os}^* = \sigma_o + I_{ol} . \quad (6.39)$$

From (6.35), (6.37), and (6.39) I_{qs}^*, I_{ds}^*, and I_{os}^* are the desired reference currents for generating the given reference voltages.

The required modulating signals need to be generated, hence from Equation (6.32) define

$$r_s I_{qs} + L_s pI_{qs} = \sigma_{qq} , \quad (6.40)$$

$$r_s I_{ds} + L_s pI_{ds} = \sigma_{dd} , \quad (6.41)$$

$$r_s I_{os} + (L_s - 3L_n) pI_{os} = \sigma_{oo} . \quad (6.42)$$

Thus (6.32) can be written in terms of the switching function as

$$S_{qs} = \frac{1}{V_{dc}}(\sigma_{qq} + V_{qs} + \omega L_s I_{ds})$$

$$S_{ds} = \frac{1}{V_{dc}}(\sigma_{dd} + V_{ds} - \omega L_s I_{qs})$$

$$S_{os} = \frac{1}{V_{dc}}(\sigma_{oo} + V_{os}) . \quad (6.43)$$

6.3.2 Controller Definition for Reference Currents

The transfer function for the feedback voltage over the desired voltage can be obtained as follows.

From (6.34)

$$CpV_{qs} = \sigma_q = K_{1q}(V_{qs}^{*} - V_{qs}) - K_{2q}V_{qs}.$$

Thus

$V_{qs}(Cp + K_{1q} + K_{2q}) = K_{1q}V_{qs}^{*}$ and hence the controller transfer function for q- axis is defined as

$$\frac{V_{qs}}{V_{qs}^{*}} = \frac{K_{1q}}{(Cp + K_{1q} + K_{2q})}$$

Where K_{1q} is the integral gain $= \frac{K_{1q}}{p}$ and K_{2q} is the proportional gain, thus we have

$$\frac{V_{qs}}{V_{qs}^{*}} = \frac{K_{1q}}{(Cp^2 + pK_{2q} + K_{1q})}. \tag{6.44}$$

From (6.36)

$$CpV_{ds} = \sigma_d = K_{1d}(V_{ds}^{*} - V_{ds}) - K_{2d}V_{ds}.$$

Thus

$V_{ds}(Cp + K_{1d} + K_{2d}) = K_{1d}V_{ds}^{*}$ and hence the controller transfer function for d- axis is defined as

$$\frac{V_{ds}}{V_{ds}^{*}} = \frac{K_{1d}}{(Cp + K_{1d} + K_{2d})}$$

where K_{1d} is the integral gain $= \frac{K_{1d}}{p}$ and K_{2d} is the proportional gain.

Thus

$$\frac{V_{ds}}{V_{ds}^{*}} = \frac{K_{1d}}{(Cp^2 + pK_{2d} + K_{1d})} \quad . \tag{6.45}$$

From (6.39)

$$CpV_{os} = \sigma_o = K_{1o}(V_{os}^{*} - V_{os}) - K_{2o}V_{os} \,.$$

Thus

$V_{os}(Cp + K_{1o} + K_{2o}) = K_{1o}V_{os}^{*}$ and hence the controller transfer function for o- axis is defined as

$$\frac{V_{os}}{V_{os}^{*}} = \frac{K_{1o}}{(Cp + K_{1o} + K_{2o})}$$

where K_{1o} is the integral gain $= \frac{K_{1o}}{p}$ and K_{2o} is the proportional gain. Thus

$$\frac{V_{os}}{V_{os}^{*}} = \frac{K_{1o}}{(Cp^2 + pK_{2o} + K_{1o})} \quad . \tag{6.46}$$

6.3.3 Controller Definition for Reference Currents

The transfer function for the feedback current over the reference current can be defined as follows.

$$r_s I_{qs} + L_s pI_{qs} = \sigma_{qq} = K_{1qq}(I_{qs}^{*} - I_{qs}) - K_{2qq}I_{qs}$$

Thus

$I_{qs}(r_s + L_s p + K_{1qq} + K_{2qq}) = K_{1qq}I_{qs}^{*}$ and hence the controller transfer function for q-axis is reference current defined as

$$\frac{I_{qs}}{I_{qs}^{*}} = \frac{K_{1qq}}{(r_s + L_s p + K_{1qq} + K_{2qq})}$$

where K_{1qq} is the integral gain $=\dfrac{K_{1qq}}{p}$ and K_{2qq} is the proportional gain. **T**hus

$$\frac{I_{qs}}{I_{qs}^{*}}=\frac{K_{1qq}}{(L_s p^2+p(r_s+K_{2qq})+K_{1qq})}\ . \tag{6.47}$$

The transfer function for the reference current can be defined as follows.

$$r_s I_{ds}+L_s pI_{ds}=\sigma_{dd}=K_{1dd}(I_{ds}^{*}-I_{ds})-K_{2dd}I_{ds}$$

Thus

$I_{ds}(r_s+L_s p+K_{1dd}+K_{2dd})=K_{1dd}I_{ds}^{*}$ and hence the controller transfer function for d-axis is reference current defined as

$$\frac{I_{ds}}{I_{ds}^{*}}=\frac{K_{1dd}}{(r_s+L_s p+K_{1dd}+K_{2dd})}$$

where K_{1dd} is the integral gain $=\dfrac{K_{1dd}}{p}$ and K_{2dd} is the proportional gain. Thus

$$\frac{I_{ds}}{I_{ds}^{*}}=\frac{K_{1dd}}{(L_s p^2+p(r_s+K_{2dd})+K_{1dd})}. \tag{6.48}$$

The transfer function for the reference current can be defined as follows

$$r_s I_{os}+(L_s-3L_n)pI_{os}=\sigma_{oo}=K_{1oo}(I_{os}^{*}-I_{os})-K_{2oo}I_{os}$$

Thus

$I_{os}(r_s+(L_s-3L_n)p+K_{1oo}+K_{2oo})=K_{1oo}I_{os}^{*}$ and hence the controller transfer function for o- axis is reference current defined as

$$\frac{I_{os}}{I_{os}^{*}}=\frac{K_{1oo}}{(r_s+(L_s-3L_n)p+K_{1oo}+K_{2oo})}$$

where K_{1oo} is the integral gain $= \frac{K_{1oo}}{p}$ and K_{2oo} is the proportional gain. Thus

$$\frac{I_{os}}{I_{os}^{*}} = \frac{K_{1oo}}{((L_s - 3L_n)p^2 + p(r_s + K_{2oo}) + K_{2oo})} \cdot \qquad (6.49)$$

The controller structure for the inverter is given in Figure 5. The manipulations in (7-8) ensures that the description of the system becomes linear making it possible to use classical control design schemes to evolve a controller that works for all anticipated operating conditions. The outputs of the three axis control loops are inverse-transformed to yield the fundamental components of the desired phase voltages, V_{ap}^{*}, V_{bp}^{*} and V_{cp}^{*}. These voltages are transformed to the stationary reference frame to determine the sector where the control voltage set lies and the identification of the expressions for the discontinuous modulation signals in Table V.

The gains of the transfer functions are selected based on a Butterworth polynomial which locates the eigen-values uniformly in the left-half S-plane on a circle of radius ω_o , with its center at the origin. The Butterworth polynomials for a transfer function with a second denominator is given as : $p^2 + \sqrt{2}p\omega_o + \omega^2{}_o = 0$.

Comparing the denominators of Equations 6.44 through 6.49 with the second ordered Butterworth polynomial $p^2 + p\sqrt{2}\omega_o + \omega_o{}^2$ and equating the coefficients for desired response time and percent overshoot the desired IP controller parameters can be found.

Thus on comparing the middle term of the polynomial equation, the K_p or the proportional parameter can be defined as

$$\frac{(r_s + K_{2qq})}{L_s} = \sqrt{2}\omega_o \text{ or}$$

$K_{2qq} = (\sqrt{2}\omega_o - r_s)L_s$ similarly $K_{2dd} = (\sqrt{2}\omega_o - r_s)L_s$ and

$$K_{2oo} = (\sqrt{2}\omega_o - r_s)(L_s - 3L_n). \tag{6.50}$$

Now similarly,

$$K_{2q} = \sqrt{2}\omega_o C, \; K_{2d} = \sqrt{2}\omega_o C, \text{ and } K_{2o} = \sqrt{2}\omega_o C \;. \tag{6.51}$$

Now comparing the last term of the polynomial equation, the K_i or the integral parameter can be defined as.

$K_{1qq} = \omega_o^{\,2} L_s$ similarly $K_{1dd} = \omega_o^{\,2} L$ and

$$K_{1oo} = \omega_o^{\,2}(L_s - 3L_n). \tag{6.52}$$

Now similarly,

$$K_{1q} = \omega_o^{\,2} C, \; K_{1d} = \omega_o^{\,2} C, \text{ and } K_{1o} = \omega_o^{\,2} C. \tag{6.53}$$

6.4 Simulation Results

Figure 6 shows the simulation results of the controlled four-leg inverter feeding three phase unbalanced loads. The filter parameters are r_s = 0.01 Ohm, L_s = 0.4 mH, C = 400 μF , Vdc = 400 V and the load impedances are r_a = 10 Ohm, L_a = 0.5 mH, r_b = 50 Ohm, L_b = 0.5 mH, r_c = 100 Ohm, L_c = 0.5 mH. The reference ‘qdo’ load voltages, which are ramped and then maintained at constant values, are seen to track closely the actual measured voltages as reflected in Figure 6(a-b). The steady state phase load voltages given in Figure 6(a), track the references and the required modulation signals for the top devices of the four legs are also given in figure 6(d). Figure 7 gives the simulation results with balanced three-phase r-l loads with $r_a = r_b = r_c = 50$ Ohms and $L_a = L_b = L_c = 0.5$ mH and also a single-phase diode bridge rectifier with a capacitor 500uF – resistance 200 Ohm on phase ‘a’ of the inverter output. The V_{qdol}^* references were kept at 150V, 100V, and 0V, respectively. The controller gains for ‘q’ and ‘d’ phases were calculated with cut off frequency chosen as 754 rad/sec for the given L_s and C, while for ‘o’ phase the ω_o was chosen as 20 rad/sec.

The I.P controller gains for voltage loop are

$K_{1qV} = K_{1dV} = 227$ and $K_{1oV} = 0.16$ and

$K_{2qV} = K_{2dV} = 0.4265$ and $K_{2oV} = 0.0113$ and

for the current loop

$K_{1qI} = K_{1dI} = 227$ and $K_{1oI} = 0.1480$

$K_{2qI} = K_{2dI} = 0.4165$ and $K_{2oI} = 4.6518e\text{-}04$.

Initially the reference ‘qdo’ voltages are ramped and later maintained at constant values. Inspite of the unbalanced and nonlinear nature of the load, the regulation of the three-phase voltage is achieved both under dynamic and steady state situations. Although there are slight differences between the reference and actual ‘qdo’ voltages the actual phase voltages closely track the reference.

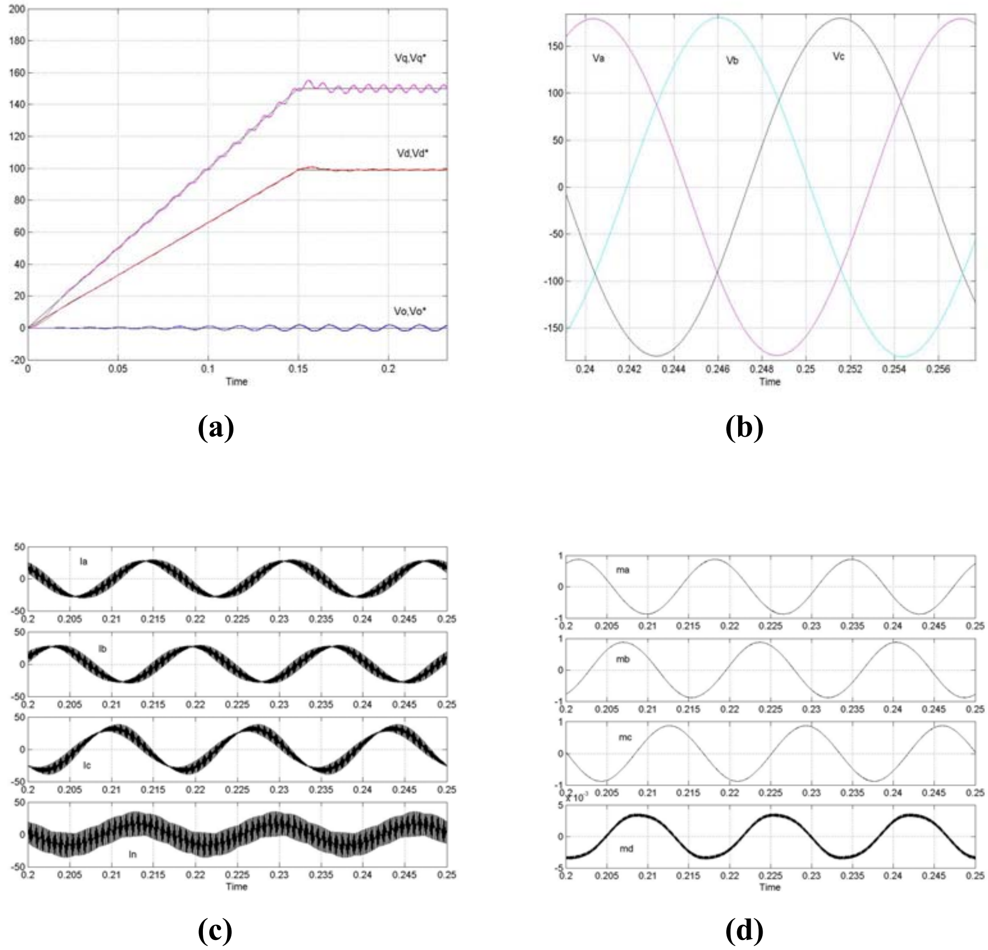
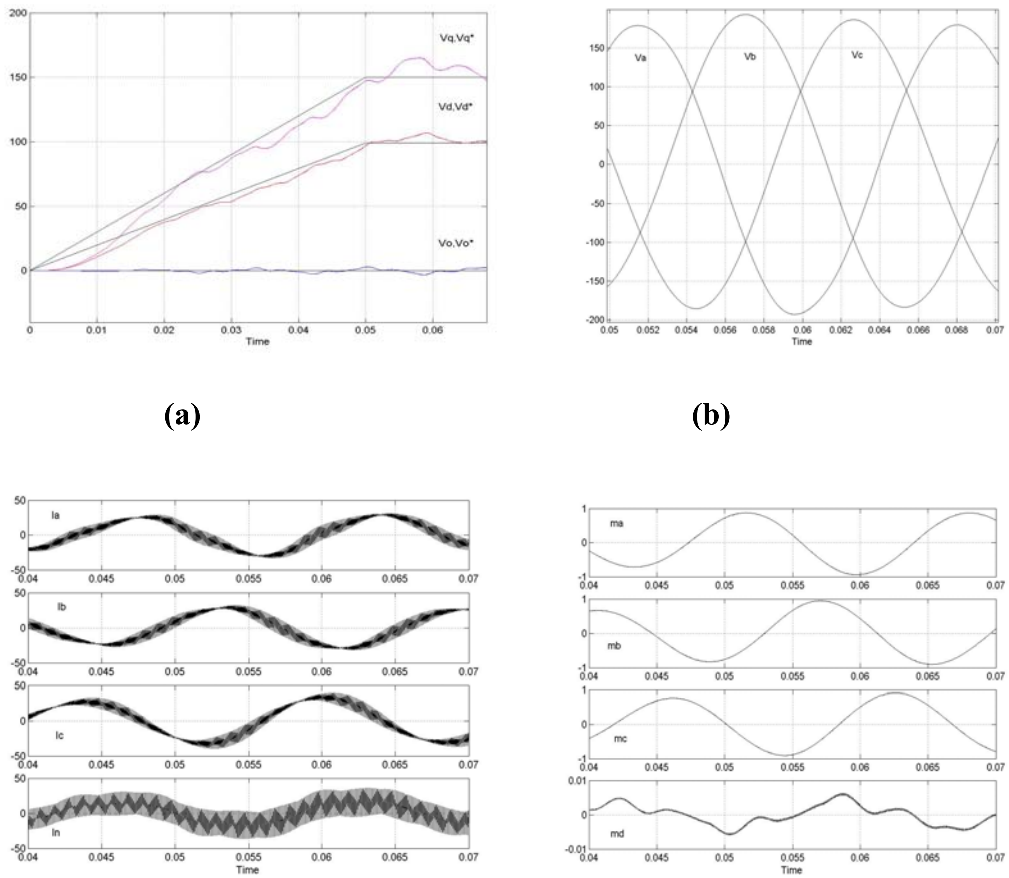


Figure 6.3 Regulation of load voltages having unbalanced load impedance. (a) Tracking of reference voltages, (b) steady-state load voltages (c) line currents, (d) modulation signals.



(a) **(b)**

(c) **(d)**

Figure 6.4 Regulation of load voltages having a parallel combination of balanced load impedance and single-phase rectifier loads. (a) Tracking of reference voltages, (b) steady-state load voltages (c) line currents, (d) modulation signals.

The **current sensor** is placed on the load side and hence the inverter currents are calculated as follows:

$$CpV_{qs} = I_{qs} - I_{ql} - \omega C V_{ds}$$

$$CpV_{ds} = I_{ds} - I_{dl} + \omega C V_{qs}$$

$$CpV_{o} = I_{os} - I_{ol}$$

At steady state $CpV_{qs} = CpV_{ds} = CpV_{qs} = 0$ hence we can assume

$$I_{qs} = \omega C V_{ds} + I_{ql}$$

$$I_{qs} = -\omega C V_{qs} + I_{dl}$$

$$I_{os} = I_{ol}.$$

Per Unit Model:

Selection of Base Values:

$V_{base} = V_{dc}$ and $I_{base} = I_{dc}$

$$Z_{base} = \frac{V_{dc}}{I_{dc}}$$

$$Z_{base} = \omega_b L_b \text{ and } Z_{base} = \frac{1}{\omega_b C_b}$$

$$V_{qsn} + R_{sn} I_{qsn} + \frac{L_{sn}}{\omega_b} pI_{qsn} + L_{sn} I_{dsn} = S_{qs}$$

$$V_{dsn} + R_{sn} I_{dsn} + \frac{L_{sn}}{\omega_b} pI_{dsn} - L_{sn} I_{qsn} = S_{ds}$$

$$V_{osn} + R_{sn} I_{osn} + \frac{L_{sn} - 3L_{nn}}{\omega_b} pI_{osn} = S_{os}$$

and

$$\frac{C_{sn}}{\omega_b} pV_{qsn} = I_{qsn} - I_{q\ln} - C_{sn}V_{dsn}$$

$$\frac{C_{sn}}{\omega_b} pV_{dsn} = I_{dsn} - I_{d\ln} + C_{sn}V_{qsn}$$

$$\frac{C_{sn}}{\omega_b} pV_{osn} = I_{osn} - I_{o\ln}$$

where

$$R_{sn} = \frac{r_s}{Z_{base}},\ L_{sn} = \frac{L_s}{L_b}, C_{sn} = \frac{C_s}{C_b}$$

$$\frac{C_{sn}}{\omega_b} pV_{qsn} = \sigma_{qn},\ \frac{C_{sn}}{\omega_b} pV_{dsn} = \sigma_{dn} \text{ and } \frac{C_{sn}}{\omega_b} pV_{osn} = \sigma_{on}$$

hence we have

$$I_{qsn}^{\ *} = \sigma_{qn} + I_{q\ln} + C_{sn}V_{dsn}$$

$$I_{dsn}^{\ *} = \sigma_{dn} + I_{d\ln} - C_{sn}V_{qsn}$$

$$I_{osn}^{\ *} = \sigma_{on} + I_{o\ln}$$

$$R_{sn}I_{qsn} + \frac{L_{sn}}{\omega_b} pI_{qsn} = \sigma_{qqn}$$

$$R_{sn}I_{dsn} + \frac{L_{sn}}{\omega_b} pI_{dsn} = \sigma_{ddn}$$

$$R_{sn}I_{osn} + \frac{L_{sn}}{\omega_b} pI_{osn} = \sigma_{oon}$$

$$S_{ds} = V_{dsn} + \sigma_{ddn} + L_{sn}I_{qsn}$$

$$S_{qs} = V_{qsn} + \sigma_{qqn} - L_{sn} I_{dsn}$$

$$S_{os} = V_{osn} + \sigma_{oon}$$