CHAPTER 3

PWM SCHEMES IN THREE PHASE VOLTAGE SOURCE INVERTERS

3.1 Three phase VSI as a *Switching Converter*

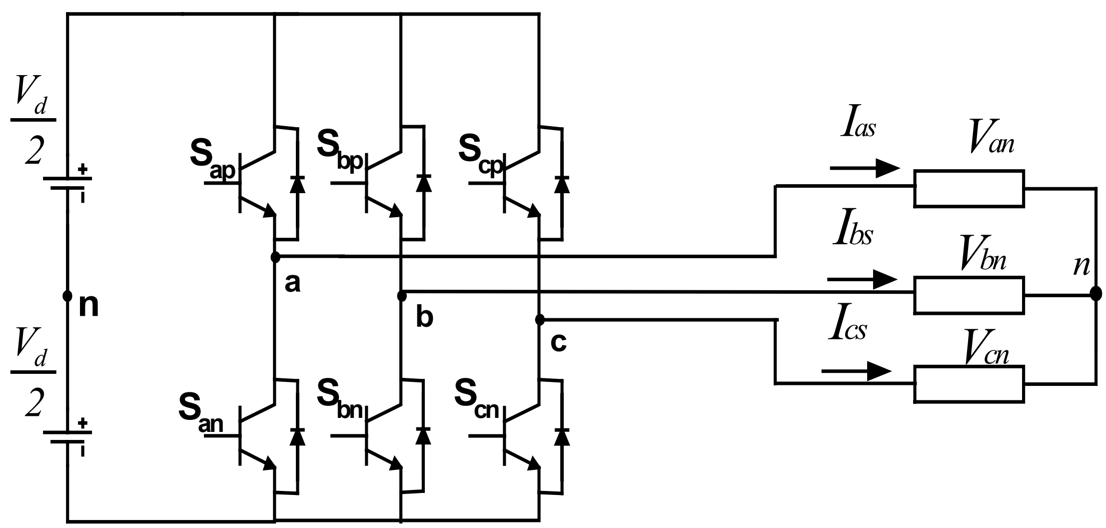


Figure 3.1 Three phase VSI

Power Electronic applications involved in synthesis of quality power essentially rely on *switching converters*. In switching converters, power semiconductor devices are operated in saturation region of operation. This is because there exists higher losses in active region operation of these semiconductor devices. Thus switching aids in achieving high efficiency, low weight, smaller dimensions, fast operation and higher power densities in power converters. Hence the switching converters are applied in following conversion techniques:

DC-DC conversion (direct current) - involves change and control of output voltage magnitude.

AC-DC (alternating current) rectification- involves control of output DC voltage and input AC current for unity power factor operation.

AC-DC inversion- involves synthesis and control of sinusoidal output voltages and currents.

AC-AC conversion- involves change and control of input voltage and frequency.

Three phase DC/AC Voltage Source Inverter (VSI) shown in Figure 3.1 is being used extensively in motor drives, active filters and unified power flow controllers in power systems and uninterrupted power supplies to generate controllable frequency and AC voltage magnitudes using various pulse-width modulation (PWM) strategies.

3.2 Desirable characteristics of three phase PWM VSI

i. Wide linearity of operation

ii. Minimum switching to ensure low switching loss

iii. Minimum voltage and current harmonics

iv. Over-modulation operation including six stepped operation

To achieve the mentioned characteristics two-implementation techniques exists

- Direct digital technique SVPWM
- Carrier based (triangle comparison) technique

The direct digital technique involves utilization of space vector approach wherein the duty cycles for the switching inverter are calculated. The gating signals are pre-sequenced and stored as lookup Table for the available switching states of a VSI.

Carrier based PWM utilizes the per cycle volt-second balance [3.14] to synthesize the desired output voltage waveform.

3.3 Sinusoidal or Continuous PWM

The turn on and turn off action of the switch produces a rectangular waveform as shown in Figure 3.2. The voltage is equal to input voltage $v_s(t)$ when the switch is turned ON while it is equal to zero whenever it is turned OFF. Thus continuous turn ON and OFF cycles produces a train of output pulses. If the switch is turned ON for D. T_s where D is the duty cycle of the switch and T_s is the switching frequency then the average value of the output voltage is given by [A.1]

$$V_s = \frac{1}{T_s}\int_0^{T_s} v_s(t)dt = DV_g \qquad (3.1)$$

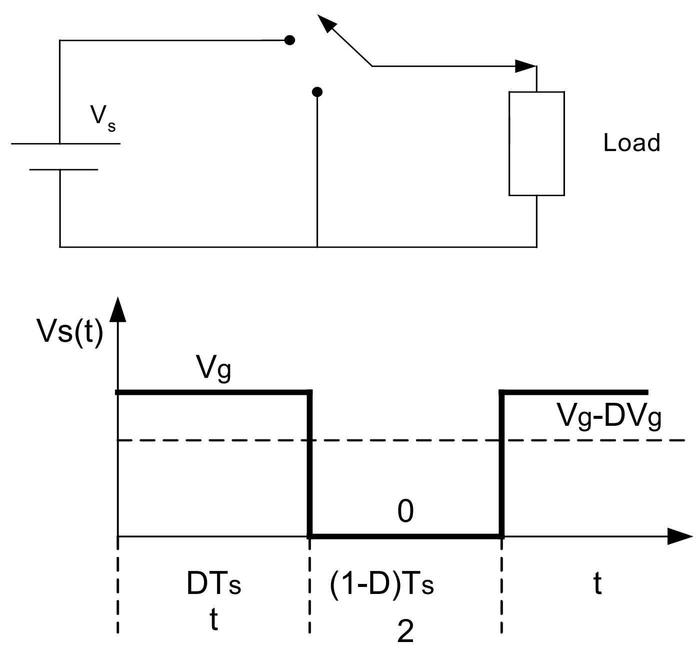


Figure 3.2 Switching converter

It is thus evident that varying the duty cycle of the switching device will result in variable output voltage. In synthesizing a sinusoidal output signal:

- High frequency sine modulated pulses are used to drive the switching device.
- The modulation is done by comparing sine signal with a triangle.
- When the output of the inverter is filtered through a low pass filter, the original modulating signal is obtained which is of higher magnitude.

This principle is known, as Sinusoidal Pulse Width Modulation (SPWM) where by comparing a sinusoidal signal with a triangle a sine weighted modulating signal is generated [1]. This is shown in Figure 3.3, which is also known as *continuous modulation*

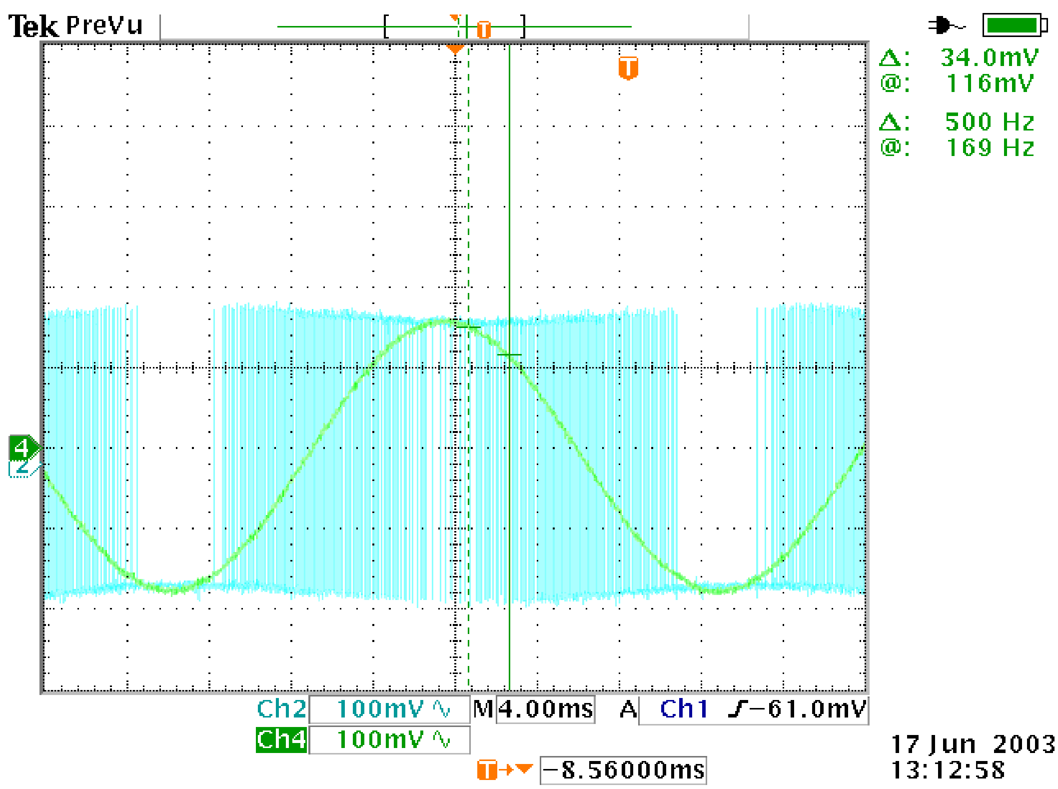


Figure 3.3 Generation of SPWM modulating index = 1

In three phase VSI, the three phase shifted reference signals are compared with the carrier signal, which define the switching instants for the power devices. The harmonics generated with this scheme are around the carrier frequency and its multiples. These can be filtered out using a low pass filter. These harmonics may not be completely suppressed. The narrow range of linearity is the limitation of SPWM because the DC bus utilization is only up to 78.5%. To maximize the DC bus utilization an alternative technique known as Space Vector Modulation [3.2] is used which will be discussed in the following section.

3.4 Space Vector Modulation SVPWM

3.4.1 Generation of the PWM switching signals

With a three-phase voltage source inverter there are eight possible operating states. Obeying Kirchoff's Voltage Law (K.V.L) and Kirchoff's Current Law (K.C.L) the generated states for the inverter are listed in Table 3.1

For KVL, no device in the same inverter leg should be turned on at the a time else the DC link would be shorted leading to damage of the inverter.

Table 3.1 Switching States in a 3 phase VSI

State	S_{ap}	S_{bp}	S_{cp}	S_{an}	S_{bn}	S_{cn}	
Null, S_0	0	0	0	1	1	1	S_{an} S_{bn} S_{cn}
S_1	0	0	1	1	1	0	S_{cp} S_{an} S_{bn}
S_2	0	1	0	1	0	1	S_{bp} S_{an} S_{cn}
S_3	0	1	1	1	0	0	S_{bp} S_{cp} S_{an}
S_4	1	0	0	0	1	1	S_{ap} S_{bn} S_{cn}
S_5	1	0	1	0	1	0	S_{ap} S_{cp} S_{bn}
S_6	1	1	0	0	0	1	S_{ap} S_{bp} S_{cn}
Null, S_7	1	1	1	0	0	0	S_{ap} S_{bp} S_{cp}

It is conspicuous that the inverter has six active states corresponding to S_1 through S_6 and two null states S_0 and S_7, The stationary reference frame 'qdo' voltages of the switching modes, also given in Table 3.2, are expressed in the complex variable form as ($a = e^{j\varsigma}$, $\varsigma = 120°$) [3.20] :

$$V_{qds} = 2/3(V_{an} + a\,V_{bn} + a^2\,V_{cn})$$

$$V_o = 1/3(V_{an} + V_{bn} + V_{cn}) \qquad (3.2)$$

Table 3.2: Switching modes of the three-phase voltage source inverter and corresponding stationary reference frame qdo voltages.

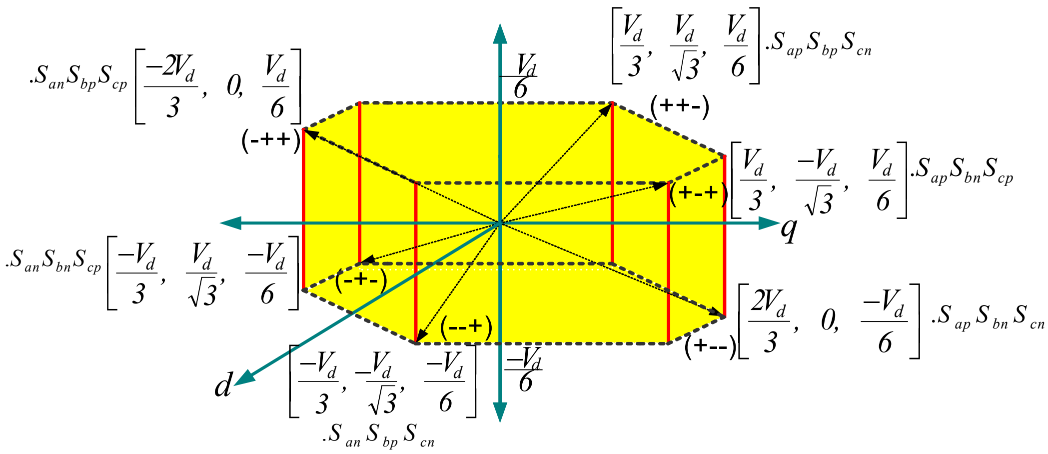
Mode	S_{ap}	S_{bp}	S_{cp}	V_{qs}	V_{ds}	V_{os}
1	0	0	0	0	0	$-V_d/2$
2	0	0	1	$-V_d/\sqrt{3}$	$V_d/\sqrt{3}$	$-V_d/6$
3	0	1	0	$-V_d/3$	$-V_d/\sqrt{3}$	$-V_d/6$
4	0	1	1	$-2V_d/3$	0	$V_d/6$
5	1	0	0	$2V_d/3$	0	$-V_d/6$
6	1	0	1	$V_d/3$	$-V_d/\sqrt{3}$	$V_d/6$
7	1	1	0	$V_d/3$	$V_d/\sqrt{3}$	$V_d/6$
8	1	1	1	0	0	$V_d/2$

This Table can be visualized as a regular hexagon and dividing it into six equal sectors denoted as I, II, III, IV, V, VI in Figure 3.4.

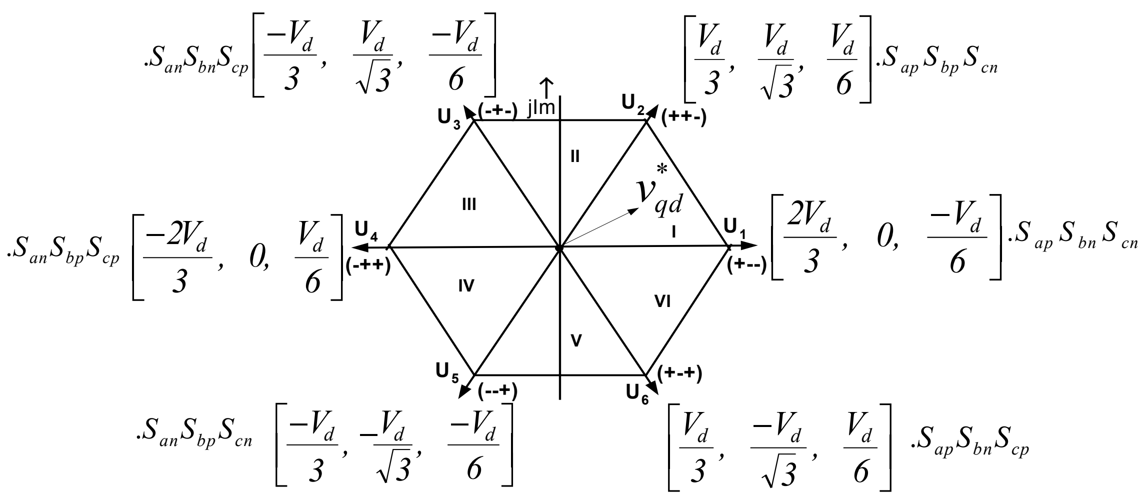
Thus a reference voltage vector in any sector can be referred to as

$V_{qd}^* = U_k = \frac{2}{3} V_d e^{j(k-1)\frac{\Pi}{3}}$ [B.1] with (k= 1,2,3,4,5, and 6)

(3.3)



(a)



(b)

Figure 3.4 (a) 3-D Plot of Stationary qdo voltages for the given states, (b) Projection of the available states on the q-d plane.

A reference signal, V_{qd}^* over a switching period T_s can be defined from the space vector. Assuming that T_s is sufficiently small, V_{qd}^* can be considered approximately constant during this interval, and it is this vector, which generates the fundamental behavior of the load.

The continuous Space Vector Modulation technique is based on the fact that every vector V_{qd}^* inside the hexagon can be expressed as a weighted average combination of the two adjacent active space vectors and the null-state vectors 0 and 7. Therefore, in each cycle imposing the desired reference vector may be achieved by switching between these four inverter states.

From Figure 3.4(b), assuming V_{qd}^* to be laying in sector k, the adjacent active vectors are U_k and U_{k+1}, where k+1 is set to 1 for k = 6. In order to obtain optimum harmonic performance and the minimum switching frequency for each of the power devices, the state sequence is arranged such that switching only one inverter leg performs the transition from one state to the next. This condition is met if the sequence begins with one zero-state and the inverter switches are toggled until the other null-state is reached.

To complete the cycle the sequence is reversed, ending with the first zero-state. If, for instance, the reference vector sits in sector I, the state sequence has to be $\ldots U_0U_1U_2U_7U_2U_1U_0\ldots$ whereas in sector IV it is $\ldots U_0U_5U_4U_7U_4U_5U_0\ldots$ The central part of the space vector modulation strategy is the computation of both the active and zero-state times for each modulation cycle. These may be calculated by equating the average voltage to the desired value.

In the following, T_k denotes half the on-time of vector U_k. T_o is half the null-state time. Hence, the on-times are evaluated by the following equations [B.1]:

$$\int_0^{\frac{T_s}{2}} V_{qd}^* dt = \int_0^{\frac{T_o}{2}} U_o dt + \int_{\frac{T_o}{2}}^{\frac{T_o}{2}+T_k} U_k dt + \int_{\frac{T_o}{2}+T_k}^{\frac{T_o}{2}+T_k+T_{k+1}} U_{k+1} dt + \int_{\frac{T_o}{2}+T_k+T_{k+1}}^{\frac{T_s}{2}} U_7 dt$$

$$T_o + T_k + T_{k+1} = \frac{T_s}{2} \tag{3.4}$$

Taking into account that $U_0=U_7\equiv 0$ and that V_{qd}^* is assumed constant and the fact that U_k, U_{k+1} are constant vectors, equation (3.4) reduces to

$$V_{qd}^* . \frac{T_s}{2} = (U_k.T_k)+(U_{k+1}.T_{k+1}) \tag{3.5}$$

Splitting this equation into real and imaginary components, from (3.3) follows that:

$$\begin{pmatrix} V_q \\ V_d \end{pmatrix} \frac{T_s}{2} = \frac{2}{3} V_d \left(T_k \begin{pmatrix} \cos\frac{(k-1)\pi}{3} \\ \sin\frac{(k-1)\pi}{3} \end{pmatrix} + T_{k+1} \begin{pmatrix} \cos\frac{k\pi}{3} \\ \sin\frac{k\pi}{3} \end{pmatrix} \right) = \frac{2}{3} V_d \begin{bmatrix} \cos\frac{(k-1)\pi}{3} & \cos\frac{k\pi}{3} \\ \sin\frac{(k-1)\pi}{3} & \sin\frac{k\pi}{3} \end{bmatrix} \begin{pmatrix} T_k \\ = \\ T_{k+1} \end{pmatrix} \tag{3.6}$$

Where k is to be determined from the argument of the reference vector

$$\alpha = \arg\begin{pmatrix} V_q \\ V_d \end{pmatrix} \quad \text{such that} \quad \frac{(k-1)\pi}{3} \le \arg\begin{pmatrix} V_q \\ V_d \end{pmatrix} \le \frac{k\pi}{3}. \tag{3.7}$$

For minimal number of commutations per cycle is met only if in every odd sector the sequence of applied vectors is U_0 U_k U_{k+1} U_k U_0 , whereas in even sector the active vectors are applied in the reversed order, hence U_0 U_{k+1} U_k U_7 U_k U_{k+1} U_0.

Solving system (3.6) :

$$\begin{pmatrix} T_k \\ T_{k+1} \end{pmatrix} = \frac{\sqrt{3}}{2} \frac{T_s}{V_d} \begin{bmatrix} \sin\frac{k\pi}{3} & -\cos\frac{k\pi}{3} \\ -\sin\frac{(k-1)\pi}{3} & \cos\frac{(k-1)\pi}{3} \end{bmatrix} \cdot \begin{pmatrix} V_q \\ V_d \end{pmatrix} \tag{3.8}$$

The total null-state time T_0 may be divided in an arbitrary fashion between the two zero states. A common solution is to divide T_0 equally between the two null-state vectors U_0 and U_7. From (3.4), T_0 results as

$$T_0=\frac{T_s}{2} - (T_k+T_{k+1}) \tag{3.9}$$

As an example for the switching scheme, in sector I one finds:

Assuming that is desired to produce a balanced system of sinusoidal phase voltages, it is known that the corresponding space vector locus is circular. Imposing $V_{qd}^{*} = |V_m|.(\cos(\omega t) + j\sin(\omega t))$, where $|V_m|$ is the magnitude and ω is the angular frequency of the desired phase voltages, it follows from (3.8) that

$$\begin{pmatrix} T_k \\ T_{k+1} \end{pmatrix} = \frac{\sqrt{3}}{2}\frac{|V_m|}{V_d}.T_s.\begin{bmatrix} \sin\frac{k\pi}{3} & -\cos\frac{k\pi}{3} \\ -\sin\frac{(k-1)\pi}{3} & \cos\frac{(k-1)\pi}{3} \end{bmatrix}.\begin{pmatrix} \cos\omega t \\ \sin\omega t \end{pmatrix} \tag{3.10}$$

While $0 \le \omega t \le \pi/3$ the reference vector lies in sectors I and equation (3.10) reduces to

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \frac{\sqrt{3}}{2}\frac{|V_m|}{V_d}T_s\begin{pmatrix} \sin(\frac{\pi}{3} - \omega t \\ \sin\omega t \end{pmatrix} \tag{3.11}$$

Thus the mentioned procedure is used for microprocessor or DSP based implementation of space vector PWM. The corresponding output is shown in Figure 3.5. The null times T_0 have to be sequenced in every sector. This sequencing is not necessary in carrier based PWM technique. This technique is discussed in following section.

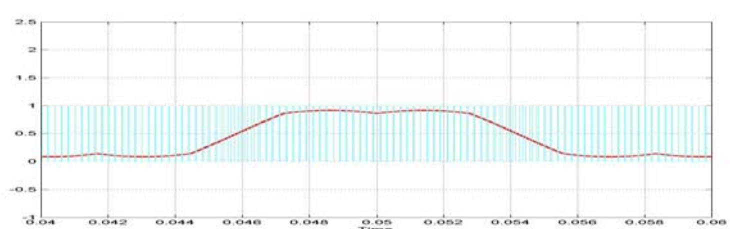


Figure 3.5 Generation of SVPWM modulating index = 1

3.4.2 Carrier based implementation of Space Vector Modulation SVPWM

In Figure 3.1 the important criteria to satisfy KVL and KCL is that.

$S_{ap} + S_{an} = 1$ where S_{ij} are switching functions and are defined as when compared with

$S_{bp} + S_{bn} = 1$ triangle so that when

$S_{cp} + S_{cn} = 1$ $V_{abc} > V_{carr}$; $S_{ij}=1$

and $V_{abcn} < V_{carr}; S_{ij}=0$

where i,j=a,b,c. (3.12)

The phase voltage equations for star-connected, balanced three-phase loads expressed in terms of the existence functions and input DC voltage V_d are given as [3.20]:

$$(S_{ap} - S_{an})\frac{V_d}{2} = V_{an} + V_{no}$$

$$(S_{bp} - S_{bn})\frac{V_d}{2} = V_{bn} + V_{no}$$

$$(S_{cp} - S_{cn})\frac{V_d}{2} = V_{cn} + V_{no} \tag{3.13}$$

In equations in (3.13), V_{an}, V_{bn}, V_{cn} are the phase voltages of the load while the voltage of the load neutral to inverter reference is V_{no}. If the reference voltage set is balanced, the load voltages from (3.12) are expressed in (3.14). The eight feasible switching modes for the three-phase voltage source inverter are enumerated in Table 3.1.

$$V_{an} = \frac{V_d}{6}(2S_{ap} - S_{bp} - S_{cp} - 2S_{an} + S_{bn} + S_{cn})$$

$$V_{bn} = \frac{V_d}{6}(2S_{bp} - S_{ap} - S_{cp} - 2S_{bn} + S_{an} + S_{cn})$$

$$V_{cn} = \frac{V_d}{6}(2S_{cp} - S_{ap} - S_{bp} - 2S_{cn} + S_{an} + S_{bn}) \tag{3.14}$$

The stationary reference frame qdo voltages of the switching modes, also given in Table 1.1, are expressed as:

$$f_q = \frac{1}{3}(2f_a - f_b - f_c)$$

$$f_d = \frac{1}{\sqrt{3}}(f_b - f_c)$$

$$f_o = \frac{1}{3}(f_a + f_b + f_c) \tag{3.15}$$

since

$$S_{ap} + S_{an} = 1$$

$$S_{ap} = 1 - S_{an}\ ,\ \ S_{bp} = 1 - S_{bn}\ ,\ \ S_{cp} = 1 - S_{cn} \tag{3.16}$$

Substituting (3.16) in (3.14) and then solving for (3.15) we have

$$V_{qs} = \frac{1}{3}\left(2[(2S_{ap} - 1)\frac{V_d}{2} - V_{no}] - [(2S_{bp} - 1)\frac{V_d}{2} - V_{no}] - [(2S_{cp} - 1)\frac{V_d}{2} - V_{no} \right)$$

$$= \frac{1}{3}\left(4S_{ap}\frac{V_d}{2} - V_d - 2V_{no} - 2S_{bp}\frac{V_d}{2} + \frac{V_d}{2} + V_{no} - 2S_{cp}\frac{V_d}{2} + \frac{V_d}{2} + V_{no} \right)$$

$$= \frac{1}{3}\left(2S_{ap}V_d - S_{bp}V_d - S_{cp}V_d \right)$$

thus

$$V_{qs} = \frac{V_d}{3}\left(2S_{ap} - S_{bp} - S_{cp} \right) \tag{3.16}$$

$$V_{ds} = \frac{1}{\sqrt{3}}\left((2S_{cp} - 1)\frac{V_d}{2} - V_{no} - (2S_{bp} - 1)\frac{V_d}{2} + V_{no} \right)$$

$$V_{ds} = \frac{1}{\sqrt{3}}(S_{cp} - S_{bp}) \tag{3.17}$$

Now,

$$V_{os} = (2S_{bp} - 1)\frac{V_d}{2} + (2S_{cp} - 1)\frac{V_d}{2} + (2S_{ap} - 1)\frac{V_d}{2} = (V_{an} + V_{bn} + V_{cn}) + 3V_{no}$$

For balanced case $V_{an} + V_{bn} + V_{cn} = 0$

$$\therefore (S_{ap} + S_{bp} + S_{cp})V_d - \frac{3V_d}{2} = 3V_{no}$$

$$\therefore\ V_{os} = \frac{V_d}{3}(S_{ap} + S_{bp} + S_{cp}) - \frac{V_d}{2} = 3V_{no} \tag{3.18}$$

In the direct digital PWM method, the complex plane stationary reference frame qd output voltage vector of the three-phase voltage source inverter is used to calculate the turn-on times of the inverter switching devices required to synthesize a reference three-phase balanced voltage set. In general, the three-phase balanced voltages expressed in the stationary reference frame; situated in the appropriate sector in Figure 3.4(b) are approximated by the time-average over a sampling period (converter switching period, T_s) of the two adjacent active qd voltage inverter vectors and the two zero states U_0 and U_7. If the normalized times (with respect to modulator sampling time or converter switching period, T_s) the set of four voltage vectors termed as V_{qda} , V_{qdb}, V_{qd0}, V_{qd7} corresponds to time signals t_a , t_b, t_0, t_7 respectively. The q and d components of the reference voltage V_{qd}^* are approximated as [3.20]:

$$V_{qd}^* = V_{qq} + jV_{dd} = V_{qda}t_a + V_{qdb}t_b + V_{qdo}t_o + V_{qd7}t_7 \tag{3.19}$$

Where we have the time spend in the null state given as,

$$t_c = t_0 + t_7 = 1 - t_a - t_b \tag{3.20}$$

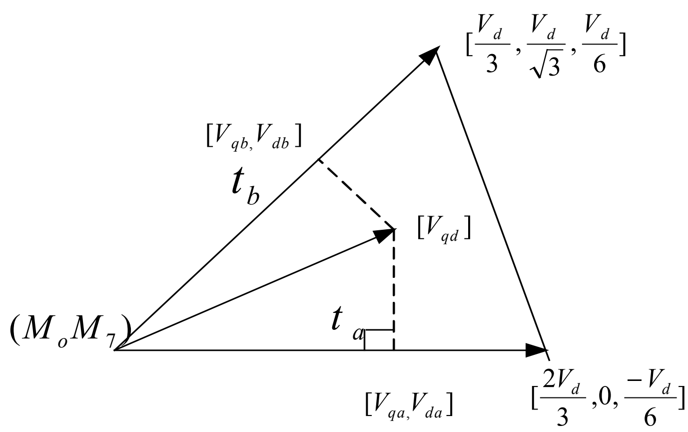


Figure 3.6 Projection of times to generate the q,d reference vectors

When separated into real and imaginary parts, (3.19) gives the expressions for t_a and t_b as :

$$\begin{bmatrix} V_{qq} \\ V_{dd} \end{bmatrix} = \begin{bmatrix} V_{qa} & V_{qb} \\ V_{da} & V_{db} \end{bmatrix} \begin{bmatrix} t_a \\ t_b \end{bmatrix}$$

Now $\Delta = V_{qa}V_{db} - V_{da}V_{qb}$ (3.21)

hence

$$t_a = \frac{1}{\Delta}\begin{bmatrix} V_{qq} & V_{qb} \\ V_{dd} & V_{db} \end{bmatrix} = \frac{V_{qq}V_{db} - V_{qd}V_{dd}}{\Delta}$$

$$t_b = \frac{1}{\Delta}\begin{bmatrix} V_{qa} & V_{qq} \\ V_{da} & V_{dd} \end{bmatrix} = \frac{V_{qa}V_{dd} - V_{qq}V_{da}}{\Delta} \qquad (3.22)$$

It is observed that both V_{qd0} and V_{qd7} do not influence the values of t_a and t_b . The times t_a and t_b are given in Table 3.2 for voltage references in the six sectors.

The expressions for the normalized times (t_a,t_b) displayed in Table 3.3 can be derived as as follows:

Consider for sector I from Figure 3.6 we have

$$V_{qa} = \frac{2V_d}{3}, V_{da} = 0$$

$$V_{qb} = \frac{V_d}{3}, V_{db} = \frac{V_d}{\sqrt{3}}$$

Now $\Delta = V_{qa}V_{db} - V_{da}V_{qb} \ = \frac{2V_d}{3}.\frac{V_d}{\sqrt{3}} - 0$

$$\Delta = \frac{2V_d^{\ 2}}{3\sqrt{3}}$$

From equation (3.22) we can write

$$t_a = \frac{1}{\Delta}(V_{qq}V_{db} - V_{db}V_{db}) \ = \frac{(\frac{V_d}{\sqrt{3}}V_{qq} - \frac{V_d}{3}V_{dd})}{\frac{2V_d^{\ 2}}{3\sqrt{3}}}$$

$$t_a = \frac{0.5(3V_{qq} - \sqrt{3}V_{dd})}{V_d} \tag{3.23}$$

and

$$t_b = \frac{1}{\Delta}(V_{qa}V_{dd} - V_{qq}V_{da}) = \frac{1}{\Delta}(V_{qa}V_{dd})$$

$$t_b = \frac{\sqrt{3}V_{dd}}{V_d} \tag{3.24}$$

The times are calculated in each sector are listed in Table 3.3. These times can be expressed in terms of the corresponding line-line voltages after inverse transformation from stationary to 'a-b-c'.

The stationary reference frame inverse transformation is given as:

$$f_a = f_q + f_o$$

$$f_b = \frac{-f_q}{2} - \frac{\sqrt{3}f_d}{2} - f_o$$

$$f_c = \frac{-f_q}{2} + \frac{\sqrt{3}f_d}{2} + f_o$$

Now

$$f_a f_b = f_{ab} = f_q + f_o + \frac{f_q}{2} + \frac{\sqrt{3}}{2} f_d - f_o \quad = \frac{0.5}{V_d}(3f_q + \sqrt{3}f_d) \tag{3.25}$$

$$f_a - f_c = f_{ac} = f_q + f_o + \frac{f_q}{2} - \frac{\sqrt{3}}{2} f_d - f_o \ = \frac{0.5}{V_d}(f_q - \sqrt{3}f_d) \tag{3.26}$$

$$f_b - f_c = f_{bc} = -\frac{f_q}{2} - \frac{\sqrt{3}}{2} f_d + f_o + \frac{f_q}{2} - \frac{\sqrt{3}}{2} f_d - f_o \ = -\sqrt{3}f_d$$

$$f_{cb} = -\sqrt{3}f_d \tag{3.27}$$

Substituting the expressions for t_a and t_b from Table 3.3 into equations (3.25), (3.26), and (3.27) we get Table 3.4 [3.20]

Kirchoff's voltage law constraints the existence functions such that $\mathbf{S_{ip}} + \mathbf{S_{in}} = 1$, where i = a, b, and c which when substituted in (3.2) are expressed as :

$$(2S_{ap} - 1)\frac{V_d}{2} = V_{an} + V_{no}$$

$$(2S_{bp} - 1)\frac{V_d}{2} = V_{bn} + V_{no}$$

$$(2S_{cp} - 1)\frac{V_d}{2} = V_{cn} + V_{no} \tag{3.28}$$

The voltage equations expressed in terms of the modulation signals in (8) are facilitated by the Fourier series approximation of the existence functions, which are approximated as [A.2][A.3]:

$$S_{ap} \cong Z_{ap} = 0.5(1 + M_{ap})$$

$$S_{bp} \cong Z_{bp} = 0.5(1 + M_{bp})$$

$$S_{cp} \cong Z_{cp} = 0.5(1 + M_{cp}) \tag{3.29}$$

Where, M_{ap} , M_{bp}, M_{cp} are the carrier-based modulation waveforms comprising of fundamental frequency components. These vary between –1 and 1 (for the linear modulation range). The approximate existence functions (Z_{ap}, Z_{bp}, Z_{cp}) which range between zero and unity can be used to generate actual existence functions by comparing them with a high frequency triangular waveform that ranges between unity and zero.

In general, the existence functions are usually generated by comparing the high frequency triangle waveform, which ranges between –1 and 1 with the modulation waveforms (M_{ap}, M_{bp}, M_{cp}). Hence the inverter switching signals which are connected to the base drives of the switching devices for the carrier-based PWM scheme can be achieved by either of these two methods.

- Using the actual modulating signals.
- Using the approximate existence functions.

3.5 Carrier based implementation with injection of generalized zero sequence voltage

The equations for the modulating signals of the top devices from (3.28) and (3.29) are expressed as [3.20]:

$$M_{ap} = \frac{V_{an}}{V_d/2} + \frac{V_{no}}{V_d/2}$$

$$M_{bp} = \frac{V_{bn}}{V_d/2} + \frac{V_{no}}{V_d/2}$$

$$M_{cp} = \frac{V_{cn}}{V_d/2} + \frac{V_{no}}{V_d/2} \qquad (3.30)$$

The neutral voltage V_{no} averaged over the switching period T_s is given as :

$$< V_{no} >= V_{oa}t_a + V_{ob}t_b + V_{oo}t_0 + V_{o7}t_7 \qquad (3.31)$$

It should be noted that t_c is partitioned into dwell times for the two null voltage vector - $t_c\alpha$ for U_o and $t_c(1-\alpha)$ for U_7.

The averaged zero sequence voltages for reference voltages in the voltage sectors are derived as follows:

We know that the total switching time from (3.20) can be expressed as:

$$\frac{t_a}{T_s}+\frac{t_b}{T_s}+\frac{t_o}{T_s}+\frac{t_7}{T_s}=1 \text{ or } t_c=1-t_a-t_b$$

Now $t_c=t_o+t_7$ if we define $\alpha t_c=t_o \;\therefore t_7=(1-\alpha)t_c$

$$\therefore <V_{no}>=V_{oa}t_a+V_{ob}t_b+\alpha t_cV_{oo}+(1-\alpha)t_cV_{o7} \tag{3.32}$$

This zero sequence voltage has various values in different sectors, which can be generalized to obtain a single expression applied to all the sectors.

3.5.1 Generalized expression for V_{no}

Consider section I we have

$$V_{oa}=\frac{-V_d}{6};V_{ob}=\frac{V_d}{6}$$

$$V_{oo}=\frac{-V_d}{2};V_{o7}=\frac{V_d}{2}$$

$$\therefore <V_{no}>=\frac{-V_d}{6}t_a+\frac{V_d}{6}t_b+\alpha t_c(\frac{-V_d}{2})+(1-\alpha)t_c\frac{V_d}{2}$$

$$=\frac{V_d}{2}(t_b-t_a)-\alpha t_c\frac{V_d}{2}+(1-\alpha)t_c\frac{V_d}{2}$$

$$<V_{no}>=\frac{V_d}{6}(t_b-t_a)+\frac{t_cV_d}{6}(1-2\alpha) \tag{3.33}$$

It is notable that a similar expression is obtained in sector III and V

Consider sector II

$$V_{oa}=\frac{V_d}{6};V_{ob}=\frac{-V_d}{6}$$

$$V_{oo}=\frac{-V_d}{2};V_{o7}=\frac{V_d}{2}$$

$$\therefore \ <V_{no}> = \frac{V_d}{6}(t_a - t_b) + \frac{t_c V_d}{2}(1-2\alpha) \qquad (3.34)$$

Similarly the above expression is obtained for sector IV and VI

Thus the expression for zero sequence voltage in each sector used for injection with the phase voltages is given in Table 3.5.

Table 3.5 Average zero sequence voltage for the sectors [3.20]

Sectors	I, III, V	II, IV, VI
<V_{no}>	$<V_{no}> = \frac{V_d}{6}(t_b - t_a) + \frac{t_c V_d}{6}(1-2\alpha)$	$<V_{no}> = \frac{V_d}{6}(t_a - t_b) + \frac{t_c V_d}{2}(1-2\alpha)$

Thus the generalized expression for <V_{no}> is obtained as follows

It is known that for balanced case sum of the phase voltages is zero hence,

$$V_{an} + V_{bn} + V_{cn} = 0$$

Consider sector I

$$<V_{no}> = \frac{V_d}{6}(t_b - t_a) + \frac{t_c V_d}{6}(1-2\alpha)$$

From Table 3.4 substituting the values of $t_a = \frac{V_{ac}}{V_d}$ and $t_b = \frac{V_{cb}}{V_d}$ and $t_c = 1 - t_a - t_b$

$$<V_{no}> = \frac{V_d}{6}\left(\frac{V_{cb}}{V_d} - \frac{V_{ac}}{V_d}\right) + \left(1 - \frac{V_{cb}}{V_d} - \frac{V_{ac}}{V_d}\right)\frac{V_d}{2}(1-2\alpha)$$

$$<V_{no}> = \frac{1}{6}(2V_c - V_b - V_a) + \frac{V_d}{2}(1-2\alpha) + 0.5(1-2\alpha)(V_b - V_a)$$

$$<V_{no}> = \frac{1}{6}(2V_c - V_b - V_a) + \frac{V_d}{2}(1-2\alpha) + 0.5(1-2\alpha)(V_b - V_a)$$

using $V_{cn} = -V_{an} - V_{bn}$

$$< V_{no} >= \frac{1}{6}(-2V_a - 2V_b - V_b - V_a) + \frac{V_d}{2}(1-2\alpha) + 0.5(1-2\alpha)(V_b - V_a)$$

$$< V_{no} >= -\frac{1}{2}(V_b + V_a) + \frac{V_d}{2}(1-2\alpha) + 0.5(1-2\alpha)(V_b - V_a)$$

$$< V_{no} >= \frac{V_d}{2}(1-2\alpha) - \frac{V_a}{2}(-1-1+2\alpha) + \frac{V_b}{2}(1-1-2\alpha)$$

$$< V_{no} >= \frac{V_d}{2}(1-2\alpha) + V_a(\alpha-1) - \alpha V_b \qquad (3.36)$$

From Table 3.4, it is observed that for Sector I the maximum phase voltage is V_a while the minimum phase voltage is V_c thus

$$< V_{no} >= \frac{V_d}{2}(1-2\alpha) + V_{\max}(\alpha-1) - \alpha V_{\min} \qquad (3.37)$$

The expression for remaining sectors is as follows:

Sector II.

$$V_{no} = \frac{1}{6}[2V_a - V_b - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_b - V_c]$$

$$= \frac{1}{6}[-2V_b - 2V_c - V_b - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_b - V_c]$$

$$= \frac{-1}{2}[V_c + V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_b - V_c]$$

$$= \frac{V_d}{2}[1-2\alpha] + [-\frac{1}{2} - \frac{1}{2} + \alpha]V_c + [-\frac{1}{2} + \frac{1}{2} - \alpha]V_b$$

$$= \frac{V_d}{2}[1-2\alpha] + V_c[\alpha-1] - \alpha V_b$$

$$= \frac{V_d}{2}[1-2\alpha] + V_{\max}[\alpha-1] - \alpha V_{\min}$$

Sector III.

$$V_{no} = \frac{1}{6}[2V_b - V_a - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_c]$$

$$= \frac{1}{6}[-2V_a - 2V_c - V_a - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_c]$$

$$= \frac{-1}{2}[V_a + V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_c]$$

$$= \frac{V_d}{2}[1-2\alpha] + [-\frac{1}{2} - \frac{1}{2} + \alpha]V_c + [-\frac{1}{2} + \frac{1}{2} - \alpha]V_a$$

$$= \frac{V_d}{2}[1-2\alpha] + V_c[\alpha - 1] - \alpha V_a$$

$$= \frac{V_d}{2}[1-2\alpha] + V_{\max}[\alpha - 1] - \alpha V_{\min}$$

Sector IV.

$$V_{no} = \frac{1}{6}[2V_c - V_a - V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_b]$$

$$= \frac{1}{6}[-2V_a - 2V_b - V_a - V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_b]$$

$$= \frac{-1}{2}[V_a + V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_a - V_b]$$

$$= \frac{V_d}{2}[1-2\alpha] + [-\frac{1}{2} - \frac{1}{2} + \alpha]V_b + [-\frac{1}{2} + \frac{1}{2} - \alpha]V_a$$

$$= \frac{V_d}{2}[1-2\alpha] + V_b[\alpha - 1] - \alpha V_a$$

$$= \frac{V_d}{2}[1-2\alpha] + V_{\max}[\alpha - 1] - \alpha V_{\min}$$

Sector V.

$$V_{no} = \frac{1}{6}[2V_a - V_b - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_b]$$

$$= \frac{1}{6}[-2V_b - 2V_c - V_b - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_b]$$

$$= \frac{-1}{2}[V_b + V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_b]$$

$$= \frac{V_d}{2}[1-2\alpha] + [-\frac{1}{2} - \frac{1}{2} + \alpha]V_b + [-\frac{1}{2} + \frac{1}{2} - \alpha]V_c$$

$$= \frac{V_d}{2}[1-2\alpha] + V_b[\alpha - 1] - \alpha V_c$$

$$= \frac{V_d}{2}[1-2\alpha] + V_{\max}[\alpha - 1] - \alpha V_{\min}$$

Sector VI.

$$V_{no} = \frac{1}{6}[2V_b - V_a - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_a]$$

$$= \frac{1}{6}[-2V_b - 2V_c - V_a - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_a]$$

$$= \frac{-1}{2}[V_a + V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[V_c - V_a]$$

$$= \frac{V_d}{2}[1-2\alpha] + [-\frac{1}{2} - \frac{1}{2} + \alpha]V_a + [-\frac{1}{2} + \frac{1}{2} - \alpha]V_c$$

$$= \frac{V_d}{2}[1-2\alpha] + V_a[\alpha - 1] - \alpha V_c$$

$$= \frac{V_d}{2}[1-2\alpha] + V_{\max}[\alpha - 1] - \alpha V_{\min}$$

This expression being the same for all the sectors, is a generalized expression [3.20] [3.14] for the zero sequence voltage.

Substituting this V_{no} in (3.30) gives the carrier based SVPWM scheme.

$$M_{ap} = \frac{V_{an}}{V_d/2} + (1-2\alpha) + \frac{V_{\max}(\alpha-1) - \alpha V_{\min}}{V_d/2}$$

$$M_{bp} = \frac{V_{bn}}{V_d/2} + (1-2\alpha) + \frac{V_{\max}(\alpha-1) - \alpha V_{\min}}{V_d/2}$$

$$M_{cp} = \frac{V_{cn}}{V_d/2} + (1-2\alpha) + \frac{V_{\max}(\alpha-1) - \alpha V_{\min}}{V_d/2} \tag{3.38}$$

The expressions in 3.38 can be used for synthesizing the reference signals for carrier based discontinuous modulating scheme.

Alternatively using the theory of existence functions, the equations for discontinuous modulating signals can be derived. It will be shown in the preceding section that both the methods yield same results.

3.6 Carrier based implementation with injection of zero sequence signal using theory of existence functions

Figure 3.7 shows the existence functions of the three top devices of the inverter when operating in the first sector and the available times for each device. It is observed from Figure 3.9 that the average (the first term of the Fourier series expansion) of an existence function is equal to the sum of the normalized times each device is turned on to realize a reference voltage. These existence functions are obtained by using the sequence 111→110→100→000→000→100→110→111. Thus the switching functions in all the six sectors are shown below in Figure 3.8 and Figure 3.9

Sap	Sbp	Scp	
1	1	1	t_7
1	1	0	t_0
1	0	0	t_a
0	0	0	t_b

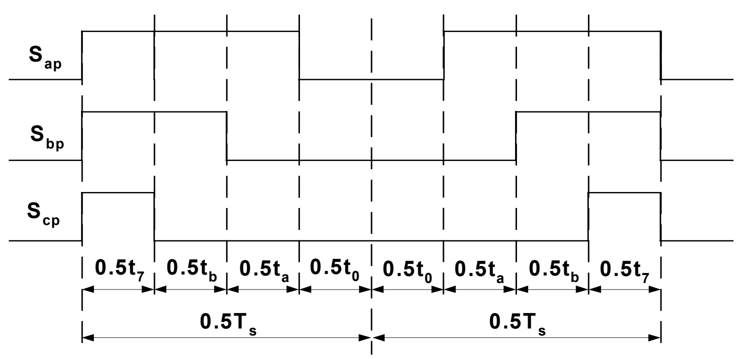
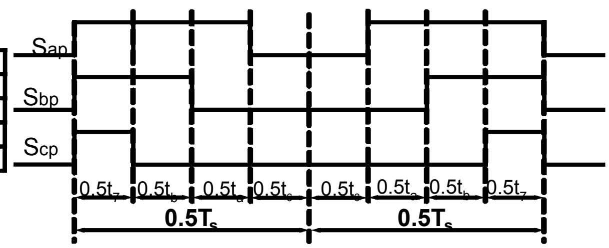


Figure 3.7 Existence functions of top devices for operation in sector I

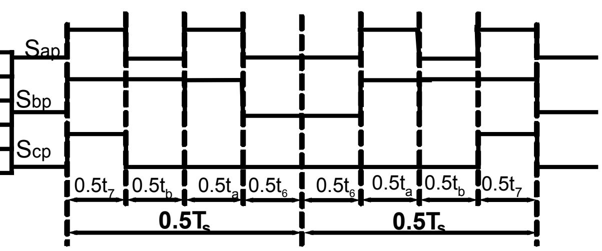
Sector I

Sap	Sbp	Scp	
1	1	1	t7
1	1	0	tb
1	0	0	ta
0	0	0	to



Sector II

Sap	Sbp	Scp	
1	1	1	t7
0	1	0	tb
1	1	0	ta
0	0	0	to



Sector III

Sap	Sbp	Scp	
1	1	1	t7
0	1	1	tb
0	1	0	ta
0	0	0	to

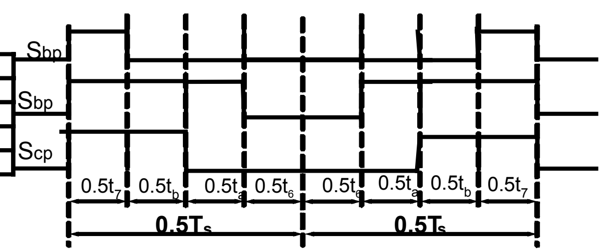
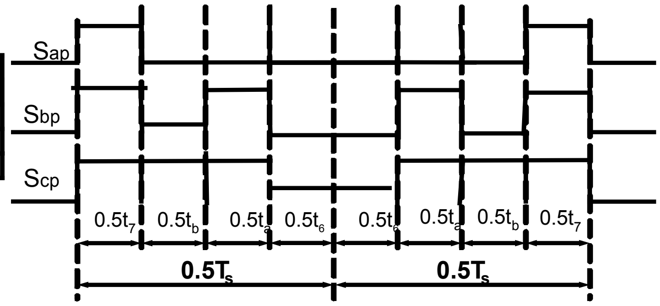


Figure 3.8 Existence functions of top devices for operation in sector I, II and III

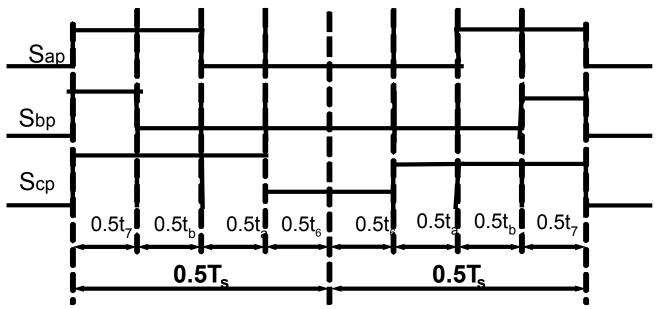
Sector IV

Sap	Sbp	Scp	
1	1	1	t7
0	0	1	tb
0	1	1	ta
0	0	0	to



Sector V

Sap	Sbp	Scp	
1	1	1	t7
1	0	1	tb
0	0	1	ta
0	0	0	to



Sector VI

Sap	Sbp	Scp	
1	1	1	t7
1	0	0	tb
1	0	1	ta
0	0	0	to

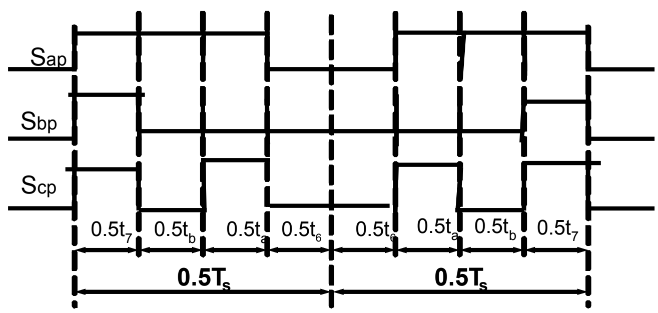


Figure 3.9 Existence functions of top devices for operation in sector IV, V and VI

Table 3.7 Normalized times for the top devices

	I	II	III	IV	V	VI
Z_{ap}	$t_a + t_b + t_c$	$t_7 + t_a$	t_7	t_7	$t_6 + t_7$	$t_a + t_b + t_7$
Z_{bp}	$t_7 + t_b$	$t_a + t_b + t_7$	$t_a + t_b + t_7$	$t_7 + t_a$	t_7	t_7
Z_{cp}	t_7	t_7	$t_6 + t_7$	$t_a + t_b + t_7$	$t_a + t_b + t_7$	$t_7 + t_a$

Thus the normalized times for the top switching devices are expressed by adding active on times from Figures 3.8 and 3.9 are summarized as in Table 3.7

If we define $\beta = (1- \alpha)$ then it is notable that, $0 \le \beta = (1- \alpha) \le 1$ which when varied introduces different weights to the times the null switching modes are used.

3.6.1 Modulating signals expressions for each sector using existence functions

It is mandatory that $t_7 + t_o + t_a + t_b = 1$

Consider $t_7 = (1-\alpha)t_c$

$$= (1-\alpha)(1-t_a-t_b)$$

$$= (1-\alpha)(1-t_a-t_b) \tag{3.39}$$

consider sector I of Table 3.7

$$Z_{ap} = t_a + t_b + (1-\alpha)(1-(t_a+t_b))$$

From Table 3.4 we have

$$Z_{ap} = \frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d} + (1-\alpha)\left(1-(\frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_a})\right)$$

$$Z_{ap} = \frac{V_a - V_c}{V_d} + \frac{V_c - V_b}{V_d} + (1-\alpha)\left(1-(\frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d})\right) \text{ since } (1-\alpha) = \beta$$

$$Z_{ap} = V_{abn} + \beta(1-V_{abn}) \text{ where } V_{abn} = \frac{V_{ab}}{V_d} \tag{3.40}$$

similarly we have

$$Z_{bp} = t_7 + t_b$$

$$Z_{bp} = t_b + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{bp} = \frac{V_{cb}}{V_d} + (1-\alpha)\left(1-(\frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d})\right)$$

$$Z_{bp} = \frac{V_{cb}}{V_d} + \beta(1-V_{abn}) = V_{cbn} + \beta(1-V_{abn}) \text{ where } V_{cbn} = \frac{V_{cb}}{V_d}$$

similarly we have

$$Z_{cp} = t_7$$

$$Z_{cp} = (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = (1-\alpha)\left(1-(\frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d})\right)$$

$$Z_{cp} = \beta(1-V_{abn}) \tag{3.41}$$

So we have the expressions for existence function in sector I can be written as

$$Z_{ap} = V_{abn} + \beta(1 - V_{abn})$$

$$Z_{bp} = V_{bcn} + \beta(1 - V_{abn})$$

$$Z_{cp} = \beta(1 - V_{abn}) \qquad (3.42)$$

Sector II of Table 3.7

$$Z_{ap} = t_a + (1-\alpha)(1-(t_a + t_b))$$

From Table 3.4 we have

$$Z_{ap} = \frac{V_{ab}}{V_d} + (1-\alpha)\left(1 - (\frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d})\right) \text{ since } (1-\alpha) = \beta$$

$$Z_{ap} = V_{abn} + \beta(1 - V_{cbn}) \text{ Where } V_{abn} = \frac{V_{ab}}{V_d} \qquad (3.42)$$

Similarly we have

$$Z_{bp} = t_a + t_b + t_7$$

$$Z_{bp} = t_a + t_b + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{ap} = \frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d} + (1-\alpha)\left(1 - (\frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d})\right)$$

$$Z_{ap} = \frac{V_a - V_b}{V_d} + \frac{V_c - V_a}{V_d} + (1-\alpha)\left(1 - (\frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d})\right)$$

$$Z_{bp} = \frac{V_{cb}}{V_d} + \beta(1 - V_{cbn}) = V_{cbn} + \beta(1 - V_{abn}) \text{ where } V_{cbn} = \frac{V_{cb}}{V_d}$$

Similarly we have

$$Z_{cp} = t_7$$

$$Z_{cp} = (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = (1-\alpha)\left(1-(\frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d})\right)$$

$$Z_{cp} = \beta(1-V_{cbn}) \text{ where } V_{cbn} = \frac{V_{cb}}{V_d} \qquad (3.43)$$

So we have the expressions for existence function in sector II can be written as

$$Z_{ap} = V_{abn} + \beta(1-V_{cbn})$$

$$Z_{bp} = V_{cbn} + \beta(1-V_{cbn})$$

$$Z_{cp} = \beta(1-V_{cbn})$$

Sector III of Table 3.7

$$Z_{ap} = t_7$$

$$Z_{ap} = (1-\alpha)(1-(t_a + t_b))$$

From Table 3.4 we have

$$Z_{ap} = (1-\alpha)\left(1-(\frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d})\right) \text{since } (1-\alpha) = \beta$$

$$Z_{ap} = \beta(1-V_{can}) \text{ where } V_{cac} = \frac{V_{ca}}{V_d} \qquad (3.44)$$

Similarly we have

$$Z_{bp} = t_a + t_b + t_7$$

$$Z_{bp} = t_a + t_b + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{bp} = \frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d} + (1-\alpha)\left(1-(\frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d})\right)$$

$$Z_{bp} = \frac{V_c - V_b}{V_d} + \frac{V_b - V_a}{V_d} + (1-\alpha)\left(1-(\frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d})\right)$$

$$Z_{bp} = \frac{V_{ca}}{V_d} + \beta(1-V_{can}) = V_{can} + \beta(1-V_{can}) \text{ where } V_{can} = \frac{V_{ca}}{V_d}$$

Similarly we have

$$Z_{cp} = t_b + t_7$$

$$Z_{cp} = \frac{V_{ba}}{V_d} + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = \frac{V_{ba}}{V_d} + (1-\alpha)\left(1-(\frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d})\right)$$

$$Z_{cp} = V_{ban} + \beta(1-V_{czn}) \text{ where } V_{ban} = \frac{V_{ba}}{V_d} \tag{3.45}$$

So we have the expressions for existence function in sector III can be written as

$$Z_{ap} = \beta(1-V_{can})$$

$$Z_{bp} = V_{can} + \beta(1-V_{can})$$

$$Z_{cp} = V_{ban} + \beta(1-V_{czn})$$

Sector IV of Table 3.7

$$Z_{ap} = t_7$$

$$Z_{ap} = (1-\alpha)(1-(t_a + t_b))$$

From Table 3.4 we have

$$Z_{ap} = (1-\alpha)\left(1-(\frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d})\right) \text{ since } (1-\alpha) = \beta$$

$$Z_{ap} = \beta(1-V_{ban}) \text{ where } V_{bac} = \frac{V_{ba}}{V_d} \tag{3.46}$$

Similarly we have

$$Z_{bp} = t_a + t_7$$

$$Z_{bp} = t_a + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{bp} = \frac{V_{ca}}{V_d} + (1-\alpha)\left(1-(\frac{V_{cs}}{V_d} + \frac{V_{bc}}{V_d})\right)$$

$$Z_{bp} = \frac{V_{ca}}{V_d} + \beta(1-V_{ban}) = V_{can} + \beta(1-V_{ban}) \text{ where } V_{can} = \frac{V_{ca}}{V_d}$$

Similarly we have

$$Z_{cp} = t_a + t_b + t_7$$

$$Z_{cp} = \frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d} + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = \frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d} + (1-\alpha)\left(1-(\frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d})\right)$$

$$Z_{cp} = V_{ban} + \beta(1 - V_{ban}) \text{ where } V_{ban} = \frac{V_{ba}}{V_d} \tag{3.47}$$

So we have the expressions for existence function in sector IV can be written as

$$Z_{ap} = \beta(1 - V_{ban})$$

$$Z_{bp} = V_{can} + \beta(1 - V_{ban})$$

$$Z_{cp} = V_{ban} + \beta(1 - V_{ban})$$

Consider sector V of Table 3.7

$$Z_{ap} = t_b + t_7$$

$$Z_{ap} = t_b + (1 - \alpha)(1 - (t_a + t_b))$$

From Table 3.4 we have

$$Z_{ap} = \frac{V_{ac}}{V_d} + (1 - \alpha)\left(1 - (\frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d})\right) \text{since } (1 - \alpha) = \beta$$

$$Z_{ap} = V_{acn} + \beta(1 - V_{bcn}) \text{ where } V_{acn} = \frac{V_{ac}}{V_d} \tag{3.48}$$

Similarly we have

$$Z_{bp} = t_7$$

$$Z_{bp} = (1 - \alpha)(1 - (t_a + t_b))$$

$$Z_{bp} = (1 - \alpha)\left(1 - (\frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d})\right)$$

$$Z_{bp} = \beta(1 - V_{bcn}) \text{ were } V_{can} = \frac{V_{bcn}}{V_d}$$

Similarly we have

$$Z_{cp} = t_a + t_b + t_7$$

$$Z_{cp} = \frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d} + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = \frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d} + (1-\alpha)\left(1 - (\frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d})\right)$$

$$Z_{cp} = V_{bcn} + \beta(1 - V_{bcn}) \text{ were } V_{bcn} = \frac{V_{bcn}}{V_d} \qquad (3.49)$$

So we have the expressions for existence function in sector V can be written as

$$Z_{ap} = V_{acn} + \beta(1 - V_{bcn})$$

$$Z_{bp} = \beta(1 - V_{bcn})$$

$$Z_{cp} = V_{bcn} + \beta(1 - V_{bcn})$$

Sector VI of Table 3.7

$$Z_{ap} = t_a + t_b + t_7$$

$$Z_{ap} = t_a + t_b + (1-\alpha)(1-(t_a + t_b))$$

From Table 3.4 we have

$$Z_{ap} = \frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d} + (1-\alpha)\left(1 - (\frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d})\right) \text{ since } (1-\alpha) = \beta$$

$$Z_{ap} = V_{acn} + \beta(1 - V_{acn}) \tag{3.41}$$

Where $V_{acn} = \dfrac{V_{ac}}{V_d}$

Similarly we have

$$Z_{bp} = t_7$$

$$Z_{bp} = (1-\alpha)(1-(t_a + t_b))$$

$$Z_{bp} = (1-\alpha)\left(1-(\frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d})\right)$$

$$Z_{bp} = \beta(1 - V_{acn}) \text{ where } V_{acn} = \frac{V_{acn}}{V_d}$$

Similarly we have

$$Z_{cp} = t_a + t_7$$

$$Z_{cp} = \frac{V_{bc}}{V_d} + (1-\alpha)(1-(t_a + t_b))$$

$$Z_{cp} = \frac{V_{bc}}{V_d} + (1-\alpha)\left(1-(\frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d})\right)$$

$$Z_{cp} = V_{bcn} + \beta(1 - V_{acn}) \text{ where } V_{bcn} = \frac{V_{bcn}}{V_d} \tag{3.43}$$

So we have the expressions for existence function in sector VI can be written as

$$Z_{ap} = V_{acn} + \beta(1 - V_{acn})$$

$$Z_{bp} = \beta(1 - V_{acn})$$

$Z_{cp} = V_{bcn} + \beta(1 - V_{acn})$

The expressions for all the sectors is summarized as follows

Table 3.8 Discontinuous existence functions for top devices with star-connected load, $V_{ijn} = V_{ij}/V_d$, where i, j = a,b,c and i ≠ j . (β = 1- α)

Sector	Z_{ap}	Z_{bp}	Z_{cp}
VI	$\beta(1-V_{acn})+V_{acn}$	$\beta(1-V_{acn})$	$\beta(1-V_{acn})+V_{bcn}$
V	$\beta(1-V_{bcn})+V_{acn}$	$\beta(1-V_{bcn})$	$\beta(1-V_{bcn})+V_{bcn}$
IV	$\beta(1-V_{ban})$	$\beta(1-V_{ban})+V_{can}$	$\beta(1-V_{ban})+V_{ban}$
III	$\beta(1-V_{can})$	$\beta(1-V_{can})+V_{can}$	$\beta(1-V_{acn})+V_{ban}$
II	$\beta(1-V_{cbn})+V_{bcn}$	$\beta(1-V_{cbn})+V_{cba}$	$\beta(1-V_{cbn})$
I	$\beta(1-V_{abc})+V_{abn}$	$\beta(1-V_{abn})+V_{can}$	$\beta(1-V_{abn})$

3.6.2 Observations on the obtained schemes

Various kinds of GDPWM waveforms, which have been reported in the literature [3.10-3.19] including DPWMIN, DPWMMAX, DPWM1, DPWM2 and DPWM3, can be generated using either equation 3.38 or Table 3.8 for an three phase VSI inverter. When β = 1, each top inverter leg connected to a phase is clamped to the upper rail of the DC source for 120 degrees and the lower inverter leg is clamped to the lower rail of the DC source when β = 0. When β = 0 and β = 1, DPWMMAX and DPWMMIN are obtained, respectively and $\beta = 0.5$, gives the SVPWM. With the definition $\beta = 0.5[1 + \text{Sgn}(\text{Cos } 3(\omega t + \delta))]$ where ω is the angular frequency of the reference voltage and δ is the modulation phase angle, an infinite number of modulation waveforms can be generated. If $\delta = 0, -\pi/6, -\pi/3$, the resulting modulation signals are the same as the DPWM1, DPWM2, and DPWM3 respectively. Figures 3.13 through 3.15 shows the comparison of the modulating signal and the corresponding gating (switching) signal. It can be noted that the device is clamped effectively for 120 degrees in a cycle for all the modulators. Thus a varied switching loss in the device can be anticipated for different load power factors. The modulator has to be chosen in order to minimize the switching loss for the given load power factor. The characteristics of these modulators have been already studied in terms of switching loss, harmonic loss factor or the distortion factor [3.10-3.19]. The superior performance of the DPWM1, DPWM2 and DPWM3 in higher modulation region has been explicated in [3.10] [3.11] [3.17]. The method for achieving these waveforms was by injection of zero sequence voltages through generalized expression of V_{no}.

The method for generation of zero sequence voltage through theory of existence function gives the same results, which are shown in the following figures.

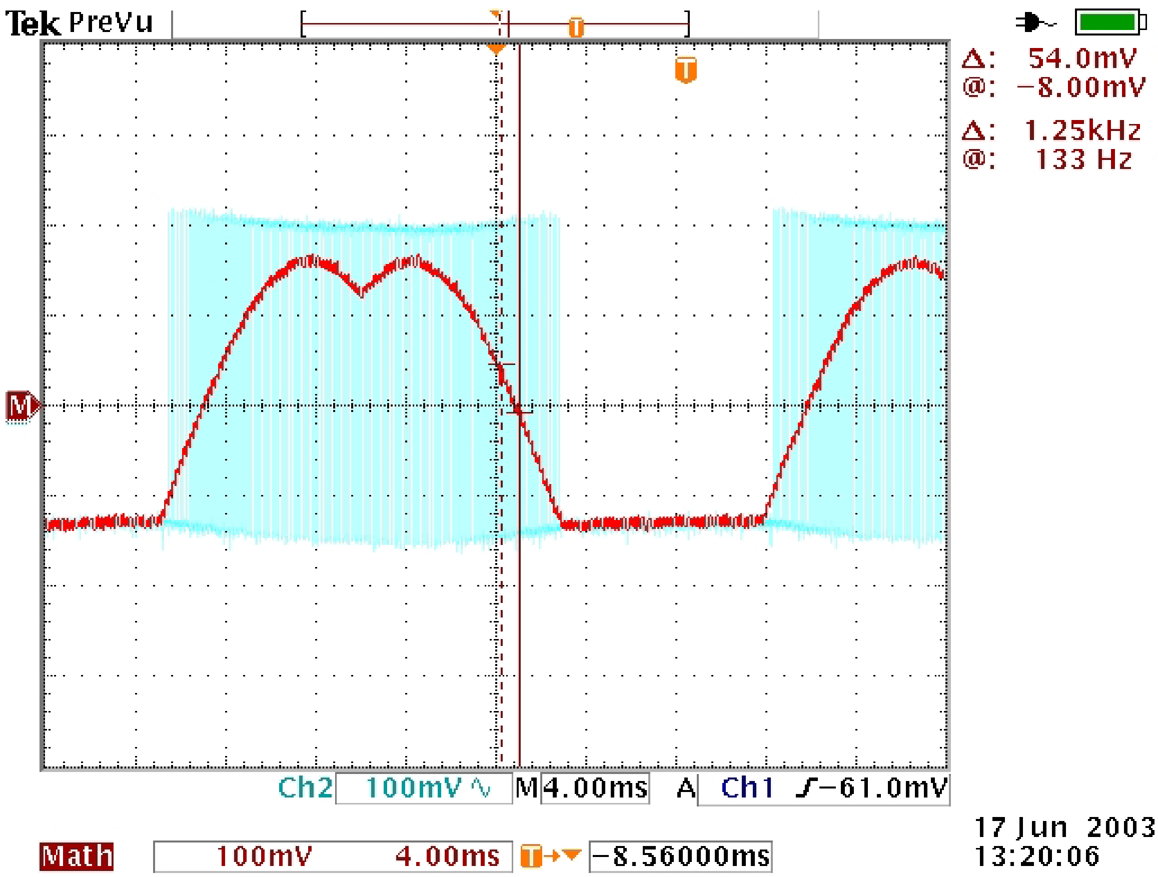


Figure 3.10 Generation of Switching function for DPWMMIN when β = 0

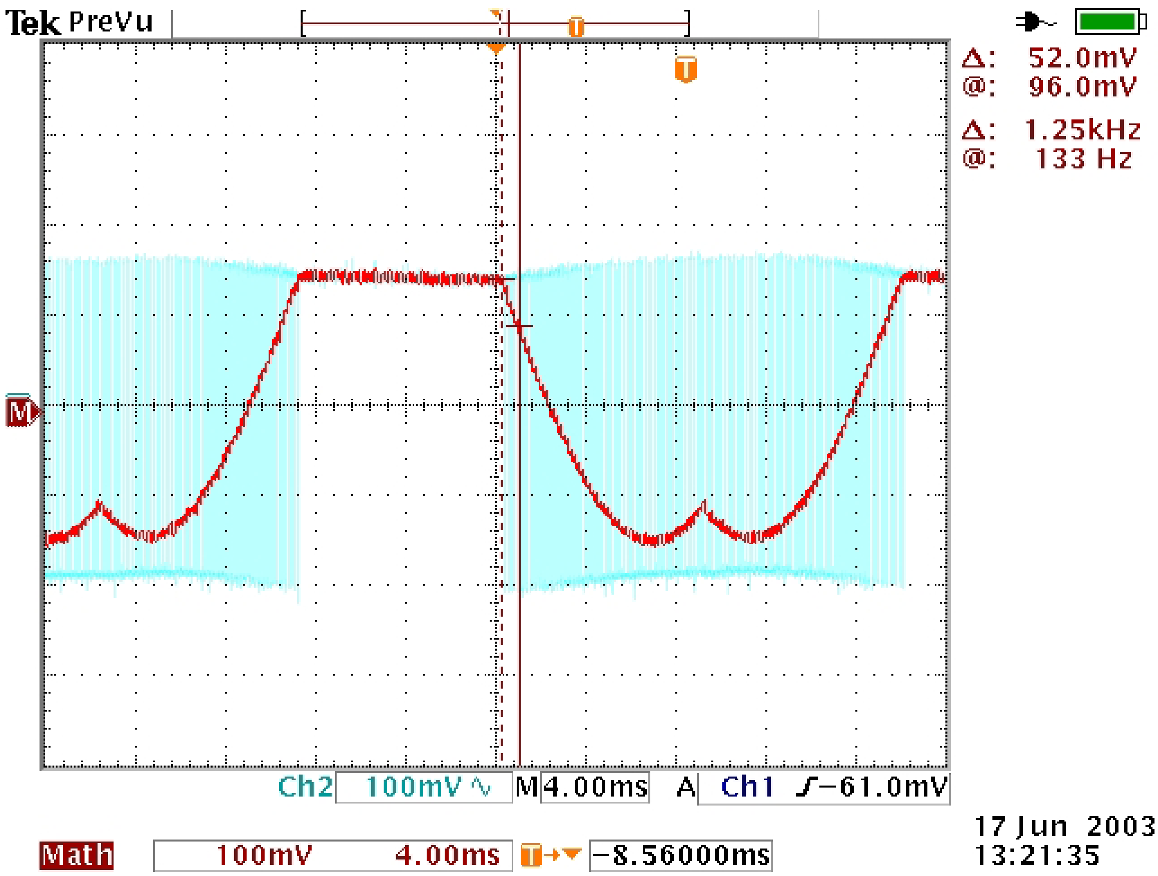


Figure 3.11 Generation of Switching function for DPWMMAX when β = 1

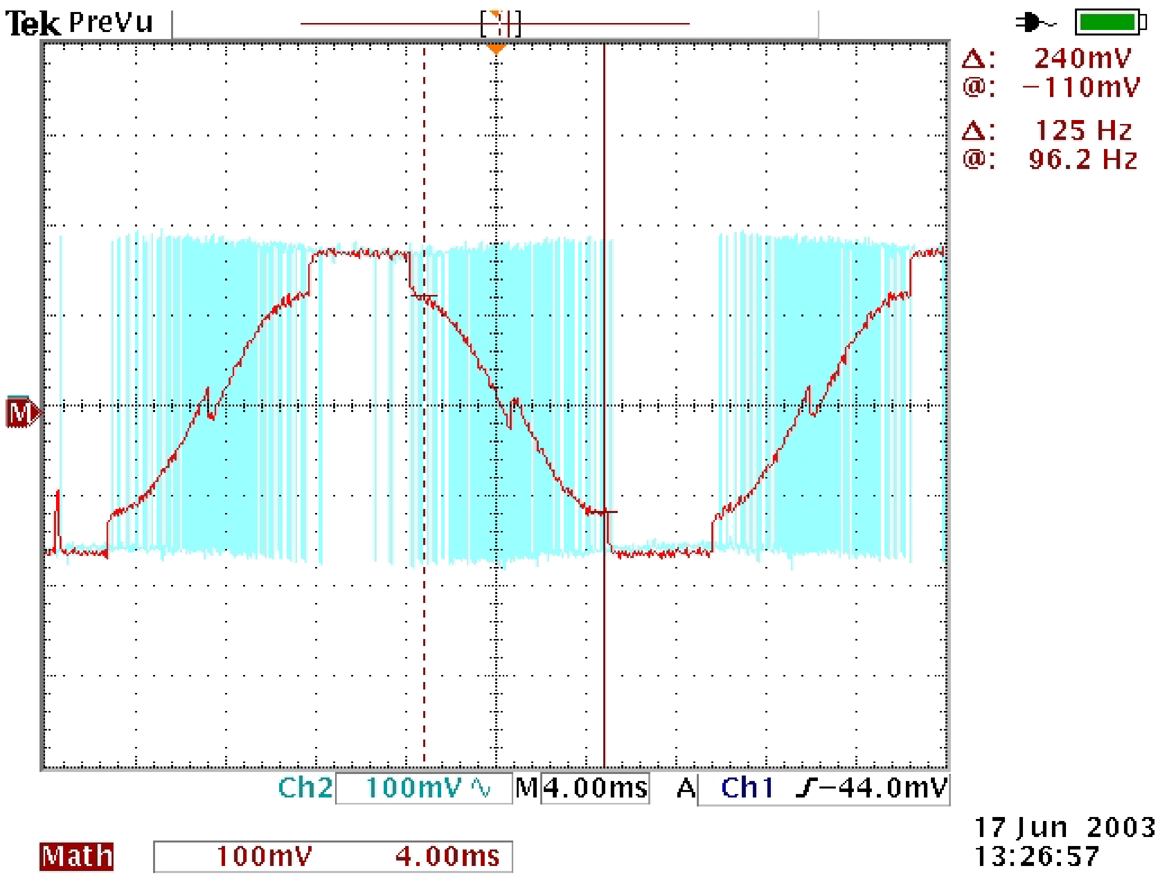


Figure 3.12 Generation of Switching function for DPWM1 when, β = 0.5[1 + Sgn(Cos 3(ωt + δ))] and δ = 0

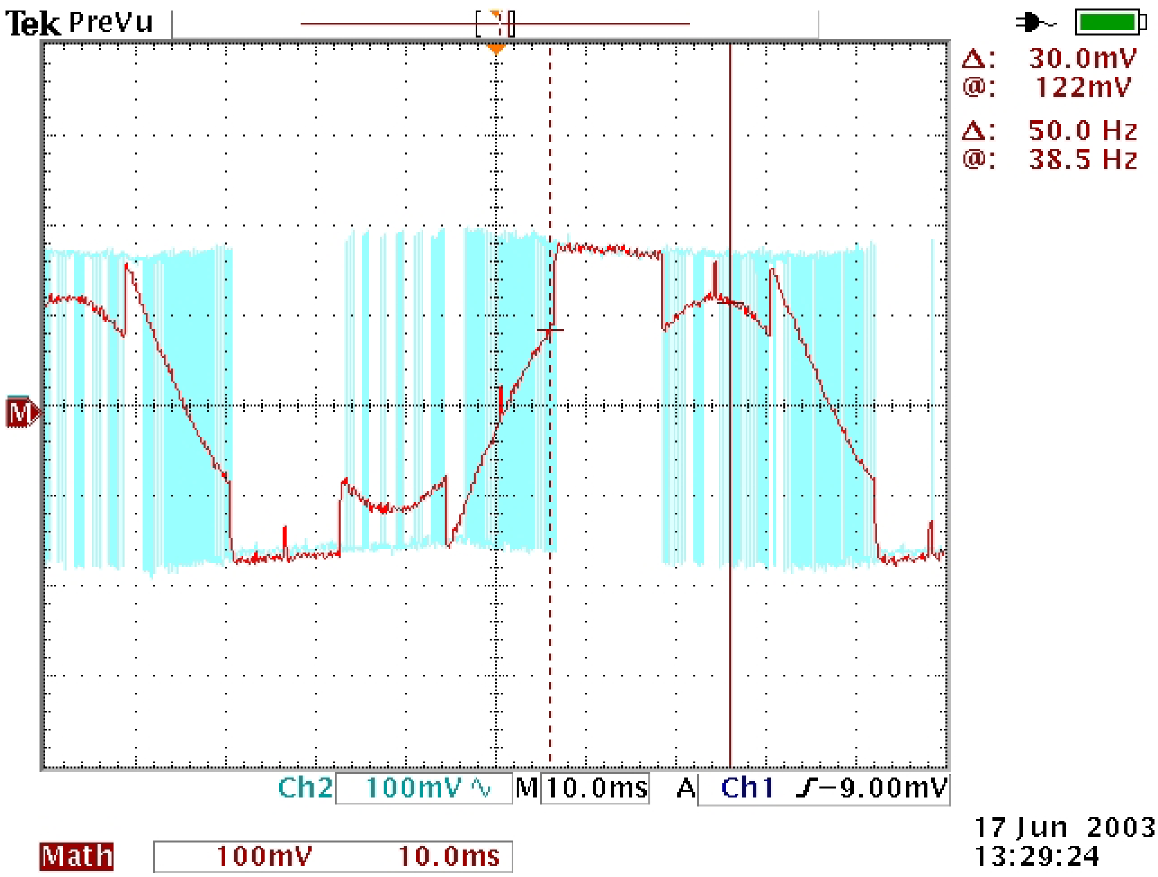


Figure 3.13 Generation of Switching function for DPWM2 when, β = 0.5[1 + Sgn(Cos 3(ωt + δ))] and δ = -π/6

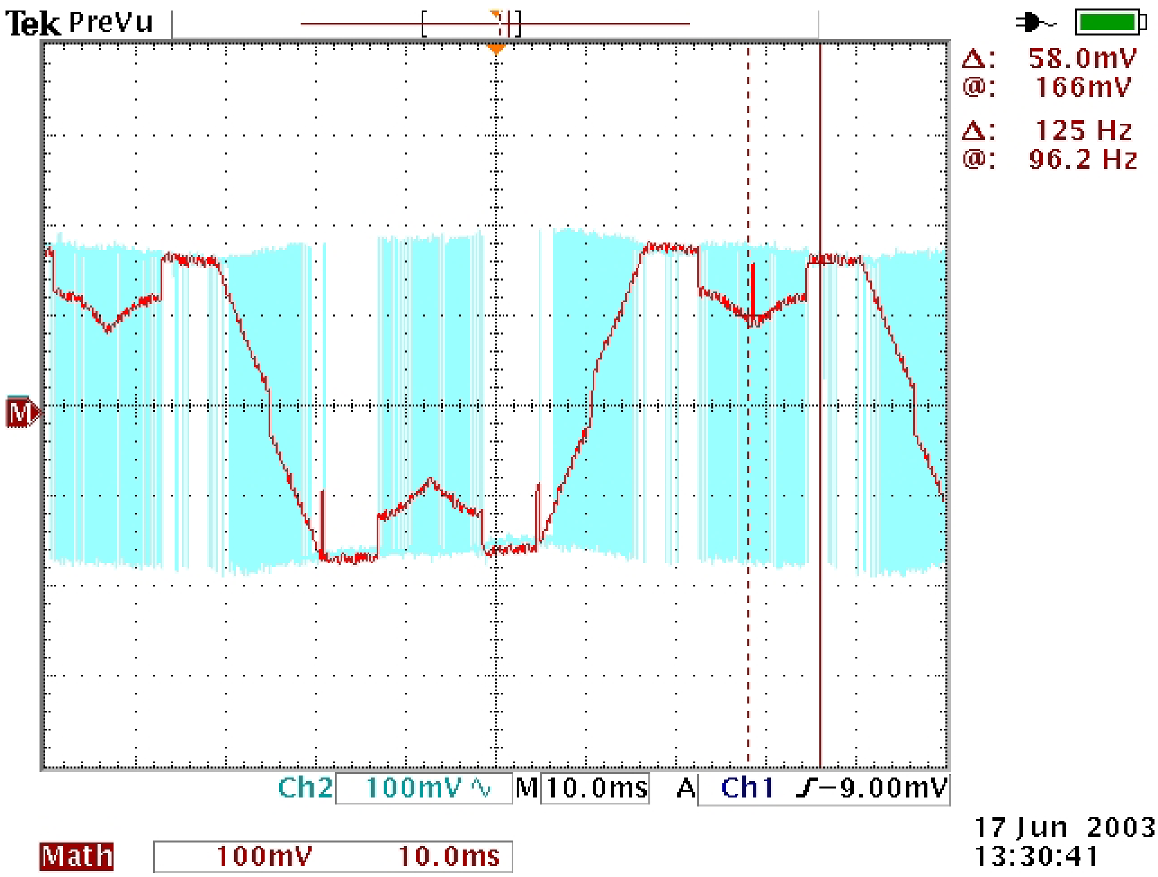
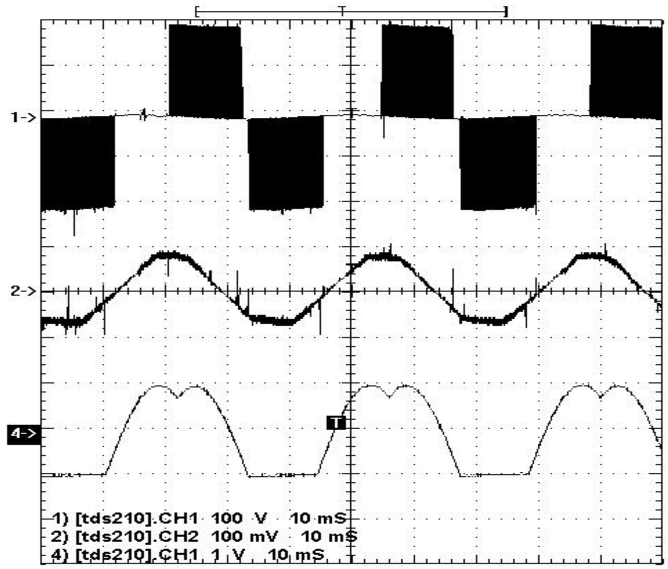


Figure 3.14 Generation of Switching function for DPWM3 when,

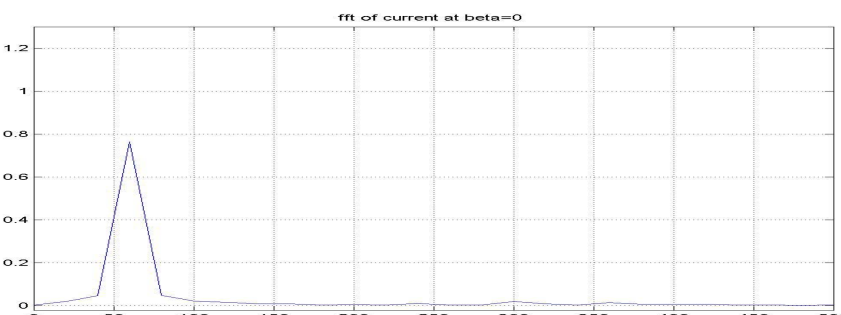
$\beta = 0.5[1 + \text{Sgn}(\text{Cos } 3(\omega t + \delta))]$ and $\delta = -\pi/3$

3.7 Experimental results

Illustrative experimental results are given in Figures 3.15 through 3.20 showing the nature of the discontinuous modulation waveforms and the corresponding voltage and current waveforms along with the FFT of the current waveforms. The load applied was a 1-hp three-phase induction machine at no load. From these Figures, the relationships between the modulation schemes derived in this paper and those already reported in the literature are established. The FFT of current shows the harmonic content generated by various modulators for the same load power factor.

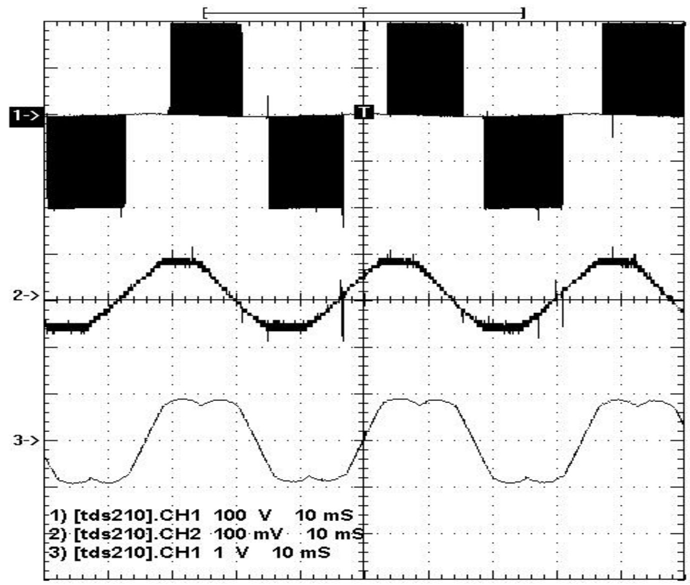


(a)

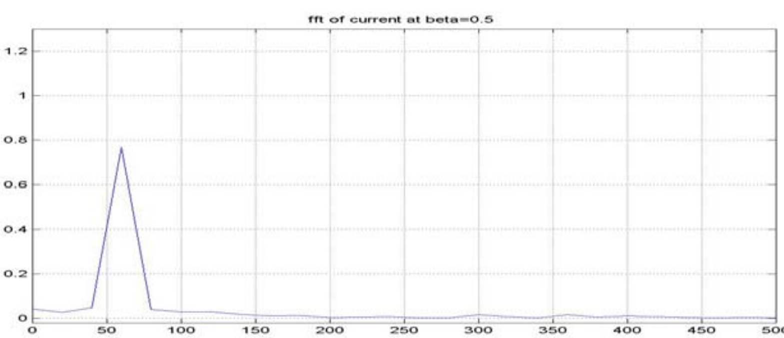


(b)

Figure 3.15 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz., Modulation magnitude = 0.9 (a) (1) Motor line-line voltage, (2) motor phase current. (4) β = 0, DPWMMAX (b) FFT of the phase current

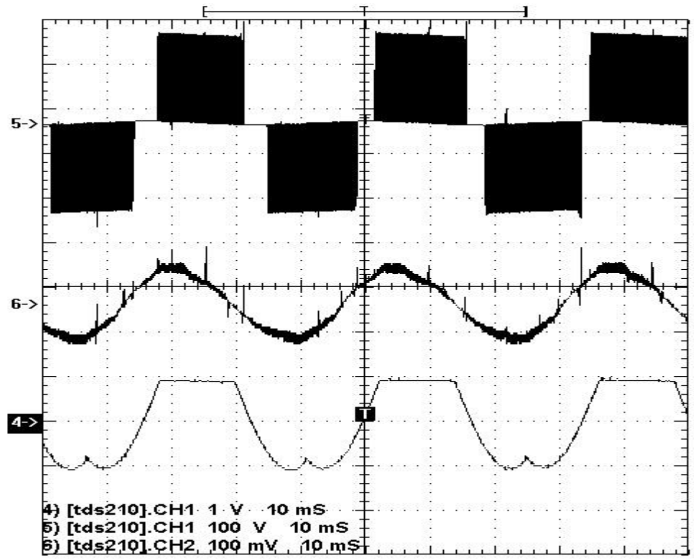


(a)

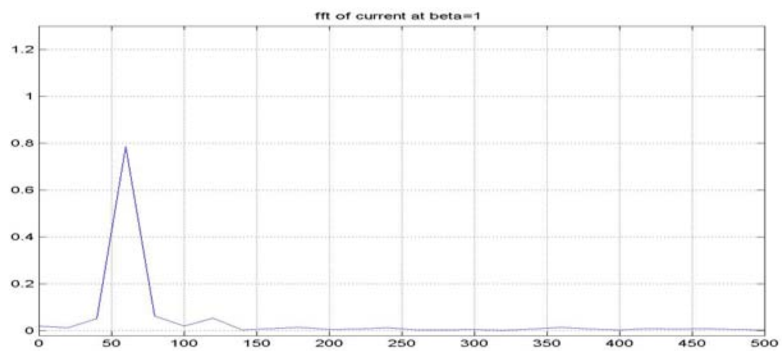


(b)

Figure 3.16 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz., Modulation magnitude = 0.9 (a) (1) Motor line-line voltage, (2) motor phase current, (b) β = 0.5, SVPWM, (b) FFT of the phase current

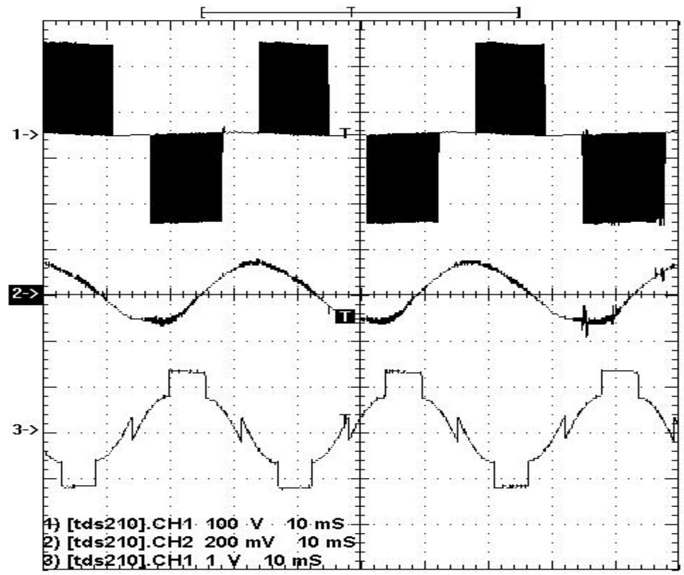


(a)

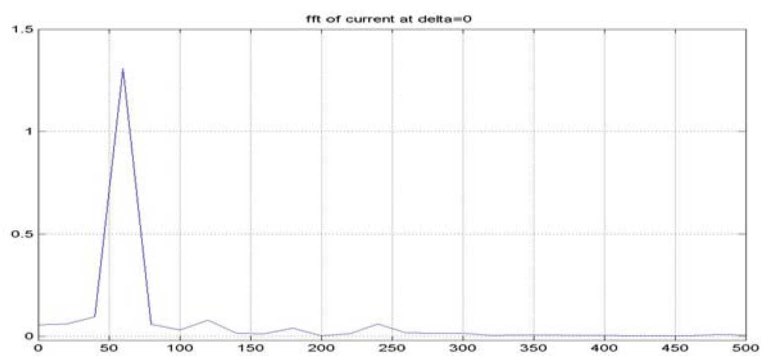


(b)

Figure 3.17 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz., Modulation magnitude = 0.9 (1) Motor line-line voltage, (2) motor phase current. β =1.0., DPWMMIN. (b) FFT of the phase current

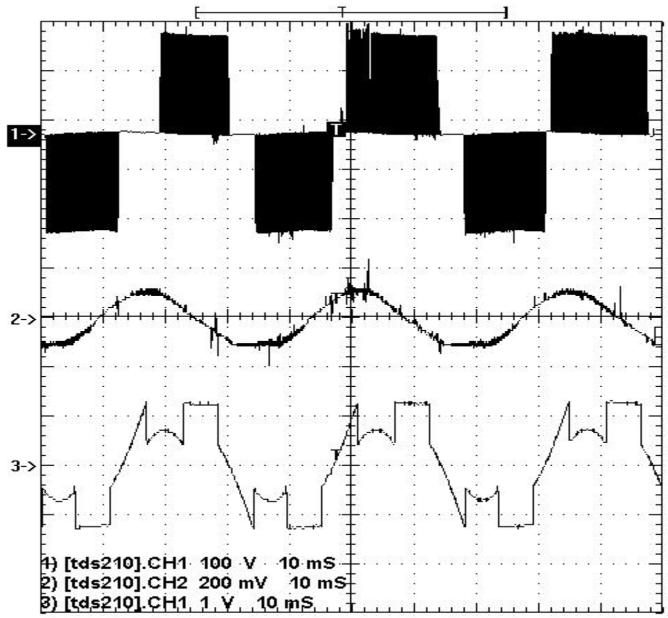


(a)

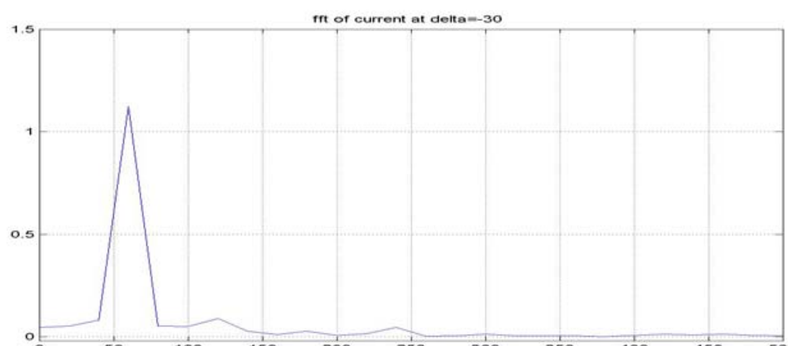


(b)

Figure 3.18 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz, modulation magnitude = 0.9 (a) (1) motor line-line voltage, (2) motor phase current. $\beta = 0.5[1 + \text{SgnCos } 3(\omega t + \delta)]$, $\delta = 0$, DPWM1 (b) FFT of the phase current

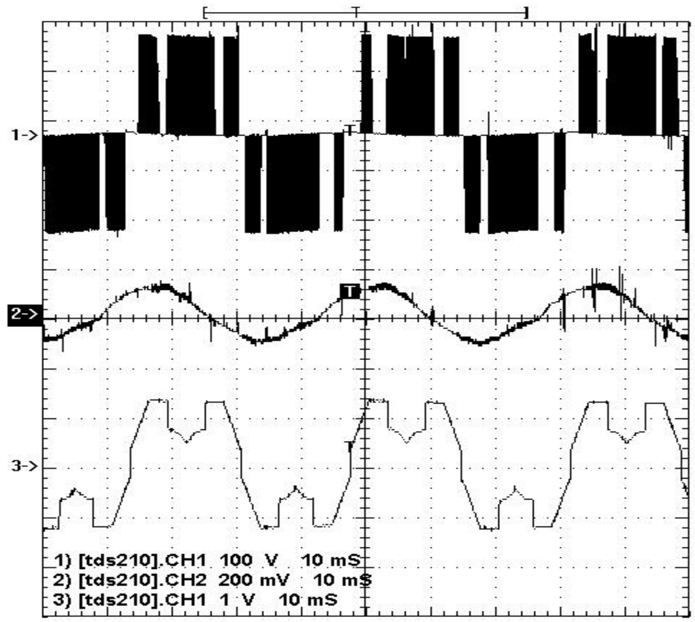


(a)

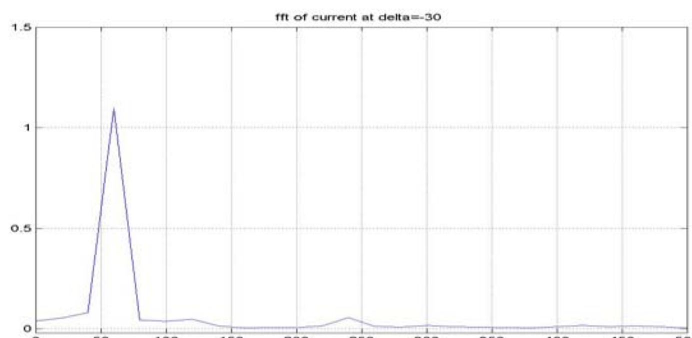


(b)

Figure 3.19 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz, modulation magnitude = 0.9 (1) motor line-line voltage, (2) motor phase current. $\beta = 0.5[1 + \text{Sgn}\text{Cos } 3(\omega t + \delta)]$, $\delta =$ = -30°, DPWM2, (b) FFT of the phase current



(a)



(b)

Figure 3.20 Experimental results for three-phase inverter under GDPWM modulation feeding an induction motor on no-load. V_d = 200V, frequency = 30 Hz, modulation magnitude = 0.9 (1) motor line-line voltage, (2) motor phase current. $\beta = 0.5[1 + \text{SgnCos } 3(\omega t + \delta)]$, $\delta = -60°$, DPWM3. (b) FFT of the phase current

3.8 Modulation for unbalanced voltages

There are situations in which it is desirable to impress an unbalanced three-phase voltage set to an unbalanced three-phase load in order to ensure a balanced three-phase load current or to use unbalanced three-phase voltage set for voltage or current compensation in active filters in distribution lines. In general, four-leg inverters are used in such applications since the phase currents are not constrained when the load is star-connected. However, when the impressed unbalanced three-phase voltage set is constrained such that the load currents add to zero in star-connected loads, a three-leg inverter can be used. Under such conditions, the expressions for the three modulation signals M_{ip} must be determined given the phase voltages V_{an} , $V_{bn,}$ V_{cn} which are not balanced in general. Since there are three linear independent equations to be solved to determine expressions for three unknown modulation signals and V_{no}, these equations are under-determined.

In view of this indeterminacy, there are an infinite number of solutions, which are obtained by various optimizing performance functions defined in terms of the modulation functions. For a set of linear indeterminate equations expressed as AX = Y, a solution which minimizes the sum of squares of the variable X is obtained using the Moore-Penrose inverse [A.3].

From the matrix properties if A is a matrix of rank (r x n) then we know that the product form A^TA has the dimension (n x n) while the product AA^T has dimension of (r x r). If r > n, then A^TA could be nonsingular but AA^T is a singular matrix. Similarly if r < n, AA^T can be a nonsingular matrix but A^TA is a singular matrix.

The solution of under-determined case in which the dimension of the matrix A (r x n) where r < n has the matrix product of AA^T is nonsingular. Thus the pseudoinverse definition can be derived as follows:

$$AX = Y$$

Using the identity: $AA^T [AA^T]^{-1} = I$

We have the following expression,

$$AX = AA^T [AA^T]^{-1}Y$$

Which gives

$$X = A^T[AA^T]^{-1}Y$$

This solution is for the minimization of the sum of the squares of the three modulation signals and the square of the normalized neutral voltage ($V^*_{pn} = V_{pn}/0.5\ V_d$). Equivalently, this is the maximization of the inverter output-input voltage gain, i.e. $M_{ap}{}^2 + M_{bp}{}^2 + M_{cp}{}^2 + V^*_{no}{}^2$ subject to the constraints in (1). The result expressions for the modulation signals are given as [3.20]:

$M_{ap} = 1/4\ (3V_{ann} – V_{bnn} –V_{cnn})$, $V_{ann} = V_{an}/ 0.5V_d$

$M_{bp} = 1/4\ (-V_{ann} + 3V_{bnn} – V_{cnn})$, $V_{bnn} = V_{bn}/ 0.5V_d$

$M_{cp} = 1/4\ (-V_{ann} - V_{bnn} + 3V_{cnn})$, $V_{cnn} = V_{cn}/ 0.5V_d$

$V^*_{pn} = 1/4\ (-V_{ann} -V_{bnn} - V_{cnn})$ (3.44)

An alternative carrier based discontinuous modulation scheme is obtained by using the Space Vector methodology to determine the expression for V_{no} in (3.44). Since the reference voltage set is unbalanced, the reference three-phase voltages mapped to the stationary reference frame has in addition to the q and d voltage components the zero sequence voltage. The reference zero sequence voltage, V_o is approximated by time-averaging the zero sequence voltages of the two active and two null modes. From (3.44), the neutral voltage V_{no} averaged over the switching period T_s is given as:

$$\langle V_{no}\rangle = V_{oa}t_a + V_{ob}\, t_b + V_{o0}\, t_0 + V_{o7}\, t_7 - V_o \qquad (3.45)$$

Table 3.9 gives the expression for the averaged neutral voltage $\langle V_{no}\rangle$ for the six sectors of the space vector. Hence, given the unbalanced voltage set at any instant, V^*_{qdo} in the stationary reference frame is found and the sector in which V_{qd}^* is located is determined. The expression for V_{no} is then selected and is subsequently used in (3.10) to determine the modulation signals for the three top devices.

For unbalanced case the expression for each sector is derived as follows:

$$V_{no} = V_{oa}t_a + V_{ob}t_b + V_{oo}t_o + V_{o7}t_7$$

Sector I

$$V_{no} = \frac{V_d}{6}[t_b - t_a] + (1-\alpha)t_c \frac{V_d}{2}$$

$$= \frac{V_d}{6}[t_b - t_a] + \frac{V_d}{2}\left[-\alpha t_c + t_c - \alpha t_c\right]$$

$$= \frac{V_d}{6}[t_b - t_a] + \frac{V_d}{2}\left[1-2\alpha\right]t_c \qquad t_c = [1 - t_a - t_b]$$

$$= \frac{V_d}{6}[V_{cb} - V_{ac}] + \frac{V_d}{2}[1-2\alpha][V_d - V_{cb} - V_{ac}]$$

$$= \frac{V_d}{6}[V_c - V_b - V_a + V_c] + \frac{V_d}{2}\left[1-2\alpha\right]\left[V_d - V_c + V_b - V_a + V_c\right]$$

$$= \frac{1}{6}[2V_c - V_b - V_a] + \frac{V_d}{2}\left[1-2\alpha\right] + 0.5(1-2\alpha)[\left[V_b - V_a\right]$$

Sector II

$$V_{no} = \frac{V_d}{6}[t_a - t_b] + \frac{V_d}{2}(1-2\alpha)t_c$$

$$= \frac{V_d}{6}[\frac{V_{ab} - V_{ca}}{V_d}] + \frac{V_d}{2}(1-2\alpha)\left[\frac{V_d - V_{ab} - V_{ca}}{V_d}\right]$$

$$= \frac{1}{6}[V_a - V_b - V_c + V_a] + \frac{1}{2}\left[1-2\alpha\right]\left[V_d - V_a + V_b - V_c + V_a\right]$$

$$= \frac{1}{6}[2V_a - V_b - V_c] + \frac{1}{2}\left[1-2\alpha\right]\left[V_d - V_c + V_b\right]$$

$$= \frac{1}{6}[2V_a - V_b - V_c] + \frac{V_d}{2}\left[1-2\alpha\right] + 0.5(1-2\alpha)[\left[V_b - V_c\right]$$

Sector III

$$V_{no} = \frac{V_d}{6}[t_b - t_a] + \frac{V_d}{2}(1-2\alpha)t_c$$

$$= \frac{V_d}{6}[\frac{V_{ba} - V_{cb}}{V_d}] + \frac{V_d}{2}(1-2\alpha)\left[\frac{V_d - V_{ba} - V_{cb}}{V_d}\right]$$

$$= \frac{1}{6}[V_b - V_a - V_c + V_b] + \frac{1}{2}[1-2\alpha][V_d - V_b + V_a - V_c + V_b]$$

$$= \frac{1}{6}[2V_b - V_a - V_c] + \frac{1}{2}[1-2\alpha][V_d + V_a - V_c]$$

$$= \frac{1}{6}[2V_b - V_a - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[[V_a - V_c]$$

Sector IV

$$V_{no} = \frac{V_d}{6}[t_a - t_b] + \frac{V_d}{2}(1-2\alpha)t_c$$

$$= \frac{V_d}{6}[\frac{V_{xa} - V_{bc}}{V_d}] + \frac{V_d}{2}(1-2\alpha)\left[\frac{V_d - V_{ca} - V_{bc}}{V_d}\right]$$

$$= \frac{1}{6}[V_c - V_a - V_b + V_c] + \frac{1}{2}[1-2\alpha][V_d - V_c + V_a - V_v + V_c]$$

$$= \frac{1}{6}[2V_c - V_a - V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[[V_a - V_b]$$

Sector V

$$V_{no} = \frac{V_d}{6}[t_b - t_a] + \frac{V_d}{2}(1-2\alpha)t_c$$

$$= \frac{V_d}{6}[\frac{V_{ac} - V_{ba}}{V_d}] + \frac{V_d}{2}(1-2\alpha)\left[\frac{V_d - V_{ac} - V_{ba}}{V_d}\right]$$

$$= \frac{1}{6}[V_a - V_c - V_b + V_a] + \frac{1}{2}[1-2\alpha][V_d - V_a + V_c - V_b + V_a]$$

$$= \frac{1}{6}[2V_a - V_b - V_c] + \frac{1}{2}[1-2\alpha][V_d + V_c - V_b]$$

$$= \frac{1}{6}[2V_a - V_b - V_c] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[[V_c - V_b]$$

Sector VI

$$V_{no} = \frac{V_d}{6}[t_b - t_a] + \frac{V_d}{2}(1-2\alpha)t_c$$

$$= \frac{V_d}{6}[\frac{V_{bc} - V_{ab}}{V_d}] + \frac{V_d}{2}(1-2\alpha)\left[\frac{V_d - V_{bc} - V_{ab}}{V_d}\right]$$

$$= \frac{1}{6}[V_b - V_c - V_a + V_b] + \frac{1}{2}[1-2\alpha][V_d - V_b + V_c - V_a + V_b]$$

$$= \frac{1}{6}[2V_b - V_a - V_b] + \frac{V_d}{2}[1-2\alpha] + 0.5(1-2\alpha)[[V_c - V_a]$$

The above equations are simplified and summarized in Table 3.9

Table 3.9 : Expressions for the neutral voltage for the six sectors

Sector	**Neutral Voltage** $\langle V_{no} \rangle$
VI	$(V_b - 2V_a - 2V_c)/3 + 0.5V_d(1-2\alpha) - \alpha(V_c - V_a)$
V	$(V_a - 2V_b - 2V_c)/3 + 0.5V_d(1-2\alpha) - \alpha(V_c - V_b)$
IV	$(V_c - 2V_a - 2V_b)/3 + 0.5V_d(1-2\alpha) - \alpha(V_a - V_b)$
III	$(V_b - 2V_a - 2V_c)/3 + 0.5V_d(1-2\alpha) - \alpha(V_a - V_c)$
II	$(V_a - 2V_b - 2V_c)/3 + 0.5V_d(1-2\alpha) - \alpha(V_b - V_c)$
I	$(V_c - 2V_a - 2V_b)/3 + 0.5V_d(1-2\alpha) - \alpha(V_b - V_a)$

3.9 Simulation Results

Figures 3.21-3.25 shows the simulation results of an inverter feeding an unbalanced load with the desired objective of balancing the load current. The resistances of the R-L load used are r_a = 0.5 Ohm, r_b = 0.5 Ohm, r_c = 0.5 Ohm and the corresponding load inductances are L_a = 0.025H, L_b = 0.02H, L_c = 0.0125H. With given reference three-phase currents, the corresponding phase voltages are determined and used in equations (3.44) to realize the continuous modulation signals, which are subsequently used to generate the required voltages. The same balanced current set can be generated by using the discontinuous modulation scheme based on Table 3.9 and equation 3.30.

With $\beta = 0.5[1 + \mathrm{Sgn}(\mathrm{Cos}\ 3(\omega t + \delta))]$ where $\beta = 1- \alpha$, Figures 3.21 through 3.24 are generated for values of $\delta = 0°, -30°, -60°$. It is observed that devices are clamped to the positive or negative rail for less than 120 degrees unlike when the reference phase voltages are balanced as in Figures 3.15 through 3.20.

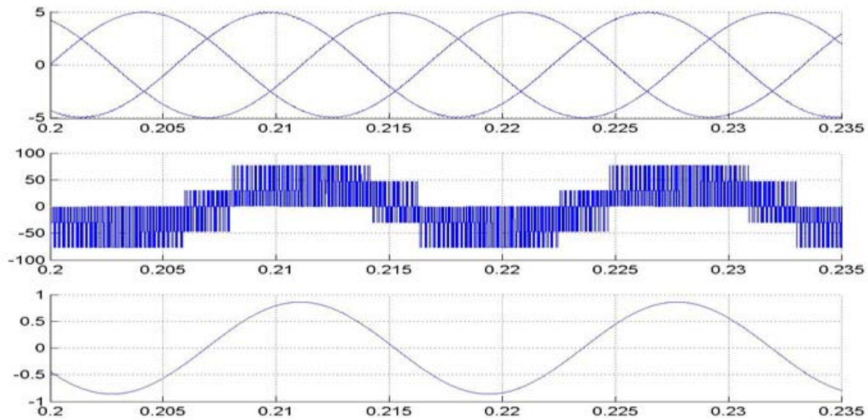


Figure 3.21 Balanced current in an unbalanced load. Reference peak current is 5A. (a) Balanced three phase actual currents (b) Phase a voltage, (c) Modulating signal.

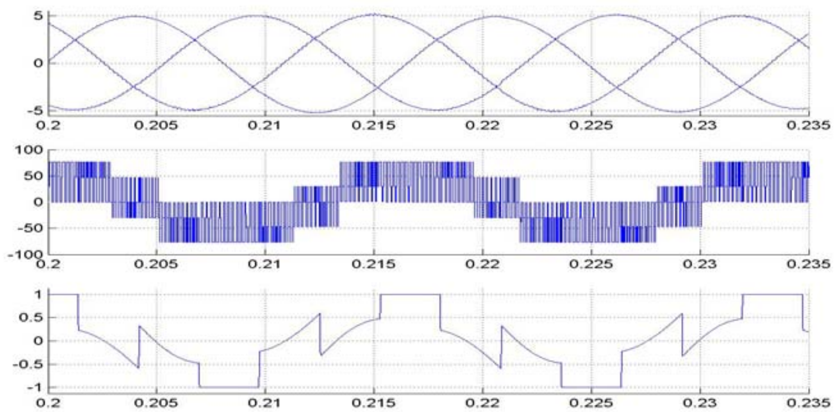


Figure 3.22 Simulation results for unbalanced three-phase voltage under GDPWM. β = 0.5[1 + SgnCos 3(ωt + δ)], δ = 0, DPWM1, (a) Balanced three-phase current (b) phase a voltage Vas (c) Modulation signal.

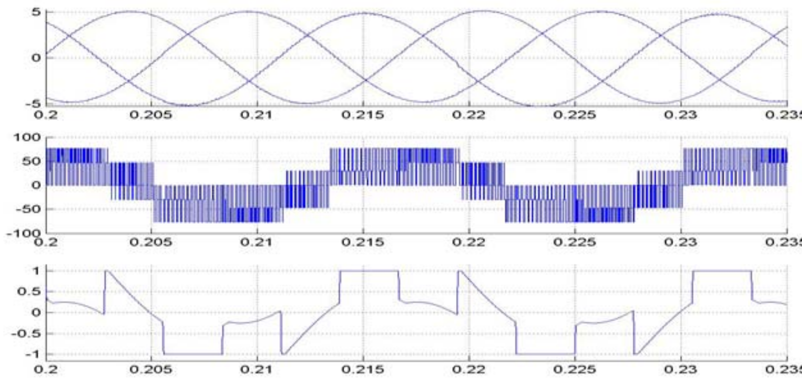


Figure 3.23 Simulation results for unbalanced three-phase voltage under GDPWM. β = 0.5[1 + SgnCos 3(ωt + δ)], δ = -30, DPWM2, (a) Balanced three-phase current (b) phase a voltage Vas (c) Modulation signal.

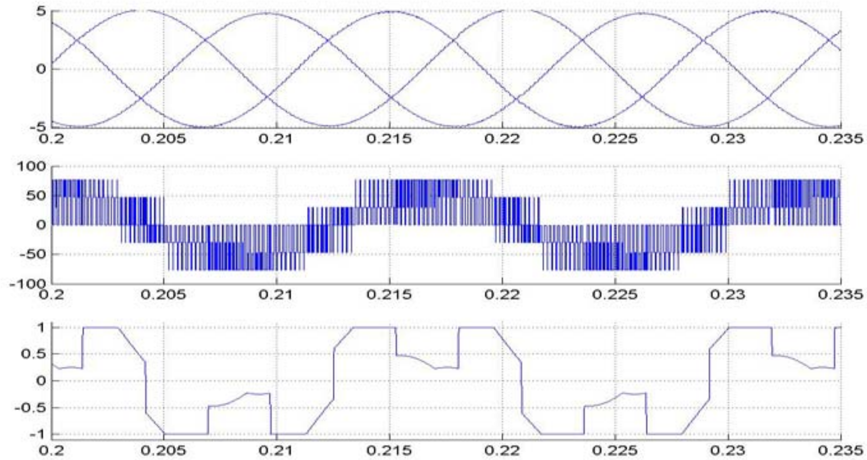


Figure 3.24 Simulation results for unbalanced three-phase voltage under GDPWM. $\beta = 0.5[1 + \text{SgnCos } 3(\omega t + \delta)]$, $\delta = -60$, DPWM3, (a) Balanced three-phase current (b) phase a voltage Vas (c) Modulation signal.