CHAPTER 6

STEADY STATE ANALYSIS AND SIMULATION OF THE IPM GENERATOR FEEDING A RECTIFIER-BOOST-RESISTIVE LOAD

6.1 Introduction

In this chapter, the steady state analysis and simulation of the IPM generator feeding a rectifier-boost- resistive load will be performed. A boost converter is normally used when the needs of the load are such that a load voltage higher than the supply voltage is required. The first part of the chapter concerns itself with developing mathematical equations which describe the switching functions of the boost converter for steady state. An equivalent resistance which is dependent upon the actual load resistance and the duty cycle of the converter will be obtained. In addition, various plots showing how an ideal voltage source system feeding a boost system behaves will be generated. The graphs are useful in conceptualizing how the boost converter works.

Next, using the mathematical model developed, the steady state experimental results will be compared with the predicted results. Finally, a comparison between the measured and simulated waveforms will be given.

6.1.1 Derivation of Boost Converter in Steady State

The derivation for the steady state equivalent circuit of a boost converter builds upon the result found for the equivalent circuit which represents the rectifier feeding a resistive load. Figure 6.1 shows the current flowing through the inductor L_p (see Figure 6.2) along with the switching functions associated with the three modes of operation for the boost converter. The first mode is when the transistor T1 is on, the second is when T1 is off and the current flowing through the inductor L_p is greater than zero, and the third mode is when the transistor T1 is off and the current flowing through the inductor L_p is zero.



Figure 6.1. Inductor current and switching functions for the boost converter



Figure 6.2. Schematic diagram of an IPM generator feeding a rectifier-boost-resistive load



Figure 6.3. Schematic diagram oa boost converter in mode 1 operation



Figure 6.4. Schematic diagram of a boost converter operating in modes 2 and 3

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Mode 1:S₁ T1 ON $0 \le t \le d_1 T$

The schematic diagram of mode 1 is shown in Figure 6.3. The systems equations can be written as

$$L_{d} \frac{dI_{1}}{dt} = V_{s} - V_{c} I$$

$$C_{1} \frac{dV_{c} I}{dt} = I_{1} - I_{p}$$

$$L_{p} \frac{dI_{l}}{dt} = V_{c} I$$

$$C_{o} \frac{dV_{co}}{dt} = -\frac{V_{co}}{R_{L}}$$

$$V_{co} = R_{L} I_{L} \cdot$$
(6.1)

$Mode \; 2 \hspace{-0.5mm}:\hspace{0.5mm} S_2 \; \; T1 \; OFF \; (\; I_p \hspace{-0.5mm}>\hspace{-0.5mm} 0) \qquad d_1T \hspace{-0.5mm}\le\hspace{-0.5mm} t \hspace{-0.5mm}\le\hspace{-0.5mm} d_2T$

For mode 2, shown in Figure 6.4, the systems equations can be written as

$$L_{d} \frac{dI_{I}}{dt} = V_{s} - V_{c} I$$

$$C_{I} \frac{dV_{c} I}{dt} = I_{I} - I_{p}$$

$$L_{p} \frac{dI_{I}}{dt} = V_{c} I - V_{co}$$

$$C_{o} \frac{dV_{co}}{dt} = I_{p} - \frac{V_{co}}{R_{L}}$$

$$V_{co} = R_{L} I_{L}.$$
(6.2)

Mode 3: S₃ T1 OFF (Ip =0) $(d_1 + d_2)$ T $\leq t \leq (d_1 + d_2 + d_3)$ T

For mode 3, shown in Figure 6.4 with the understanding that the current in the inductor $L_p = 0$, the systems equations can be written as

$$L_{d} \frac{dI_{1}}{dt} = V_{s} - V_{c} I$$

$$C_{1} \frac{dV_{c} I}{dt} = I_{1}$$

$$L_{p} \frac{dI_{1}}{dt} = 0$$

$$C_{o} \frac{dV_{co}}{dt} = -\frac{V_{co}}{R_{L}}$$

$$V_{co} = R_{L} I_{L} .$$
(6.3)

The objective is to establish global equations for the system containing modes 1 and 2 and 3. Recognizing that the sum of the switching functions is

$$S_1 + S_2 + S_3 = 1 , (6.4)$$

then S3 may be written as

$$S_2 = I - S_1 - S_3 . (6.5)$$

The global state equations for the boost converter can be written as

$$L_{d} \frac{dI_{1}}{dt} = V_{s} - V_{c} I$$

$$C_{1} \frac{dV_{c} I}{dt} = I_{1} - I_{p} + S_{3} I_{p}$$

$$L_{p} \frac{dI_{1}}{dt} = V_{c} I (I - S_{3}) - V_{co} (I - S_{1} - S_{3})$$

$$C_{o} \frac{dV_{co}}{dt} = I_{p} (I - S_{1} - S_{3}) - \frac{V_{co}}{R_{L}}.$$
(6.6)

Following the same procedure as outlined in Chapter 5, if the state equations are perturbed and separated into their dc, ac, and higher order terms, at steady state the rate of change of the states for the dc component is zero, so the left side of the equations becomes zero. Thus, the dc components of the equations can be reduced to

$$V_{s} = V_{c}I$$

$$I_{I} = I_{p} - s_{3}I_{p}$$

$$V_{c}I = \frac{(I - s_{I} - s_{3})V_{co}}{I - s_{3}}$$

$$\frac{V_{co}}{R_{L}} = (I - s_{I} - s_{3})I_{p}$$

$$V_{co} = R_{L}I_{L},$$
(6.7)

where the average values of the switching functions (i.e. s_1 , s_2 , and s_3) are their respective areas divided by the total time T. Thus,

$$s_{I} = \frac{d_{I}T}{T} = d_{I}$$

$$s_{2} = \frac{d_{2}T}{T} = d_{2}$$

$$s_{3} = \frac{d_{3}T}{T} = d_{3} .$$
(6.8)

Substituting Equation (6.8) into Equation (6.7) gives

$$V_{s} = V_{c} I$$

$$I_{1} = I_{p} - d_{3} I_{p}$$

$$V_{c} I = \frac{(I - d_{1} - d_{3})V_{co}}{I - d_{3}}$$

$$\frac{V_{co}}{R_{L}} = (I - d_{1} - d_{3}) I_{p}$$

$$V_{co} = R_{L} I_{L} .$$
(6.9)

The effective resistance for the boost can be defined as

$$R_{eff} = \frac{V_s}{I_l} . \tag{6.10}$$

Using the steady state equations, the equivalent resistance in terms of the duty cycle and the load resistance can be found as follows:

$$R_{eff} = \frac{V_s}{I_1} = V_c \frac{1}{I_p (1 - d_3)} = \frac{V_{co} (1 - d_1 - d_3)}{I_p (1 - d_3) (1 - d_3)} = \frac{R_L I_p (1 - d_1 - d_3)^2}{I_p (1 - d_3)^2}.$$
 (6.11)

Therefore, for a boost converter, the effective resistance can be written as

$$R_{effbst} = \frac{R_L (1 - d_1 - d_3)^2}{(1 - d_3)^2} .$$
(6.12)

Referring to Figure 6.1, and assuming that the converter is operating in discontinuous conduction mode (meaning that the current in the inductor becomes zero for a time) then, when the boost converter is operating in mode one, the current i_p (t) rises linearly from a zero value at the beginning of the mode to a maximum value at time t=d₁T of

$$I_{pmax} = \frac{V_c l d_1 T}{L_p} .$$
 (6.13)

The converter will then switch into mode two operation and the current will fall linearly from I_{pmax} to a zero value at a time $t = (d_1 + d_2) T$. The average value of the inductor current Ip over a complete cycle may be found by taking the area under the two triangles, and dividing by the total time T. Thus,

$$I_{p} = I_{pmax} \frac{(d_{1}T + d_{2}T)}{2T} = V_{c} I d_{1}T \frac{(d_{1} + d_{2})}{2L_{p}} .$$
(6.14)

Referring to Figures 6.4 and 6.5, the average load current may be found by first recognizing that

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$$I_{L} = \begin{bmatrix} -I_{co} & For \ 0 \ t \ d_{1}T \\ I_{1} - I_{co} & For \ (d_{1})T \ t \ (d_{1} + d_{2})T \\ -I_{co} & For \ (d_{1} + d_{2})T \ t \ T \end{bmatrix}.$$
(6.15)

Hence, the average current may be found as

$$I_{L} = \frac{1}{T} \int_{0}^{T} (-I_{co}(d_{1} + d_{2} + d_{3}) + I_{p} d_{2}) dt = \frac{1}{T} (-I_{co} + I_{p} d_{2}) .$$
 (6.16)

Recognizing that the average current in a capacitor is zero, then I_{co} is equal to zero and the equation may be reduced to

$$I_L = I_p d_2 . \tag{6.17}$$

The average load voltage may now be found as

$$V_{co} = I_L R_L = d_2 I_p R_L . (6.18)$$

Substituting the value of I_p found in Equation (6.14) into Equation (6.18) gives

$$V_{co} = R_L \frac{V_c I d_1 T (d_1 + d_2) d_2}{2 L_p} .$$
(6.19)

From Equation (6.9),

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$$V_{c} I = \frac{(I - d_{1} - d_{3})V_{co}}{I - d_{3}} = \frac{d_{2}V_{co}}{d_{1} + d_{2}} , \qquad (6.20)$$

so Equation (6.19) may be written as

$$V_{co} = \frac{d_2}{d_1 + d_2} R_L \frac{V_{co} d_1 T (d_1 + d_2) d_2}{2 L_p} .$$
(6.21)

Rearranging Equation (6.21) gives

$$I = \frac{T R_L d_2^2 d_1}{2 L_p} .$$
 (6.22)

Therefore

$$d_2 = \sqrt{\frac{2 L_p}{T R_L d_1}} .$$
 (6.23)

In terms of d_3 , the solution may be written as

$$d_{3} = 1 - \sqrt{\frac{2 L_{p}}{T R_{L} d_{1}}} - d_{1} .$$
 (6.24)

If the solution for d_3 is zero or negative, then the converter is operating in continuous conduction mode.

At the boundary condition between continuous and discontinuous conduction mode, d_3 is equal to zero. Solving Equation (6.24) in terms of the inductor L_p at the boundary condition gives

$$L_{p} = \frac{T R_{L} d_{1}}{2} (1 - d_{1})^{2} = \frac{T R_{L}}{2} (d_{1}^{3} - 2 d_{1}^{2} + d_{1}) .$$
 (6.25)

It can be seen from Equation (6.25) that when d_1 is equal to either zero or one, the minimum inductor value needed is zero. This observation is valid for the case when d_1 is zero because, for this condition, the circuit is just an LC circuit with no switching taking place. However, for the case when the duty cycle is equal to one, two things need to be pointed out. The first is that, if the duty cycle is 1, then there is an effective short circuit and extremely large currents will be flowing through the transistor. The second point, and the one more relevant to the discussion at hand, is that, for large duty cycles, the demands placed on the capacitor C_0 increase. If the assumption that the load voltage remains relatively constant is to remain valid, the size of the capacitor must necessarily increase.

Nevertheless, assuming that the capacitor is sufficiently large, the duty cycle which requires the largest inductance value L_p may be found by differentiating Equation (6.25) with respect to d_1 , setting the equation equal to zero, and solving for d_1 , i.e.

$$3 d_1^2 - 2 d_1 + l = 0 {.} {(6.26)}$$

The solutions for this quadratic equation are d_1 equals 1/3 and 1. Clearly d_1 equals 1 is one

of the solutions for a minimum L_p , and the duty cycle which is most likely to cause discontinuous condution mode is 1/3. Substituting this value back into Equation (6.25) and simplifying gives

$$L_p = \frac{T R_L}{2} (.1481) = \frac{2}{27} T R_L .$$
 (6.27)

Thus, to ensure that the boost converter is always operating in continuous conduction mode, a good rule of thumb is that

$$L_{p} - \frac{2}{27} T R_{L} . (6.28)$$

In the steady state analysis of the boost converter presented in this thesis, only the continuous conduction mode was considered. Thus, d_3 is equal to zero and Equation (6.12) may be written as

$$R_{effbst} = R_L (1 - d_1)^2 . (6.29)$$

Equation (6.29) represents the equivalent resistance at the output of the rectifier and is equivalently the same as the value R_0 given in Equation (4.31). Substituting Equation (6.29) into Equation (4.31) gives

$$R_{effbstrec} = \frac{\pi^2 R_L (1 - d_1)^2}{12} .$$
 (6.30)

The equation given in (6.30) will be used to predict the performance of the IPM feeding the rectifier-boost topology.

From the result given in Equation (6.29) it can be seen that, as the duty cycle is changed, the Thevinin equivalent is not constant; therefore, one cannot assume a constant resistive load model. For an **ideal** voltage source supplying the boost converter, the implications of this may not be that substantial; however, for a "weak" autonomous power supply, such as the permanent magnet machine used in the experiments described in this thesis, the effect is that the generator will "see" a nonlinear resistor which changes as the duty cycle of the converter is changed. Since the value of the impedance presented to the IPM machine will change the voltage output of the generator, the boost converter may not necessarily raise the load voltage.

6.1.2 Examination of Ideal Boost Converter

In order to gain an appreciation of the significance of Equation (6.29), various graphs are generated with the assumption that the dc voltage into a boost convertor is a constant 10 volts dc, and the load resistance is a constant 10 ohms.



Figure 6.5. Effective resistance vs duty cycle for a boost converter

Figure 6.5 displays how the effective resistance presented to a source decreases as the duty cycle increases. The highest impedance possible is when the boost is not switching at all and is always off. In other words, the circuit is operating in mode 2 as seen in Figure 6.4. If the duty cycle is increased to 1, then the load resistance would be zero and an effective short circuit would be created.



Figure 6.6. Source (input) current vs duty cycle for a boost converter

Figure 6.6 shows how the input current increases as the duty cycle increases. This is perfectly understandable since the load demand increases as the effective resistance decreases. It can be seen that there is an exponential rise in the load current which would tend towards infinity as the duty cycle approaches 1.

Figure 6.7 shows the exponential rise in input power into the system as the duty cycle increases. This rise, is of course, caused by the exponential rise of the input current.



Figure 6.7. Input power vs duty cycle for a boost converter



Figure 6.8. Load voltage vs duty cycle for a boost converter

Figure 6.8 displays the intended effect of the boost converter topology, that being to increase the voltage above the supply voltage. It can be seen from Figure 6.8 that the load voltage at a duty cycle of 0.8 is approximately 5 times the 10 volt constant dc source voltage.

6.2 Steady State Performance of an IPM Generator Feeding a Rectifier-Boost-Resistive Load

6.2.1 Introduction

In this section, the measured steady state performance of the IPM generator feeding a rectifier-boost-resistive load will be compared with the predicted performance of the system. In order to obtain a full performance curve (meaning that the performance of the IPM generator is tested for loads ranging from a light load to a large load) for the boost converter, it is not the load resistance R_L which is varied from a small to a large value. Rather, it is the duty cycle D which is varied from 0 to almost 1.

Referring to Figure 6.5, and remembering that the actual load resistance for this graph was 10 ohms, it can be seen that the largest resistance which the generator will see is the actual load resistance R_{L} . Thus, if one wants to be able to study the boost system operating under a light load, a large load resistance must be chosen. As the duty cycle is increased from zero, the effective resistance decreases and, therefore, it is possible to test the topology under the condition when the IPM generator is feeding a large load.

The load resistance chosen to test the system was 88 Ω and the system was tested for the generating frequencies of 30 and 45 Hz. The values of the rectifier filter components were L_d = 9.3 mH and C₁ = 10mF. The values of the boost filter were Lp = 10 mH and C_o = 10mF.

6.2.2 Experimental and Predicted Performance Results

Figure 6.9 shows how measured and calculated line to neutral voltage of the generator varies as a function of the power out of the generator. As was pointed out in Chapter 4 concerning the IPM-rectifier system, if the rectifier-boost system truly appeared as a purely resistive load to the IPM, then the measured results would fall almost exactly on the calculated results line as they did in Figure 3.5.



Figure 6.9. Measured and calculated generator line to neutral voltage vs generator output power for the IPM feeding a rectifier-boost-resistive load

Figures 6.10 and 6.11 show how the generator voltage and rectifier voltage (which is a function of the generator voltage) behave as the duty ratio increases. The rectifier voltage is the dc source to the boost converter and it is clear to see that, as the duty ratio is changed, this dc supply voltage is not constant.



Figure 6.10. Measured and calculated generator line to neutral voltage vs duty ratio for IPM generator feeding rectifier-boost-resistive load



Figure 6.11. Measured and calculated rectifier voltage vs duty ratio for IPM generator feeding rectifier-boost-resistive load

Figure 6.12 plots the generator output power vs the duty ratio. It can be seen that the maximum power produced at 45 Hz occurs when the duty ratio is approximately 0 .6, and, for 30 Hz operation, the maximum power occurs at a duty cycle of approximately 0.65. Referring back to Figure 6.7, it can be seen that for an ideal boost converter the power will continue to increase as the duty cycle is raised. However, this result is based on the

assumption that the input voltage into the boost is constant. As Figures 6.9 and 6.10 demonstrate, this assumption is not valid for the IPM generator feeding the system.



Figure 6.12. Measured and calculated generator output power vs duty ratio for IPM generator feeding rectifier-boost-resistive load



Figure 6.13. Measured and calculated generator current vs ratio for IPM generator feeding rectifier-boost-resistive load

The graph shown in Figure 6.13 plots how the generator current increases as the duty cycle is increased. There is an exponential rise in the current up to the points where the maximum power occurs (duty cycles of 0.6 and 0.65 for frequencies of 45 and 30 Hz respectively), however, after the maximum power point is reached, the currents, although they are still increasing, begin to level off.



Figure 6.14. Measured and calculated load voltage vs duty ratio for IPM generator feeding a rectifier-boost-resistive load

The measured and calculated load voltage vs duty ratio is shown in Figure 6.14. Remembering that for an ideal boost converter (see Figure 6.8) the load voltage will increase exponentially as the duty cycle is increased, it can be seen in Figure 6.14 that the IPM-rectifier-boost-resistive load topology behaves like an ideal boost converter up to the maximum power point, but after the maximum power point the load voltage actually decreases. A decrease in load voltage as the duty cycle is raised is probably not the outcome intended when a boost converter is introduced into the system. Therefore, for a practical operating scheme, the duty cycle may be restrained from going above the maximum power point.



Figure 6.15. Measured power loss of rectifier-boost system vs the duty ratio



Figure 6.16(a). Measure converter efficiency vs duty ratio for IPM generator feeding a rectifier-boost-resistive load

Up to the duty cycle corresponding to the maximum power point, the topology is demonstrating a very good ability to control the load output voltage even though the generator voltage is changing. For example, at 45 Hz operation, even though the generator line to neutral terminal voltage drops from approximately 55 Volts at zero duty cycle to approximately 36 volts (see Figure 6.9) at a duty cycle of 0 .6, the load voltage continues to increase.

Also, assuming that the duty cycle is not constrained at some maximum value, then, in order to achieve a desired output load voltage, there are often two duty cycles possible for each operating frequency. For example, one could achieve an output load voltage of 140 volts at a duty cycle of either 0.2 or 0.85 for 45 Hz operation, and at a duty cycle of either 0.46 or 0.78 for 30 Hz operation.

Figure 6.15 shows the power loss measured in the rectifier-boost topology as a function of the duty ratio. The power loss of course being the power measured going into the resistive load subtracted from the power measured going into the rectifier. It can be seen that the power loss increases as a function of the duty ratio. Up to the duty cycle where maximum power is flowing, this is to be expected since more power is flowing through the device; however, it is important to note that the power loss continues to increase even after the power being generated begins to decrease. The efficiency of the system (P_{out}/P_{in}), seen in Figure 6.16(a), therefore will begin to decrease markedly.

Finally, Figure 6.16(b) plots the measured and the calculated effective resistance vs the duty cycle for both operating frequency. It is a bit of a misnomer to refer to the "measured" effective resistance since it is not the effective resistance which is measured; rather it is the actual load resistance which is measured and the corresponding effective resistance is calculated from this value.



Figure 6.16(b). Measured and calculated rectifier-boost effective resistance vs duty ratio

6.3 Comparison of Measured and Simulated Waveforms of IPM Machine Feeding a Rectifier- Boost-Resistive Load

This section includes the comparison between simulation and measured waveforms for the IPM generator feeding a rectifier-boost-resistive load. The two cases which will be looked at are when the buck converter is operating in continuous conduction mode, and when the converter is operating in discontinuous conduction mode.

6.3.1 Boost Converter in Continuous Conduction Mode

This section looks at measured and simulated waveforms when the IPM is feeding a rectifier-boost-resistive load topology for the case when the boost is operating in continuous conduction mode. The frequency of operation of the IPM machine is 30 Hz. The rectifier filter inductor and capacitor values are $L_d=10$ mH and $C_1 = 5.6\mu$ F, the boost filter inductor and capacitor values are $L_p=10$ mH and $C_0=5.8\mu$ F, and the load resistance value is $R_L=77\Omega$. The duty cycle was set at 0.8.

Figures 6.17 and 6.18 show the simulated and measured line to line voltage and line current waveforms of the IPM generator. The comparison between the simulated and measured waveforms is favorable.

Figure 6.19 shows the measured and simulated current in the inductor L_p . It can be seen from the figure that, similar to the current I_p depicted in Figure 6.1, the current rises almost linearly when the transistor is turned on and falls linearly when the transistor is turned off. Since the current I_p always has a positive value, the converter is operating in



Measurement



Figure 6.17. Measured and simulated line to line voltage waveforms for the generator feeding a rectifier-boost-resistive 77Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: voltage: 20v/div, time 10ms/div



Figure 6.18. Measured and simulated generator current waveforms for the generator feeding a rectifier-boost-resistive 77 Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 2A/div, time 10ms/div



Figure 6.19. Measured and simulated boost inductor current waveforms for the generator feeding a rectifier-boost-resistive 77 Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: .2A/div, time .2ms/div



Figure 6.20. Measured and simulated rectifier output current waveforms for the generator feeding a rectifier-boost-resistive 77 Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: .2A/div, time 2ms/div

continuous conduction mode. Figure 6.20 shows the measured and simulated current through the rectifier filter inductor L_d .

6.3.2 Boost Converter in Discontinuous Conduction Mode

In this section, the inductor L_p is changed from the value of 10mH used in section 6.3.1 to the value of 0.25mH. In addition, the duty cycle is changed from 0.80 to a value of 0.5. With these changes, the boost converter no longer operates in continuous conduction mode; rather, it operates in discontinuous conduction mode.



Measurement

Simulation

Figure 6.21. Measured and simulated generator line to line voltage waveforms for the generator feeding a rectifier-boost-resistive 77Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: voltage: 20v/div, time 10ms/div



Measurement

Simulation

Figure 6.22. Measured and simulated generator line current waveforms for the generator feeding a rectifier-boost-resistive 77Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 2A/div, time 10 ms/div

Figures 6.21 and 6.22 show the measured and simulated line to line voltage and line Current of the generator. The simulated and measured waveforms of the voltages and currents compare favorably with each other in both magnitude and form.

It can be seen in Figure 6.23 that the converter is clearly operating in discontinuous conduction mode. As was done concerning the buck converter, it is also worthwhile to note the difference in the current in the measured waveform of Figure 6.23 (where the converter is in discontinuous mode) to the measured current of Figures 6.19. When the converter is in discontinuous mode, it can be seen that both the peak current and the change in current from its minimum to maximum value is much larger than when the converter is operating in continuous conduction mode. The high stresses placed on the transistors due to the large



Measurement

Simulation

Figure 6.23. Measured and simulated inductor current waveforms for the generator feeding a rectifier-boost-resistive 77Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: 5A/div, time .2ms/div

currents and large rate of change in currents (di/dt) is one of the main reasons why, for

practical designs, the discontinuous mode of operation is normally avoided.

Finally, Figure 6.24 shows the measured and simulated current waveforms through

the rectifier filter inductor L_d . The simulation is fairly close to the measured value.



Figure 6.24. Measured and simulated rectifier output current waveforms for the generator feeding a rectifier-boost-resistive 77Ω resistive load. Rotor speed=900 rpm. Measured waveform scale: current: .2A/div, time 2ms/div