

Tennessee Technological University
Mathematics Department

MATH 6010-6020: Functional Analysis I-II

I. COURSE DESCRIPTION FROM CATALOG:

Metric spaces, normed and Banach spaces inner product and Hilbert spaces. Fundamental theorems for normed and Banach spaces and their applications. Linear operators on normed and Hilbert spaces. Lec. 3. Cr. 3.

II. PREREQUISITE(S):

MATH 6010: C or better in MATH 4120 or MATH 5120.

MATH 6020: C or better in MATH 6010.

III. COURSE OBJECTIVE(S) MATH 6010/6020:

One of the main objectives of the course is to familiarize the students with the basic concepts, principles and methods of functional analysis and its applications. Main topics include Normed spaces, Banach spaces, Dual spaces, Hilbert spaces, Hahn- Banach Theorem, Operator theory on Hilbert spaces, and Spectral theory of linear operators.

IV. STUDENT LEARNING OUTCOMES:

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Students will:

know, understand and be able to apply fundamental theorems about normed spaces.

know, understand and be able to apply theorems about bounded operators on Hilbert spaces.

know, understand and be able to apply Banach Fixed Point Theorem.

MATH 6020

Students will:

know, understand and be able to apply approximation theory in normed spaces.

know, understand and be able to apply spectral theory of compact linear operators.

know, understand and be able to apply spectral theory of bounded self-adjoint linear operators on Hilbert spaces.

V. TOPICS TO BE COVERED:

MATH 6010

Chapter 1: Metric Spaces

1.1 Metric Space

1.2 Further Examples of Metric Spaces

1.3 Open Set, Closed Set, Neighborhood

1.4 Convergence, Cauchy Sequence, Completeness

Chapter 2: Normed Spaces. Banach Spaces.

2.1 Vector Space

2.2 Normed Space, Banach Space

2.3 Further Properties of Normed Space

2.4 Finite Dimensional Normed Spaces and Subspaces

2.5 Compactness and Finite Dimension

- 2.6 Linear Operation
- 2.7 Bounded and Continuous Linear Operators
- 2.8 Linear Functionals
- 2.9 Linear Operators and Functionals on Finite Dimensional Spaces
- 2.10 Normed Spaces of Operators, Dual Space

Chapter 3: Inner Product Spaces. Hilbert Spaces.

- 3.1 Inner Product Space. Hilbert Space
- 3.2 Further Properties of Inner Product Spaces
- 3.3 Orthogonal Complements and Direct Sums
- 3.4 Orthonormal Sets and Sequences
- 3.5 Series Related to Orthonormal Sequences and Sets
- 3.6 Total Orthonormal Sets and Sequences
- 3.7 Legendre, Hermite and Laguerre Polynomials
- 3.8 Representation of Functionals on Hilbert Spaces
- 3.9 Hilbert-Adjoint Operator
- 3.10 Self-Adjoint, Unitary and Normal Operators

Chapter 4: Fundamental Theorems for Normed and Banach Spaces

- 4.1 Zorn's Lemma
- 4.2 Hahn-Banach Theorem
- 4.3 Hahn-Banach Theorem for Complex Vector Spaces and Normed Spaces
- 4.4 Application to Bounded Linear Functionals on $C[a,b]$
- 4.5 Adjoint Operator
- 4.6 Reflexive Spaces
- 4.7 Category Theorem. Uniform Boundedness Theorem
- 4.8 Strong and Weak Convergence
- 4.9 Convergence of Sequences of Operators and Functionals
- 4.10 Application to Summability of Sequences
- 4.11 Numerical Integration and Weak* Convergence
- 4.12 Open Mapping Theorem
- 4.13 Closed Linear Operators. Closed Graph Theorem

Chapter 5: Further Applications: Banach Fixed Point Theorem

- 5.1 Banach Fixed Point Theorem
- 5.2 Application of Banach's Theorem to Linear Equations
- 5.3 Applications of Banach's Theorem to Differential Equations
- 5.4 Application of Banach's Theorem to Integral Equations

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Chapter 6: Further Applications: Approximation Theory

- 6.1 Approximation in Normed Spaces
- 6.2 Uniqueness, Strict Convexity
- 6.3 Uniform Approximation
- 6.4 Chebyshev Polynomials
- 6.5 Approximation in Hilbert Space
- 6.6 Splines

Chapter 7: Spectral Theory of Linear Operators in Normed Spaces

- 7.1 Spectral Theory in Finite Dimensional Normed Spaces
- 7.2 Basic Concepts
- 7.3 Spectral Properties of Bounded Linear Operators
- 7.4 Further Properties of Resolvent and Spectrum
- 7.5 Use of Complex Analysis in Spectral Theory
- 7.6 Banach Algebras
- 7.7 Further Properties of Banach Algebras

- Chapter 8: Compact Linear Operators on Normed Spaces and Their Spectrum
 - 8.1 Compact Linear Operators on Normed Spaces
 - 8.2 Further Properties of Compact Linear Operators
 - 8.3 Spectral Properties of Compact Linear Operators on Normed Spaces
 - 8.4 Further Spectral Properties of Compact Linear Operators
 - 8.5 Operator Equations Involving Compact Linear Operators
 - 8.6 Further Theorems of Fredholm Type
 - 8.7 Fredholm Alternative

- Chapter 9: Spectral Theory of Bounded Self-Adjoint Linear Operators
 - 9.1 Spectral Properties of Bounded Self-Adjoint Linear Operators
 - 9.2 Further Spectral Properties of Bounded Self-Adjoint Linear Operators
 - 9.3 Positive Operators
 - 9.4 Square Roots of a Positive Operator
 - 9.5 Projection Operators
 - 9.6 Further Properties of Projections
 - 9.7 Spectral Family
 - 9.8 Spectral Family of a Bounded Self-Adjoint Linear Operator
 - 9.9 Spectral Representation of Bounded Self-Adjoint Linear Operators
 - 9.10 Extension of the Spectral Theorem to Continuous Functions
 - 9.11 Properties of the Spectral Family of a Bounded Self-Adjoint Linear Operator

VI. ADDITIONAL INFORMATION:

VII. POSSIBLE TEXTS AND REFERENCES:

Introductory to Functional Analysis with Applications, by Kreyszig

VIII. ANY TECHNOLOGY THAT MAY BE USED:

IX. STUDENT ACADEMIC MISCONDUCT POLICY:

Maintaining high standards of academic integrity in every class at Tennessee Tech is critical to the reputation of Tennessee Tech, its students, alumni, and the employers of Tennessee Tech graduates. The Student Academic Misconduct Policy describes the definitions of academic misconduct and policies and procedures for addressing Academic Misconduct at Tennessee Tech. For details, view the Tennessee Tech's Policy 217 – Student Academic Misconduct at [Policy Central](#).

X. DISABILITY ACCOMMODATION:

Students with a disability requiring accommodations should contact the Office of Disability Services (ODS). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The ODS is located in the Roaden University Center, Room 112; phone 372-6119. For details, view the Tennessee Tech's Policy 340 – Services for Students with Disabilities at [Policy Central](#).